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**HIGHER SCHOOL IN APPLIED SCIENCES –TLEM CEN**

## **Course Handout**

### **Power Electronics**

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**This course is intended for 4th year students in Electrical Engineering, Automation and  
Electronics**

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### **Abstract:**

Power electronics is a fundamental discipline of electrical engineering concerned with the static conversion of electrical energy using semiconductor components operating in switching mode. It enables the efficient adaptation of the voltage, current, frequency, or waveform of a power source to the requirements of a load or system. The main conversion structures include rectifiers (AC-DC), inverters (DC-AC), choppers (DC-DC), and matrix converters (AC-AC). Power electronics relies on a close interaction between three fields: electrical engineering, electronics, and automation. Thanks to its performance in terms of energy efficiency, compactness, and flexibility, it is now ubiquitous in industrial systems, smart grids, electric transportation, renewable energy installations, and household appliances. Its ongoing development is also driven by advances in semiconductor materials and advanced control techniques, thus actively contributing to the challenges of energy transition and energy efficiency.

This handout is intended for fourth-year engineering students in electrical engineering, automation, and electronics, and follows the official curriculum outlined by the Ministry of Higher Education and Scientific Research.

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## **Introduction**

Electrical energy is, in most cases, produced and distributed in AC form, generally at a standard frequency of 50 Hz (in Europe) or 60 Hz (in North America). However, at the load level, many applications—particularly variable speed drive systems—require specific forms of electrical energy, different from those supplied by the grid.

To meet these needs, it is necessary to integrate energy conversion devices capable of adapting the characteristics of the source to those required by the load. Historically, this conversion was handled by electromechanical converters (based on rotating machines), which were relatively bulky and inefficient. The advent of power semiconductors in the 1960s paved the way for the widespread use of static converters, allowing for more compact, reliable, and efficient conversion.

Static converters are now the standard solution for shaping electrical energy. They adapt different forms of energy (DC or AC sources) to the load requirements, ensuring efficient energy transfer through semiconductor electronic switches (IGBTs, MOSFETs, thyristors, etc.) combined with passive components (inductors, capacitors, transformers).

These converters allow, in particular, the voltage and/or frequency of the electrical wave to be modified while maintaining good energy efficiency.

We generally distinguish two types of electrical sources based on their nature:

- Direct voltage sources: characterized by a constant voltage value  $V$ .
- Alternating voltage sources: defined by an effective voltage  $V$  and a frequency  $f$ .

Depending on the input and output types, there are four main converter families:

1. AC/DC converter:

→ Rectifier, used to convert an AC voltage into a DC voltage.

2. DC/DC converter:

→ Chopper, used to lower or raise a DC voltage.

3. DC/AC converter:

→ Inverter, used to generate an AC voltage from a DC source.

4. AC/AC converter:

→ Dimmer: modifies only the effective value of the AC voltage, without changing its frequency.

→ Cycloconverter: modifies both the effective value and the frequency of the output voltage.

This course is divided into seven chapters. Chapter 1 begins with a comprehensive overview of the basic concepts related to converter electrical circuits, containing the theoretical foundations

and important mathematical backgrounds. These constitute the "vital minimum" that the reader will need to have in mind to confidently approach the subsequent chapters.

The components associated with power electronic circuits are also presented in Chapter 2. For example, "diodes, thyristor" are presented in the chapter on rectification, and "power transistors" in the chapter on choppers, inverters, etc.

The "central" part of the handout, consisting of Chapters 3 to 6, is devoted to an organized and relatively traditional presentation of the major families of "static converters." It is in this section that the reader will find all the circuit developments and calculations used to study or design classic power electronic systems.

Chapter 7 presents conversion structures not covered in Chapters 3 to 6, but whose examination is necessary for an overview of power electronics converters. This chapter is devoted to another application of choppers: switching power supplies. Chapter 7 also covers converters whose development is relatively recent: matrix converters and modular multilevel converters.

all simulations are performed by `simpowersystem` from `simulink/Matlab`.

Reading this course does not require any prior knowledge other than that possessed by any student in scientific or technical higher education in mathematics and general physics.

This course is designed to be accessible to students in scientific or technical higher education, requiring only basic knowledge of mathematics and general physics acquired during the first years of study.

Consistent with the official curriculum of the Ministry of Higher Education and Scientific Research, this teaching material was developed to meet the needs of learners, teachers, and practitioners involved in using or integrating power electronics solutions.

We hope that this work will effectively contribute to the dissemination and mastery of this essential discipline in the contemporary energy and technology landscape.

# **Chapter 1: Mathematical Review of Power Electronics Signals**

### 1.1. Introduction

In power electronics, steady-state operation can be considered as a succession of transient states. To correctly model the behavior of converters, it is essential to apply solution techniques adapted to the different types of states:

- Transient states
- Periodic states

Why is it said that steady-state operation in power electronics is a succession of transient states?

When a partial circuit diagram is established, a discontinuous operating diagram is obtained. This is due to the non-continuous application of the current or voltage source, since the switch (whether a transistor, diode, thyristor, etc.) alternates between ON and OFF states. Since the circuit includes capacitive and inductive elements, transient states naturally appear during switch switching. This behavior is modeled using differential equations.

### 1.2. Reminders on Transient Conditions [1,2]

#### 1.2.1. First-Order Differential Equation:

A first-order differential equation is an equation that describes the behavior of an electrical circuit with a single reactive element (either a capacitor or an inductor). Depending on the type of reactive element, the differential equation appears differently:

- If the reactive element is a capacitor, the differential equation is formulated as a function of voltage.
- If the reactive element is an inductor, the differential equation is formulated as a function of current.

In all cases, the differential equation takes the following form:

$$a \frac{df(t)}{dt} + b \cdot f(t) = g(t)$$

present in the circuit.  $g(t)$  can be either:

- a sinusoidal function (e.g., for rectifiers, whether controlled or not),
- a step function (e.g., choppers).

#### 1- Solution:

The solution is performed in 3 steps:

- determination of a general solution (equation without the right-hand side),
- determination of a specific solution,
- determination of the integration constants.

### a- General Solution:

We attempt to solve the previous equation without the right-hand side.

$$a \frac{df(t)}{dt} + b.f(t) = 0$$

The solution to this equation is

$$f_h(t) = A.e^{\frac{-b.t}{a}}$$

$A$  is a constant of integration.

### b- Specific solution:

We will only consider two cases here:

1– the function can be written in the form  $g(t) = k$ , with  $k = \text{constant}$ . We are looking for a specific solution in the form  $f_p(t) = g(t) = k$ . We replace in the differential equation

$$a \frac{df_p}{dt}(t) + b.f_p(t) = k \Rightarrow b.f_p(t) = k \Rightarrow f_p(t) = \frac{k}{b}, \text{ so the specific solution is:}$$

$$f_p(t) = \frac{k}{b}$$

2– the function  $g(t)$  can be written in the form or a sum of sinusoidal functions  $g(t) = k.\sin(\omega.t)$ , with  $k = \text{constant}$ . We are looking for a particular solution of the form  $f_p(t) = A_1.\sin(\omega.t) + A_2.\cos(\omega.t)$ .

By replacing in the differential equation.

$$a \frac{d}{dt}(A_1.\sin(\omega.t) + A_2.\cos(\omega.t)) + b.(A_1.\sin(\omega.t) + A_2.\cos(\omega.t)) = k.\sin(\omega.t)$$

$$(a.A_1.\omega + b.A_2).\cos(\omega.t) + (-A_2.\omega + b.A_1).\sin(\omega.t) = k.\sin(\omega.t)$$

We can determine the constants  $A_1$  and  $A_2$  by identification. In the case proposed here:

$$a.A_1.\omega + b.A_2 = 0$$

$$-A_2.\omega + b.A_1 = k$$

which allows the two constants to be determined without any problem.

$$A_1 = \frac{b.k}{a^2.\omega^2 + b^2} \quad \text{and} \quad A_2 = \frac{-a.\omega.k}{a^2.\omega^2 + b^2}$$

### c- Solution to the differential equation:

The solution to the problem is the sum of the general solution  $f_h(t)$  and the particular solution  $f_p(t)$ .

$$f_{finale}(t) = f_h(t) + f_p(t)$$

### d- Integration constant:

We now need to determine the constant  $A$  of the general solution. To do this, it is essential to know the value of the current in the coil or the voltage across the capacitor. This value is determined by the principle that the current or voltage must not exhibit any discontinuities during the different operating phases.

### 1.2.2. Second-order differential equation: [1, 2]

A second-order differential equation models the operation of an electrical circuit comprising two reactive elements (capacitor and inductor, two inductors, or two capacitors). Depending on the case, the differential equation takes the following form:

- For two capacitors: the differential equation is expressed as a function of voltage,
- For two inductors: the differential equation is expressed as a function of current.

In all cases, the differential equation is an equation of the form:

$$a \frac{d^2 f(t)}{dt^2} + b \frac{df(t)}{dt} + c \cdot f(t) = g(t)$$

$a$ ,  $b$ , and  $c$  are constants, and  $g(t)$  is a function expressed as a function of the various sources present in the circuit.  $g(t)$  can be either:

- a sinusoidal function (e.g., controlled or uncontrolled rectifiers),
- a step function (e.g., choppers).

### 1- Solution:

The solution is performed in three steps:

- determination of a general solution (equation without the right-hand side),
- determination of a specific solution,
- determination of the integration constants.

### a- General Solution:

We attempt to solve the previous equation without the right-hand side.

$$a \frac{d^2 f(t)}{dt^2} + b \frac{df(t)}{dt} + c \cdot f(t) = 0$$

To do this, we write the characteristic equation of the system  $a \cdot x^2 + b \cdot x + c = 0$ . This is a classical quadratic equation where  $\Delta = b^2 - 4 \cdot a \cdot c$ . The solutions are:

- if  $\Delta > 0$ , there are 2 real roots  $r_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2 \cdot a}$ , the solution is written in the form

## Chapter 1: Mathematical Review of Power Electronics Signals

$$f_h(t) = k_1 \cdot e^{-r_1 \cdot t} + k_2 \cdot e^{-r_2 \cdot t}$$

– if  $\Delta = 0$  there is 1 double root  $r_0 = \frac{-b}{2 \cdot a}$ , the solution is written in the form,

$$f_h(t) = (k_1 + k_2 \cdot t) e^{-r_0 \cdot t}$$

– if  $\Delta < 0$  there are 2 complex conjugate roots  $r_{1,2} = \frac{-b \pm i \cdot \sqrt{\Delta}}{2 \cdot a} = \alpha + \beta \cdot i$ , the solution is written in the form.

$$f_h(t) = (k_1 \cdot \cos(\beta \cdot t) + k_2 \cdot \sin(\beta \cdot t)) \cdot e^{-\alpha \cdot t}$$

In all cases,  $k_1$  and  $k_2$  are constants of integration.

### b- Specific solution:

We will only consider the two cases commonly used in power electronics here:

1– the function  $g(t)$  can be written in the form  $g(t) = k$ , with  $k = \text{constant}$ . We are looking for a specific solution of the form  $f_p(t) = g(t) = k$ . We replace in the differential equation

$a \frac{d^2 f_p}{dt^2}(t) + b \frac{df_p}{dt}(t) + c \cdot f_p(t) = k \Rightarrow f_p(t) = \frac{k}{c}$ , so the specific solution is equal to

$$f_p(t) = \frac{k}{c}.$$

2– the function  $g(t)$  can be written in the form  $g(t) = k \cdot \sin(\omega t)$  or a sum of sinusoidal functions, with  $k = \text{constant}$ . We are looking for a particular solution of the form:

$$f_p(t) = A_1 \cdot \sin(\omega t) + A_2 \cdot \cos(\omega t).$$

Substituting into the differential equation:

$$a \frac{d^2}{dt^2} (A_1 \cdot \sin(\omega t) + A_2 \cdot \cos(\omega t)) + b \frac{d}{dt} (A_1 \cdot \sin(\omega t) + A_2 \cdot \cos(\omega t)) + c \cdot (A_1 \cdot \sin(\omega t) + A_2 \cdot \cos(\omega t)) = k \cdot \sin(\omega t)$$

$$(-a \cdot A_1 \cdot \omega^2 - b \cdot A_2 \cdot \omega + c \cdot A_1) \cdot \cos(\omega t) + (-A_2 \cdot \omega^2 - b \cdot A_1 \cdot \omega + c \cdot A_1) \cdot \sin(\omega t) = k \cdot \sin(\omega t)$$

We can determine the constants  $A_1$  and  $A_2$  by identification. In the case proposed here:

$$(c - a \cdot \omega^2) A_1 - b \cdot \omega A_2 = k$$

$$b \cdot \omega A_1 + (c - a \cdot \omega^2) A_2 = 0$$

This allows the two constants to be determined without any problem.

$$A_1 = \frac{(c - a \cdot \omega^2) \cdot k}{(c - a \cdot \omega^2)^2 + (b\omega)^2}$$
$$A_2 = \frac{-b \cdot \omega \cdot k}{(c - a \cdot \omega^2)^2 + (b\omega)^2}$$

### c- Solution to the differential equation:

The solution to the problem is the sum of the general solution  $f_h(t)$  and the particular solution.  $f_p(t)$ .

$$f_{finale}(t) = f_h(t) + f_p(t)$$

### d- Integration constant:

The remaining task is to determine the constant  $k$  of the general solution. To do this, it is essential to know the value of the current in the coil or the voltage across the capacitor. This value is calculated based on the fact that the current or voltage must not undergo discontinuities during the various operating phases.

## 1.3. A reminder about non-sinusoidal periodic quantities. [1, 2]

### 1.3.1. Mean values:

The mean value of a periodic signal is the average of the instantaneous values measured over a complete period. It is denoted by  $\bar{v}$ .

If  $T$  denotes the signal period  $v(t)$  and  $i(t)$ , then the mean value is given by:

$$V_{avg} = \frac{1}{T} \int_0^T v(t) \cdot dt$$

### 1.3.2. Effective Values:

The effective value (also called RMS or Root Mean Square) of a current or voltage, which varies over time, corresponds to the value of the direct current or direct voltage that produces identical heating in a resistor. This effective value can only be calculated if the current or voltage are periodic quantities.

It is read on a voltmeter with the switch in the AC position or on an alternating voltage gauge.

It is denoted  $V_{eff}$ .

$$V_{eff} = \sqrt{\frac{1}{T} \int_0^T v(t)^2 \cdot dt}$$

## Chapter 1: Mathematical Review of Power Electronics Signals

In single-phase in sinusoidal signal and using the previous relationship, the effective voltage and maximum voltage are linked by the relationship:

$$V_{eff} = \frac{V_{max}}{\sqrt{2}} \dots\dots\dots$$

### 1.3.3. Inductors and capacitors

Inductors and capacitors have some particular characteristics that are important in power electronics applications. For periodic currents and voltages, average values are given by:

$$I_{Cavg} = \frac{1}{T} \int_0^T i_C(t).dt = 0$$

$$V_{Lavg} = \frac{1}{T} \int_0^T v_L(t).dt = 0$$

### 1.3.4. Power:

Instantaneous electrical power, often denoted  $P(t)$  and united by the watt (symbol W), is the product of the instantaneous voltage (in volts) and the instantaneous current (in amperes).

Power varies over time. The previous formula for average voltage and current also applies to power, replacing  $v(t)$  with  $p(t)$ . If we replace  $p(t)$  with its own value in this formula, we have:

$$P = \frac{1}{T} \int_0^T p(t).dt = \frac{1}{T} \int_0^T v(t).i(t).dt$$

Single-phase apparent power is the product of the effective voltage and the effective current.

$$S = V_{eff} \cdot I_{eff}$$

### 1.3.5. Power factor and distorting power:

Power factor  $\cos(\varphi)$  is the ratio of active (average) power to apparent power in sinusoidal signals.

$$\cos(\varphi) = \frac{P}{S}$$

In the case where the current and/or voltage are sinusoidal:

$$S = \sqrt{P^2 + Q^2}$$

And in the case where the current and/or voltage are not sinusoidal:

$$S = \sqrt{P^2 + Q^2 + D^2}$$

With

$P$  Active power

$$P = V_{eff} \cdot I_{eff} \cdot \cos(\varphi)$$

$Q$  Reactive power

$$Q = V_{eff} \cdot I_{eff} \cdot \sin(\varphi)$$

$D$  Distorting power

$$D = \sqrt{\sum_{h=2}^{\infty} V_{effh}^2 \cdot I_{effh}^2} \quad \text{where } h \text{ is the harmonic order}$$

### 1.3.6. Form Factor

The form factor value characterizes the rectified voltage. The closer this value is to unity, the closer the resulting voltage is to a continuous quantity. This coefficient is used to compare different rectifier assemblies. By definition, the form factor is the ratio:

$$F = \frac{V_{eff}}{V_{avg}}$$

### 1.3.7. Ripple Rate factor

The peak ripple rate is equal to the ratio of the effective value of the alternating component of a rippled quantity to the effective value of the quantity itself and is calculated using the following relationship:

$$\tau = \sqrt{F^2 - 1}$$

### 1.3.8. Fourier Series

A periodic signal of frequency  $f$  (period  $T$ ) can be decomposed into a Fourier series:

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$$

with

$$a_0 = \frac{1}{T} \int_0^T f(x) dx$$

$$a_n = \frac{2}{T} \int_0^T f(x) \cos(nx) dx$$

$$b_n = \frac{2}{T} \int_0^T f(x) \sin(nx) dx$$

Some properties of the Fourier series:

- Symmetry of the input signal with respect to the origin (Odd function)

$$f(x) = -f(-x)$$

## Chapter 1: Mathematical Review of Power Electronics Signals

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$$a_n = 0 \quad \text{with } n=0,1,\dots$$

– Symmetry with respect to the middle of the signal period (Even function)

$$f(x) = f(-x)$$

$$b_n = 0 \quad \text{with } n=0,1,\dots$$

### 1.4. Conclusion

As mentioned in the introduction, in power electronics, steady-state operation consists of a periodic succession of transient states. Before beginning the study of the different converter structures, we provided a few reminders regarding:

– transient states, – non-sinusoidal periodic quantities.

# **Chapter 2: Power Semiconductor Basics**

## Chapter 2: Power Semiconductor Basics

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### 2.1. Introduction

Power electronics utilizes semiconductor devices that operate in a switching mode, i.e., as switches. These devices can consist of a single component or result from the combination, in series or parallel, of several semiconductors.

The objective of this chapter is not to provide an exhaustive and in-depth analysis of the internal workings of these components. Rather, it is intended to provide a concise overview of the fundamental properties of the main semiconductors used in power electronics. For a detailed examination of their structure and operating mode, the reader may refer to the specialized literature.

### 2.2. Different Types of Electrical Energy Conversion [3, 4, 5]

Power electronics is a branch of electrical engineering whose purpose is the study of the static conversion of electrical energy.

This discipline is distinguished by the processing of electrical energy using entirely static means, without the use of moving parts. It enables, in particular:

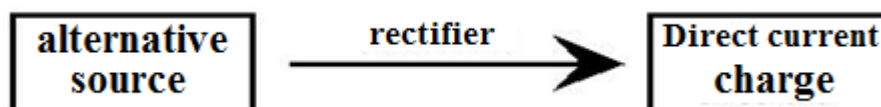
- more flexible and better adapted use of electrical energy based on the specific needs of loads,
- optimization of energy management, transmission, and distribution,
- significant reduction in size (weight and volume), as well as silent operation, often in the ultrasonic range, of associated devices.

Static conversion is achieved through static converters, fundamental devices in power electronics.

**Definition:** A static converter is an electronic device designed to transform an available form of electrical energy into a form more suitable for powering a load.

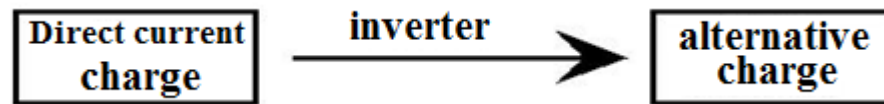
#### 2.2.1. AC-DC Conversion

The rectifier converts electrical energy supplied in AC form into DC energy intended for a load.



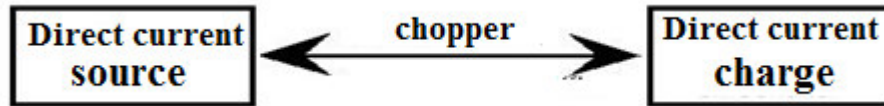
#### 2.2.2. DC-AC Conversion

Inverter: It transforms electrical energy delivered in direct current into alternating current suitable for powering a load.



### 2.2.3. DC-DC Conversion

The converter that transforms electrical energy delivered in DC form to supply a DC load is the chopper.



### 2.2.4. AC-AC Conversion

When electrical energy is delivered in AC form and also powers an AC load, the conversion can be performed with or without frequency modification:

- With frequency modification, known as the cycloconverter.
- Without frequency modification, known as the dimmer.



## 2.3. Power Sources

### 2.3.1. Modeling Electrical Sources

A static converter is inserted between a generator and a load, designated respectively as the input source and output source. From a functional perspective, regardless of the type of generator (DC power supply, single-phase or three-phase network, etc.) or load (DC motor, synchronous motor, induction motor, etc.), these elements will be represented by their equivalent electrical model.

Two approaches to modeling electrical sources are generally adopted:

- the voltage source, which imposes a voltage across its terminals independently of the current output,
- the current source, which imposes a constant current independently of the voltage across its terminals.

### 2.3.2. Definition of a Voltage Source [3, 4, 5]

A generator (or load) will be modeled as a voltage source if the voltage it delivers varies little depending on the current output. The figures below illustrate the behavior of an ideal voltage source (without losses), as well as its representation in a generator or receiver configuration.



Figure.2.1 Voltage source

2.3.3. Definition of a Current Source

The generator (or load) will be modeled by a current source if the current output has little dependence on the voltage across its terminals. The following figures present the behavior model of an ideal current source (without losses) and the representation of a current source in a generator or receiver configuration.

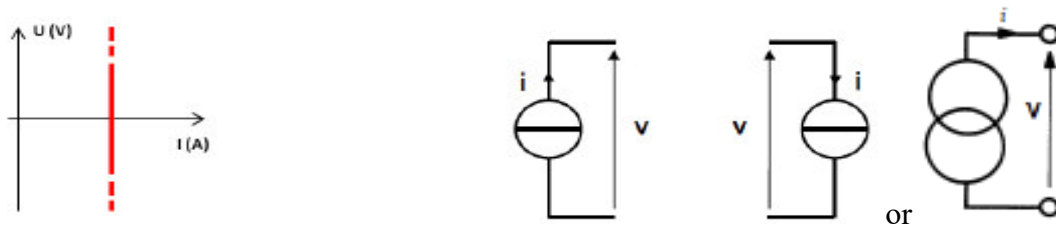


Figure 2.2 Current source

2.3.4. Combination and Transformation of Electrical Sources

2.3.4.1. Rules for Combining Electrical Sources [3, 4, 5]

A static converter consists of switches that periodically interconnect an input source and an output source.

Based on the basic principles of electrical circuit theory, it is easy to understand that not all combinations of sources are permitted (otherwise the components will be damaged), in particular:

- A voltage source must never be short-circuited, but it can be opened,
- A current source must never be open, but it can be short-circuited,
- Never connect two sources of the same type (voltage or current) directly in parallel if they have different values.; this means that only a current source and a voltage source can be connected together.

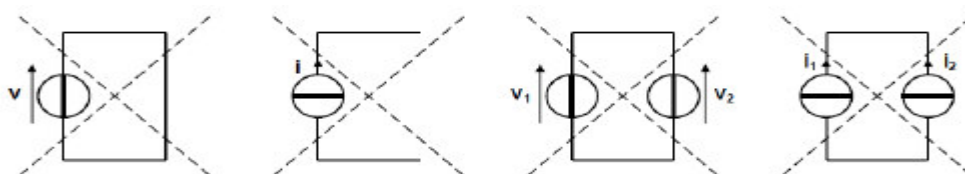


Figure 2.3 Forbidden source connection

but

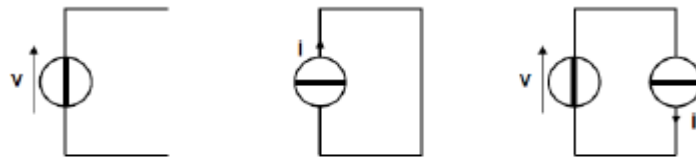


Figure 2.4 Source connection

### 2.3.4.2. Transforming the Behavior of an Electrical Source

Given these rules and the fact that the nature of a source can change over the course of its operation (for example, a DC machine acting as a current source when in receiver mode and as a voltage source when operating in generator mode), it may become necessary to adapt or modify the behavior of a source.

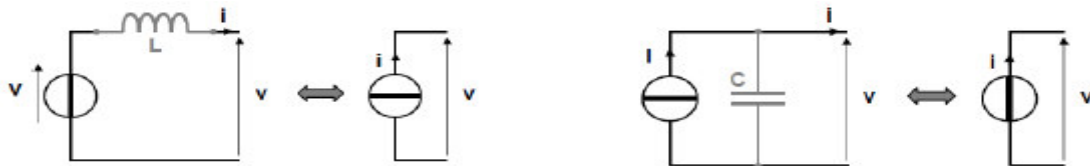


Figure 2.5 Transforming Source

## 2.4. Switches

### 2.4.1. Perfect Switch [3, 4, 5]

A switch has two states: open or closed:

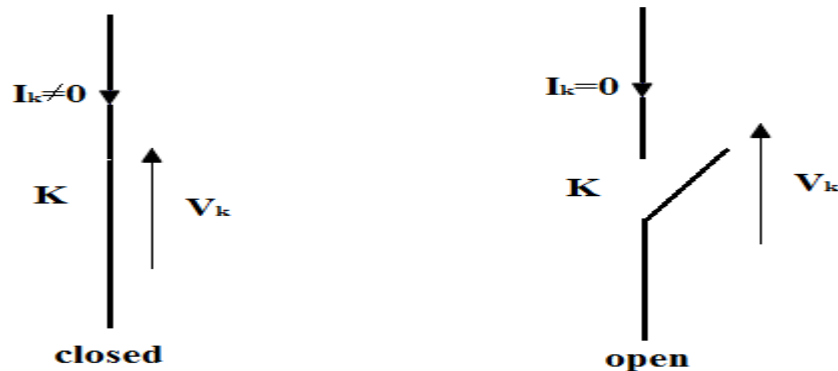


Figure 2.6 Switch states

In the closed state, the switch is said to be ON. In the open state, the switch is said to be OFF. The static characteristic, which is an intrinsic property of a switch, is therefore formed by four segments merged with the  $v$  and  $i$  axes.

### 2.4.2. Semiconductor Switch

The switch is considered a dipole using receiver conventions. A semiconductor switch consists of one or more semiconductor components.

## Chapter 2: Power Semiconductor Basics

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Its resistance  $r_k$  can vary between a very high value (when it is in the open or off state) and a very low value (when it is in the closed or on state).

Several types of semiconductor components are used in power electronics.

### 2.4.3. Two-Segment Switch [3, 4, 5]

The switch is unidirectional in voltage and current. There are two different 2-segment static characteristics, as shown in figure 2.7, namely the diode and the transistor.

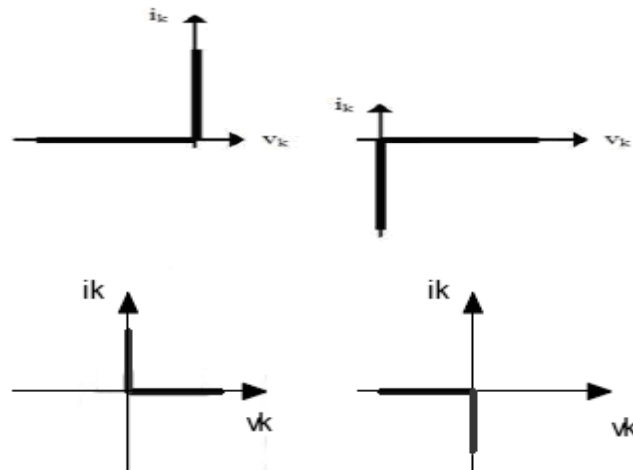


Figure 2.7 Two-Segment Switch

### 2.4.4. Three-Segment Switch

The switch is bidirectional in voltage or current, as shown in the figure. Therefore, there are only two static three-segment characteristics: the thyristor or the GTO thyristor and the combination of a transistor and a diode.

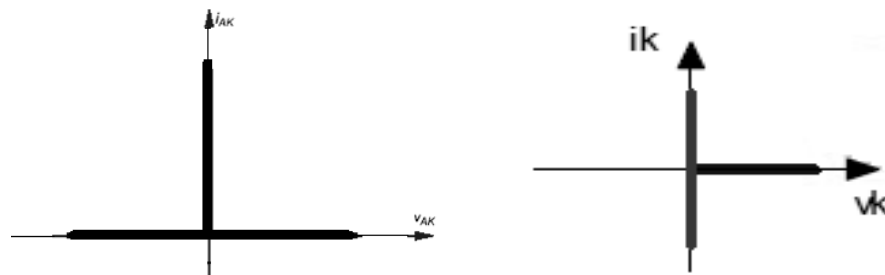
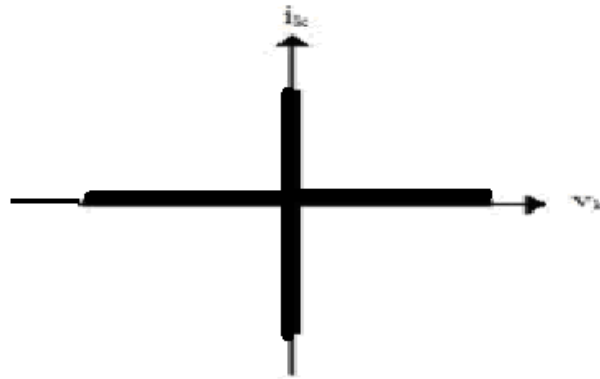


Figure 2.8 Three-Segment Switch

### 2.4.5. Four Segment Switch

The switch is bidirectional in voltage and current, as shown in figure 2.9. The static characteristic is achieved by combining the two previous types, namely the triac.



**Figure 2.9** Four-Segment Switch

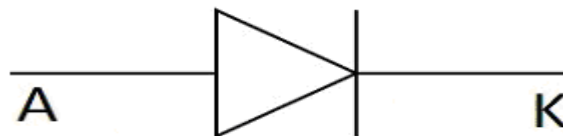
### 2.5. Semiconductor Components [3, 4, 5]

#### 2.5.1. Diode

The power diode (Figure 2.10) is a unidirectional component that cannot be controlled (either in on or off state).

It is not voltage-reversible and can only support a negative anode-cathode voltage ( $V_{AK} < 0$ ) in the off state.

It is not current-reversible and can only support a positive anode-cathode current in the on state ( $I_{AK} > 0$ ).



**Figure 2.10** Symbol of Diode

#### 2.5.1.1. Perfect Component Operation

The diode operates in two modes:

- Passing diode, anode-cathode voltage = 0 for  $I_{AK} > 0$
- Blocking diode, anode-cathode current = 0 for  $V_{AK} < 0$

This is an automatic switch that closes as soon as  $V_{AK} > 0$  and opens as soon as  $V_{AK} < 0$ .

#### 2.5.1.2. Voltage-Current Characteristic:

This is the graph that shows the current flowing through a real diode as a function of the voltage across its terminals.

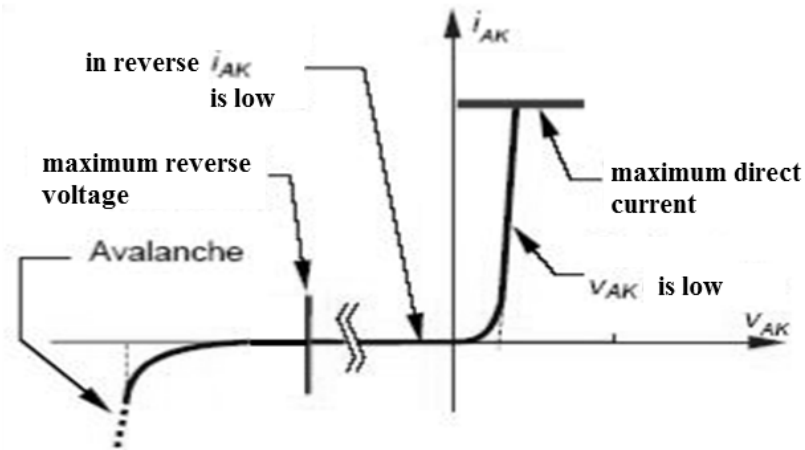


Figure 2.11 A real Diode characteristic [1,2]

The static characteristic of an ideal diode is given by the following figure:

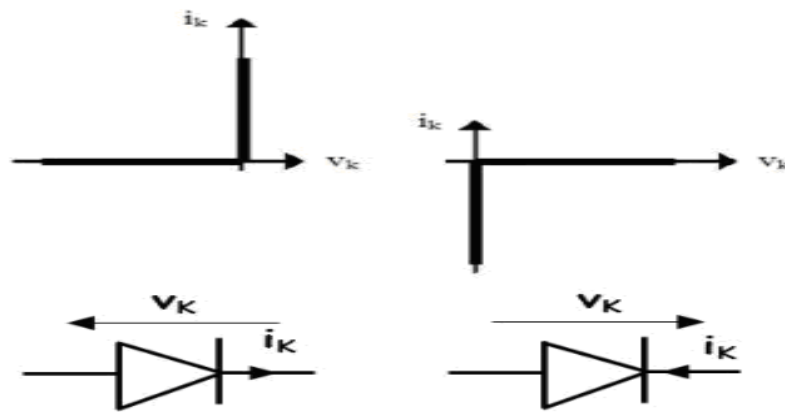


Figure 2.12 Ideal Diode characteristic [1,2]

Different diode shapes



Figure 2.13 Packaging of Diodes

### 2.5.2. Thyristor

The thyristor is a component that is controlled to close, but not to open (Figure 2.13).

It is voltage-reversible and can withstand both positive and negative  $V_{AK}$  voltages when off.

It is not current-reversible and only allows positive  $I_{AK}$  currents, i.e., in the anode-cathode direction, in the on state.

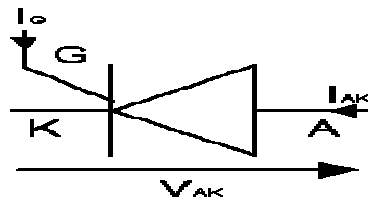


Figure 2.14 Symbol of Thyristor

2.5.2.1. Voltage-Current Characteristic:

This graph shows the current flowing through a real thyristor as a function of the voltage across its terminals.

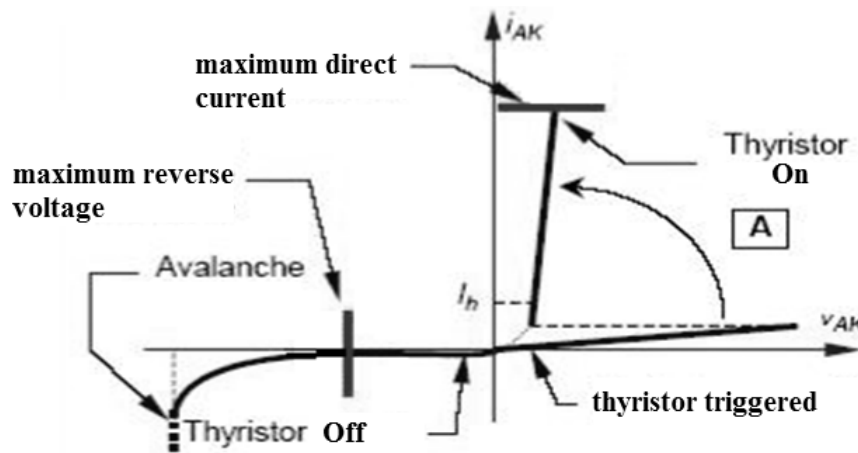


Figure 2.15 Real Thyristor characteristic [1,2]

The static characteristic of a perfect thyristor is given by the following figure:

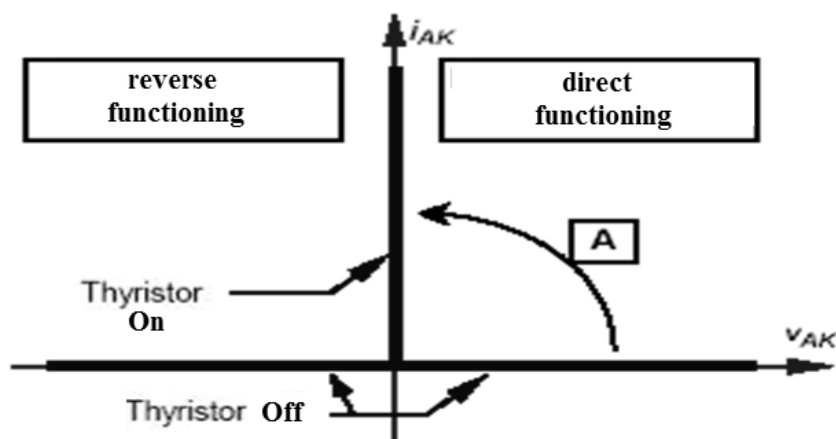


Figure 2.16 Ideal Thyristor characteristic [1,2]

Different forms of thyristor

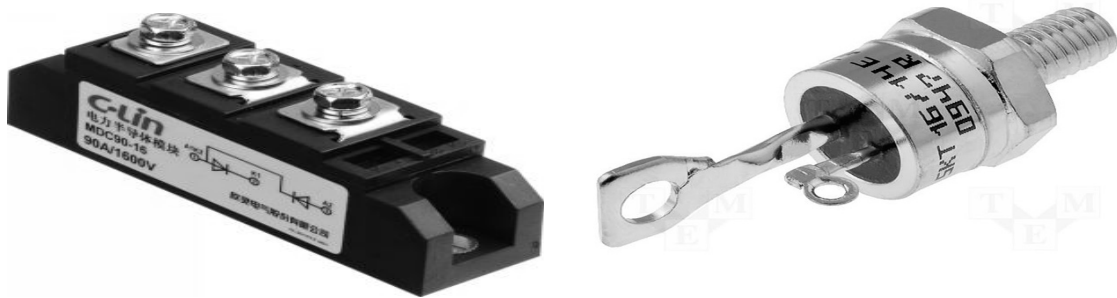


Figure 2.17 Packaging of thyristor

### 2.5.3. Triac

The triac (for Triode Alternating Current) is a three-electrode semiconductor device that allows control of the conduction and blocking of the two alternations of an alternating voltage. The triac can switch from a blocked state to a conductive state in both polarization directions. It can also return to the blocked state, either by voltage inversion (when passing through zero crossing) or by decreasing the holding current. Thus, the triac is a bidirectional component, both in current and voltage.

By analogy, a triac could be considered to consist of two thyristors connected head-to-tail.

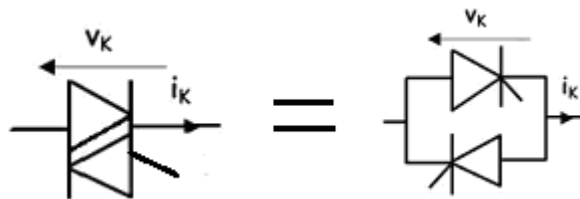


Figure 2.18 Symbol of Triac

#### 2.5.3.1. Voltage-Current Characteristic:

This graph shows the current flowing through the triac as a function of the voltage across its terminals.

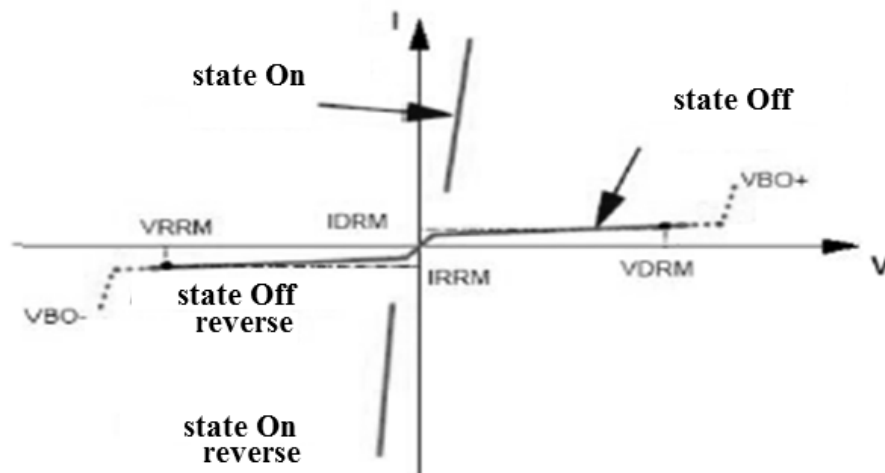


Figure 2.19 A real Triac characteristic [1,2]

## Chapter 2: Power Semiconductor Basics

The static characteristic of an ideal triac is given by the following figure:

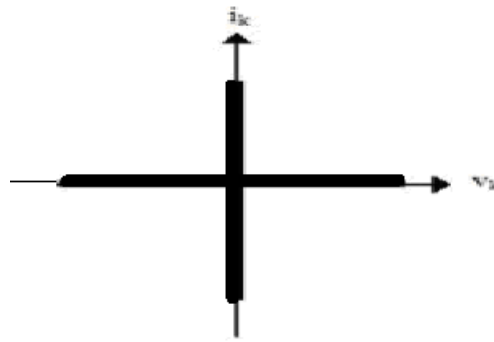


Figure 2.20 Ideal Triac characteristic [1,2]

Different triac shapes



Figure 2.21 Packaging of Triac

### 2.5.4. GTO Thyristor

The GTO (Gate Turn-Off) thyristor is a component controlled by both the on and off switches. It is voltage-reversible and can withstand both positive and negative voltages  $v_{AK}$  when in the off state. However, it is not current-reversible and only allows positive currents  $i_{AK}$ , i.e., in the anode-cathode direction, when in the on state. Thus, the GTO thyristor is a bidirectional voltage component and a unidirectional current component.

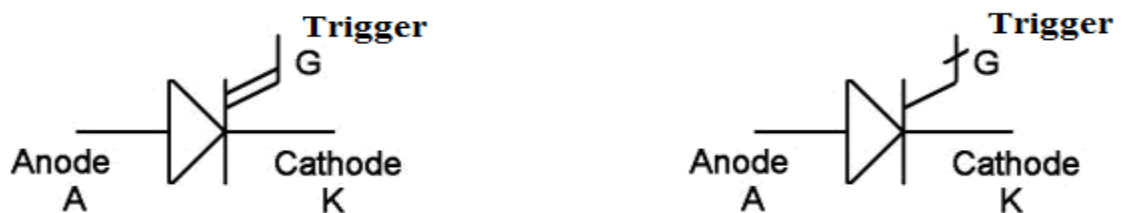


Figure 2.22 Symbol of GTO Thyristor

#### 2.5.4.1. Voltage-Current Characteristic:

This graph shows the current flowing through the GTO thyristor as a function of the voltage across its terminals.

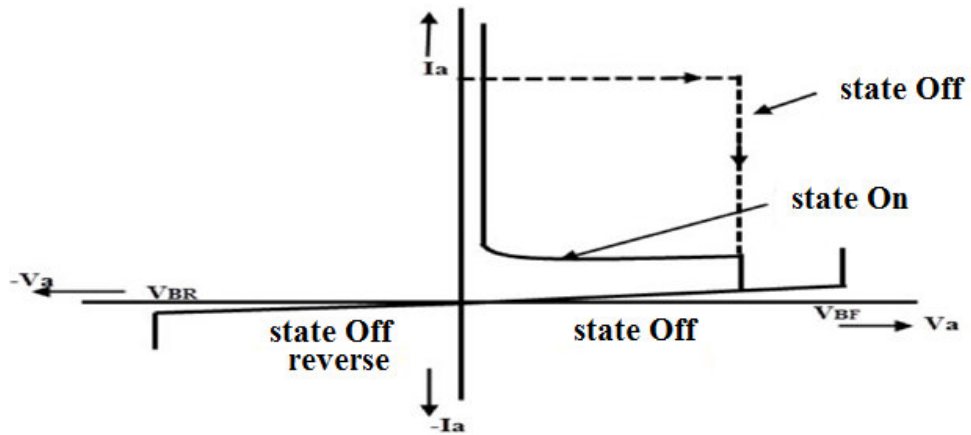


Figure 2.23 A real GTO Thyristor characteristic [1, 2]

The static characteristic of a perfect GTO thyristor is given by the following figure:

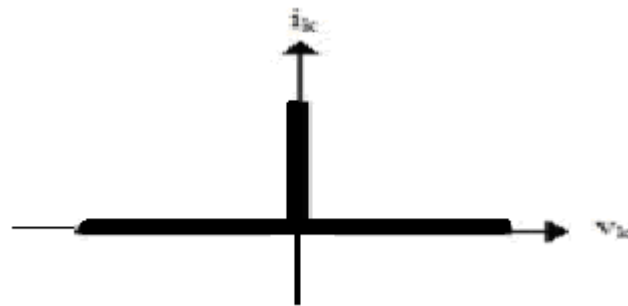


Figure 2.24 Ideal GTO Thyristor characteristic [1, 2]

Different forms of GTO thyristor



Figure 2.25 Packaging of GTO thyristor

### 2.5.5. Power Bipolar Transistor

Of the two types of transistors, NPN and PNP, the power transistor primarily falls into the NPN category. It is a fully controlled component, both on and off. It is not current-reversible,

## Chapter 2: Power Semiconductor Basics

allowing only positive collector currents ( $I_C$ ). It is not voltage-reversible, allowing only positive  $V_{CE}$  voltages when in the off state.

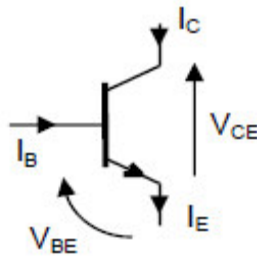


Figure 2.26 Symbol of Transistor

### 2.5.5.1. Voltage-Current Characteristic:

This graph shows the current flowing through the power transistor as a function of the voltage across its terminals.

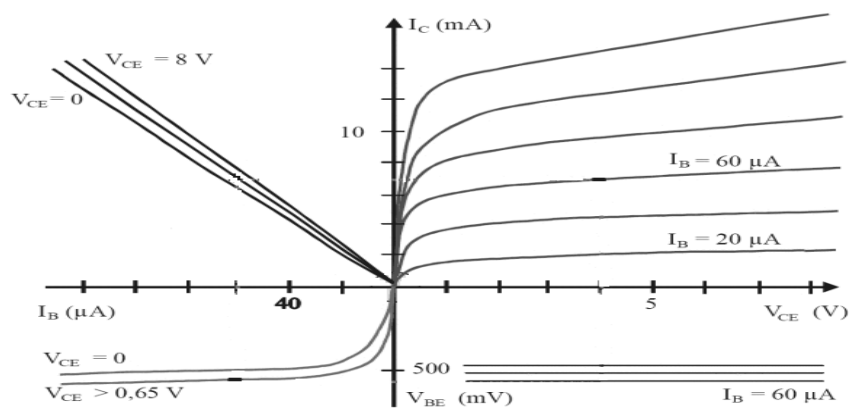


Figure 2.27 Ideal Transistor characteristic [1,2]

The static characteristic of a perfect power transistor is given by the following figure:

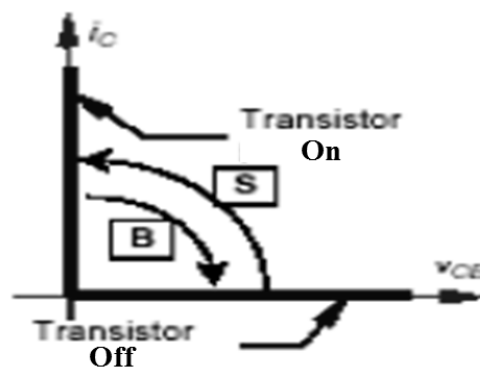
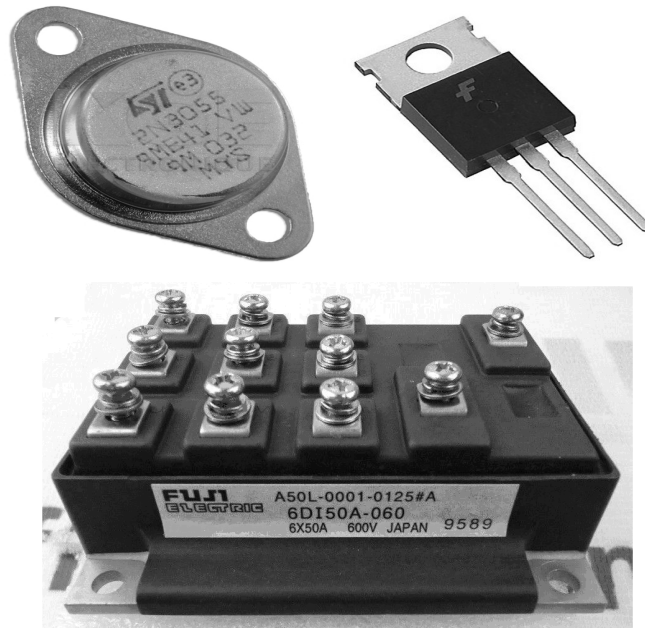


Figure 2.28 Ideal Transistor characteristic [1,2]

Different forms of power transistor

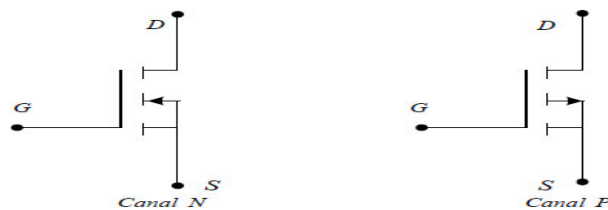


**Figure 2.29** Packaging of Transistor

### 2.5.5.2. MOS Field Effect Transistors

Manufacturers are developing field effect power (or switching) transistors. These are generally insulated gate components. These components offer performance comparable to that of bipolar transistors, while benefiting from the advantages specific to field effect transistors, including:

- Very high input impedance, meaning that the transistor's operating state is determined by the input voltage,
- Very short switching time, with no delay or stored charge discharge period, in principle.



**Figure 2.30** Symbol of MOSFET

### 2.5.2.3. IGBT (Insulated-Gate Bipolar Transistor)

An IGBT transistor is a combination of a bipolar transistor and a field-effect transistor, as shown in the following figures:



**Figure 2.31** Symbol of IGBT

### 2.6. Conclusion

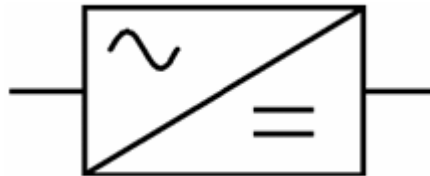
In this chapter, we have limited our presentation to the essential elements needed to explain the different types of electrical energy conversion, sources, switches, and a simple introduction to semiconductor components. This approach allowed us to explain the main static and dynamic characteristics of diodes, thyristors, as well as MOSFETs and IGBTs.

# **Chapter 3: Conversion of AC-DC electrical energy**

### 3.1. Introduction :

The needs of electrical receivers require adapting the form of energy supplied by the distribution network. This is the role of energy converters. Electrical networks and receivers absorb energy in two forms: direct current or alternating current. To adapt supply to demand, four types of converters are necessary. Among these, AC-DC conversion within uncontrolled or controlled rectifiers. Rectifications allow, using diodes and thyristors, to convert an alternating voltage (or current) into another direct voltage (or current).

### 3.2. Symbol:



#### 3.2.1. Definition:

Rectification is the process by which an alternating voltage is converted into a direct voltage. This function is found in AC-DC converters.

If the rectification uses only uncontrolled components, it is called uncontrolled rectification. Its role is to power only the DC loads from an AC source.

### 3.3. Uncontrolled Rectifiers

The study of the rectifier leads to the discussion of the existence of the current  $i(t)$  in the load as a function of the state of the diode. The voltage appears across the load terminals only if the source voltage is positive, resulting in a positive output voltage. As soon as the network voltage is negative, the diode is blocked: the voltage across the load terminals and the current are zero.

#### 3.3.1. Single-phase half-wave rectification:

##### 3.3.1.1. Purely resistive load:

Consider the circuit shown in figure 3.1 powering a resistive load. The diode is assumed to be ideal. Figure 3.1 includes two circuits, one is the electrical circuit and the other uses Simulink/Matlab [3, 4, 5, 9, 10].

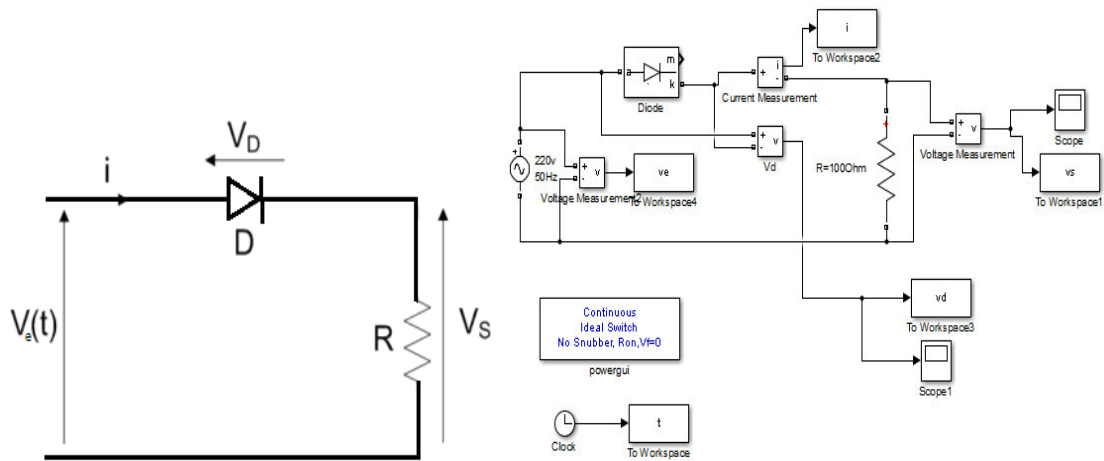


Figure 3.1 Assembly of a single-phase rectifier

### 3.3.1.1.a. Operational Analysis:

We consider diode D to be perfect.

$$v_e(t) = V_{\max} \sin(\omega t) = V_{\text{eff}} \sqrt{2} \sin(\omega t)$$

Regardless of the diode's state, we have:  $V_e = V_D + V_s$

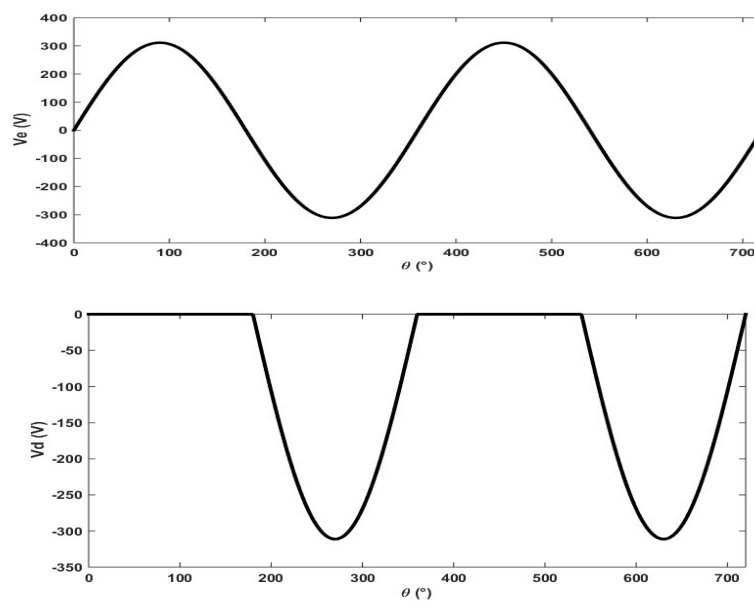
For an input voltage  $V_e(t) > 0$   $V_A > V_K$ , diode D is conductive.

$$V_D = 0V, V_s = V_e. I_R = \frac{V_e}{R}$$

For an input voltage  $V_e(t) < 0$   $V_A < V_K$ , diode D is blocked.

$$V_D = V_e, V_s = 0V. I_R = 0A$$

This operation is illustrated by the timing diagrams in figure 3.2.



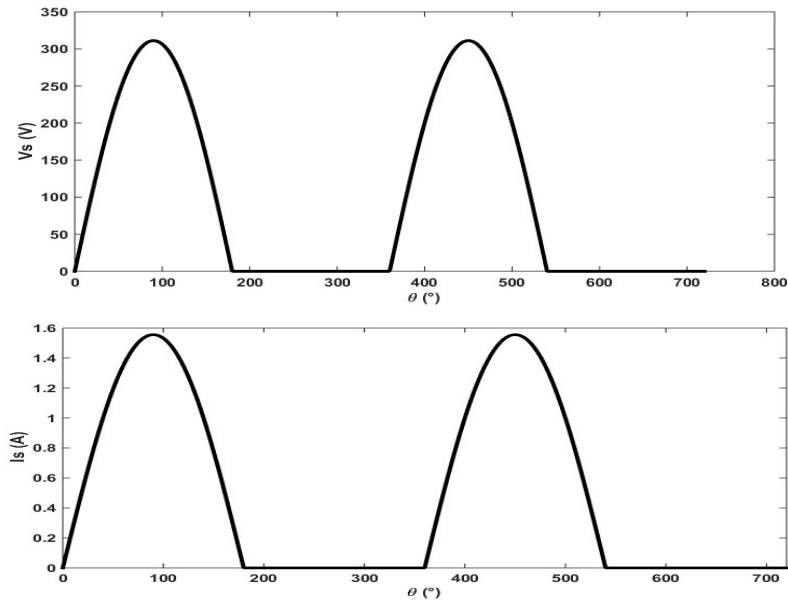


Figure 3.2 Voltage and current waveforms for a 50Hz voltage source.

### 3.3.1.1.b. Calculating the average voltage and current:

The average values of the voltage across the load terminals and the average current are:

$$V_{savg} = \frac{1}{T} \int_0^T V_e(t).dt = \frac{1}{T} \int_0^T V_{max} \sin(\omega t).dt = \frac{V_{max}}{T} \left[ -\frac{1}{\omega} \cos(\omega t) \right]_0^T = \frac{V_{max}}{T} \left[ -\frac{1}{\omega} \cos(\omega T) \right]_0^T = \frac{V_{max}}{\pi} \text{ and}$$

$$I_{savg} = \frac{V_{savg}}{R} = \frac{V_{max}}{\pi.R}$$

The effective voltage is given by the following relation:

$$V_{seff} = \sqrt{\frac{1}{T} \int_0^T (V_{max} \sin(\omega t))^2 .dt}$$

$$V_{seff} = \sqrt{\frac{V_{max}^2}{T} \int_0^T \frac{1 - \cos(2\omega t)}{2} .dt} = \sqrt{\frac{V_{max}^2}{2T} \left[ t - \frac{\sin(2\omega t)}{2\omega} \right]_0^T} = \frac{V_{max}}{2}$$

$$\text{Or : } I_{eff} = \frac{V_s}{R} = \frac{V_{eff}}{R} = \frac{V_{max}}{2.R}$$

### 3.3.1.2.Changing the nature of the load:

In electrical engineering, loads are often combined: inductive and resistive. The diagram allowing for this new study is shown in the following figure [3,4,5].

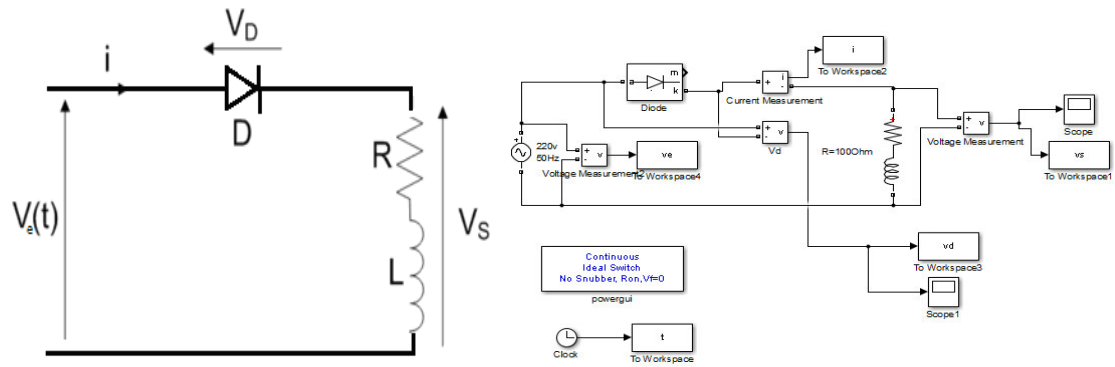


Figure 3.3 Diode rectifier with an inductive load.

The operation is illustrated by the timing diagrams in figure 3.4 for two inductance values.

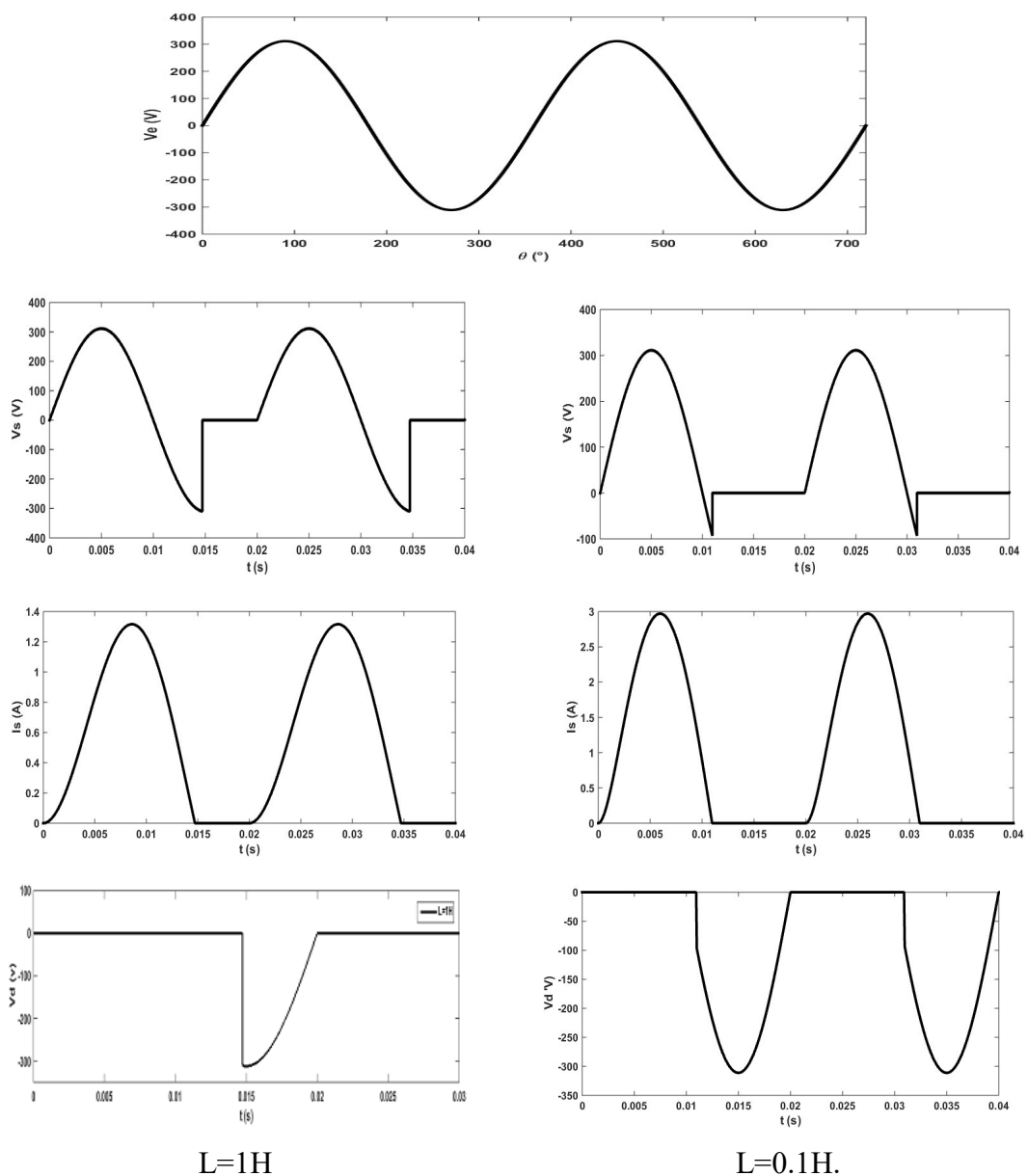


Figure 3.4 Timing diagrams of input and output voltages and current for  $L=1H$  and  $L=0.1H$  for 50Hz voltage source.

## Chapter 3: Conversion of AC-DC electrical energy

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### 3.3.1.2.a. Operational analysis:

The load stops conducting when the current through it vanishes. The current extinction  $i_L = 0$  at angle  $\beta$ , once all the energy stored in the inductor has returned to the voltage source.

When diode D begins conducting, the circuit is described by the following differential equation:

$$L \frac{di}{dt} + Ri = V_e(t) = V_{\max} \cdot \sin(\omega t)$$

This equation has two solutions: permanent  $i_p(t)$  and transient  $i_h(t)$ .

$$i_p(t) = \frac{V_{\max}}{Z} \sin(\omega t - \varphi) \quad \text{with } \varphi = \arctg\left(\frac{L\omega}{R}\right)$$

$$i_h(t) = k \cdot e^{-\frac{R}{L}t}$$

$k$  is a constant that is determined from the initial conditions.

at  $\omega t = \beta$ ,  $i_L(t) = 0$

$$0 = \frac{V_{\max}}{Z} \sin(\beta - \varphi) + k \cdot e^{-\frac{R}{L}\beta} \Rightarrow k = -\frac{V_{\max}}{Z} \sin(\beta - \varphi) \cdot e^{\frac{R}{L}\beta}$$

So solving the equation leads to:

$$i_L(t) = \frac{V_{\max}}{Z} \sin(\omega t - \varphi) - \frac{V_{\max}}{Z} \sin(\beta - \varphi) \cdot e^{\frac{R}{L}\beta} \cdot e^{-\frac{R}{L}t}$$

$$i_L(t) = \frac{V_{\max}}{Z} \sin(\omega t - \varphi) - \frac{V_{\max}}{Z} \sin(\beta - \varphi) \cdot e^{-\frac{R}{L}(t - \frac{\beta}{\omega})}$$

### 3.3.1.2.b. Calculating the average voltage and current:

The average values of the voltage across the load and the current are:

$$V_{savg} = \frac{1}{T} \int_0^{\beta} V_s(t) \cdot dt = \frac{1}{T} \int_0^{\beta} V_e \sin(\omega t) \cdot dt = \frac{V_{\max}}{2\pi} (1 - \cos(\beta))$$

$$\text{and } I_{savg} = \frac{V_{savg}}{R}$$

The effective voltage is given by:

$$V_{eff} = \sqrt{\frac{1}{T} \int_0^{\beta} (V_{\max} \sin(\omega t))^2 \cdot dt}$$

$$V_{eff} = \sqrt{\frac{V_{\max}^2}{T} \int_0^{\beta} \frac{1 - \cos(2\omega t)}{2} \cdot dt} = \sqrt{\frac{V_{\max}^2}{4\pi} \left( \beta - \frac{\sin(2\beta)}{2} \right)}$$

The only way to calculate the effective value of the current is to take the integral of the current expression.

Because the effective current in the load:

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$$I_{eff} \neq \frac{V_{eff}}{Z} \neq \frac{V_{eff}}{R}$$

And we can decompose the current in the form:

$$I_{eff}^2 = I_{avg}^2 + I_{ond}^2$$

so

$$I_{ond}^2 = \sum_{h=1}^{\infty} \left( \frac{V_{effh}}{Z_h} \right)^2$$

with

$$Z_h = \sqrt{R^2 + (jwhL)^2} \quad \text{with } h=1,2,\dots$$

$V_{effh}$  The RMS value of the h-order harmonic in the Fourier series of the rectified voltage

#### 3.3.1.3. Inductive load with a freewheeling diode:

This device reduces current ripple in the load and allows for continuous conduction if the load is highly inductive. To achieve this, a diode is added in antiparallel to the load (Figure 3.5) [3,4,5].

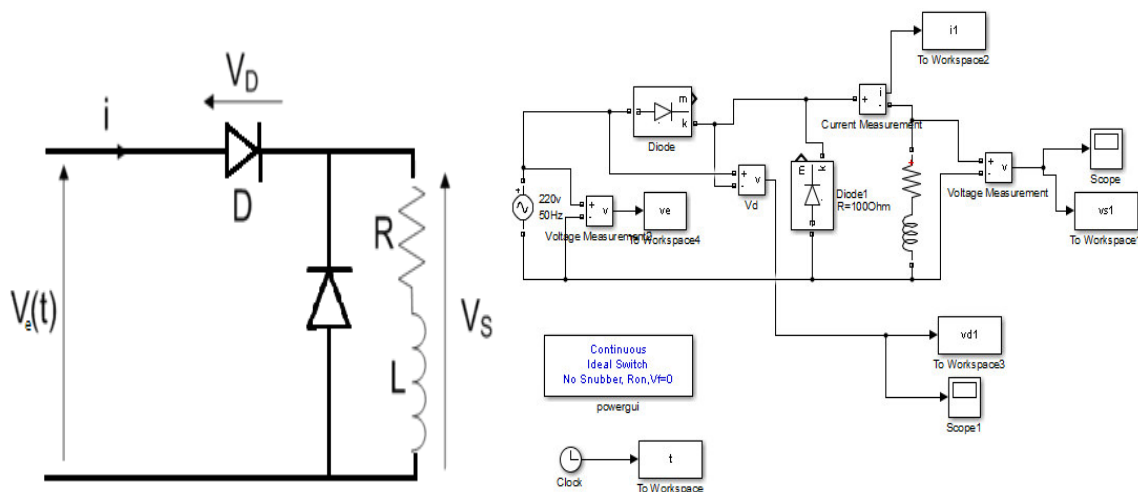
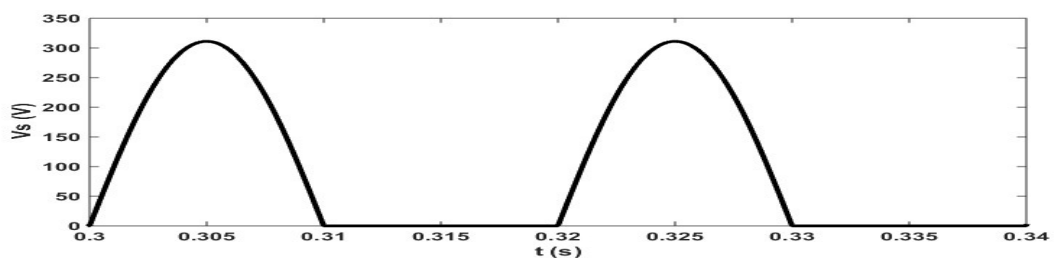


Figure 3.5: Thyristor rectifier with inductive load and a free-floating diode

The voltage and current simulation is done with Matlab (Figure3. 6).



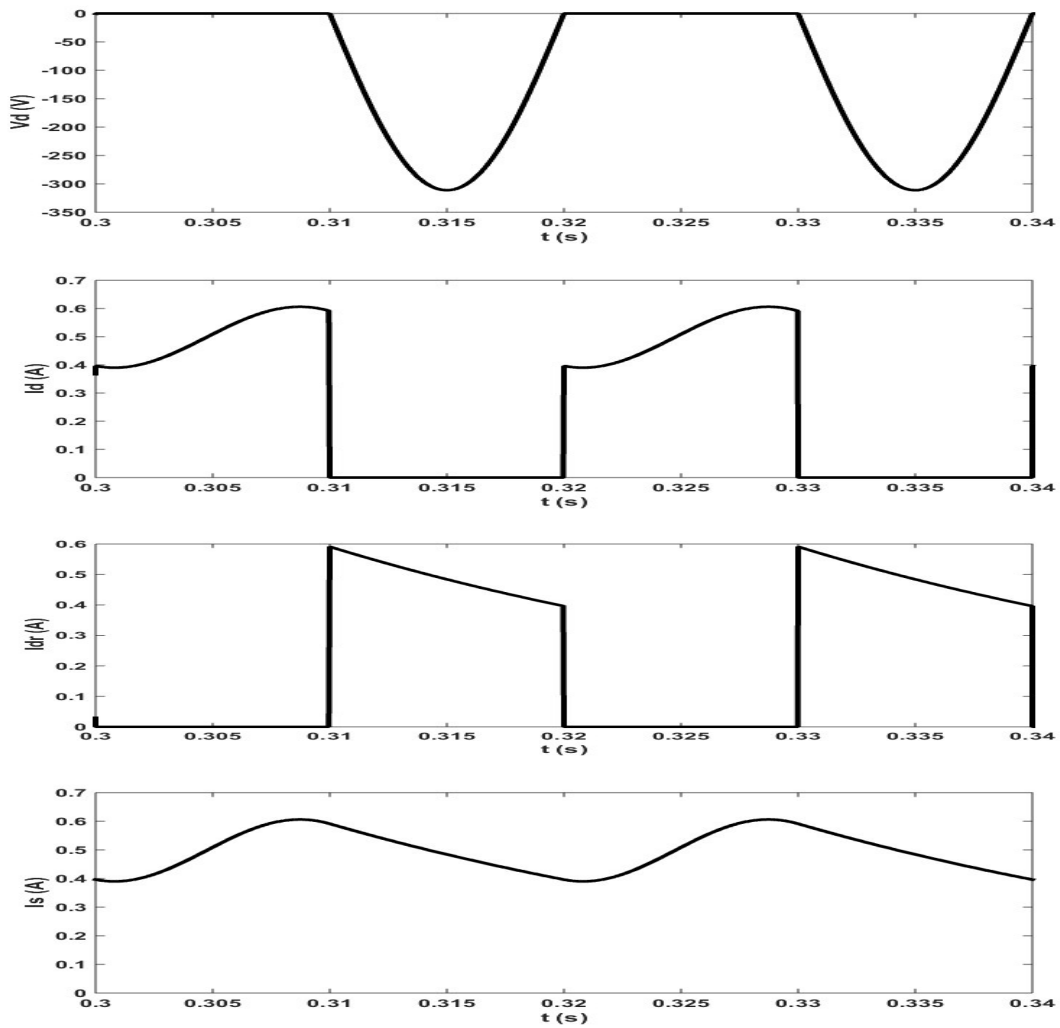


Figure 3.6 Timing diagrams of input and output voltages and current for 50Hz voltage source.

### 3.3.1.3.a. Operational analysis:

For  $0 \leq t \leq T/2$ ,

When diode D conducts and the other is blocked, the circuit is described by the following differential equation:

$$L \frac{di}{dt} + Ri = V_e(t) = V_{\max} \cdot \sin(\omega t)$$

A solution with initial condition ( $t = 0, i_L(0) = I_{\min}$ ) will be

$$i_L(t) = \frac{V_{\max}}{Z} \sin(\omega t - \varphi) + \left( I_{\min} - \frac{V_{\max}}{Z} \sin(\varphi) \right) e^{-\frac{R}{L}t}$$

For  $T/2 \leq t \leq T$ ,

Diode D is blocked and the other goes into conduction, the circuit is described by the following differential equation:

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$$L \frac{di}{dt} + Ri = 0$$

A solution with initial condition ( $t = T/2, I_{\max} = I_L(T/2)$ ) will be:

$$i_L(t) = K e^{\frac{-R}{L}(t - \frac{T}{2})}$$

$$i_L\left(\frac{T}{2}\right) = \frac{V_{\max}}{Z} \sin\left(\omega \frac{T}{2} - \varphi\right) + \left(I_{\min} - \frac{V_{\max}}{Z} \sin(\varphi)\right) e^{\frac{-R T}{L}} = K = I_{\max}$$

#### 3.3.1.4. Capacitive Filtering or R/C Load

To improve the rectified voltage, capacitive filtering is used by placing a capacitor with capacitance  $C$  in parallel with  $R$  [3, 4, 5].

Figure 3.7 represents single-phase rectification for a capacitive load.

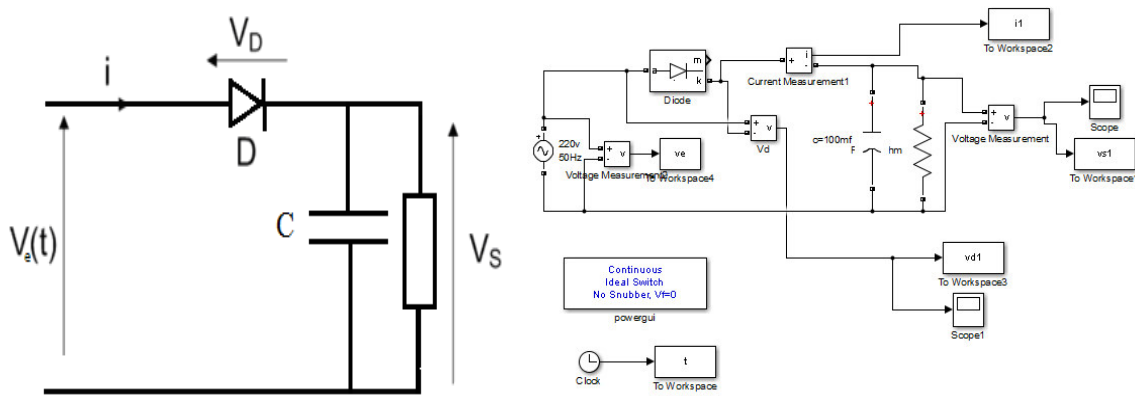
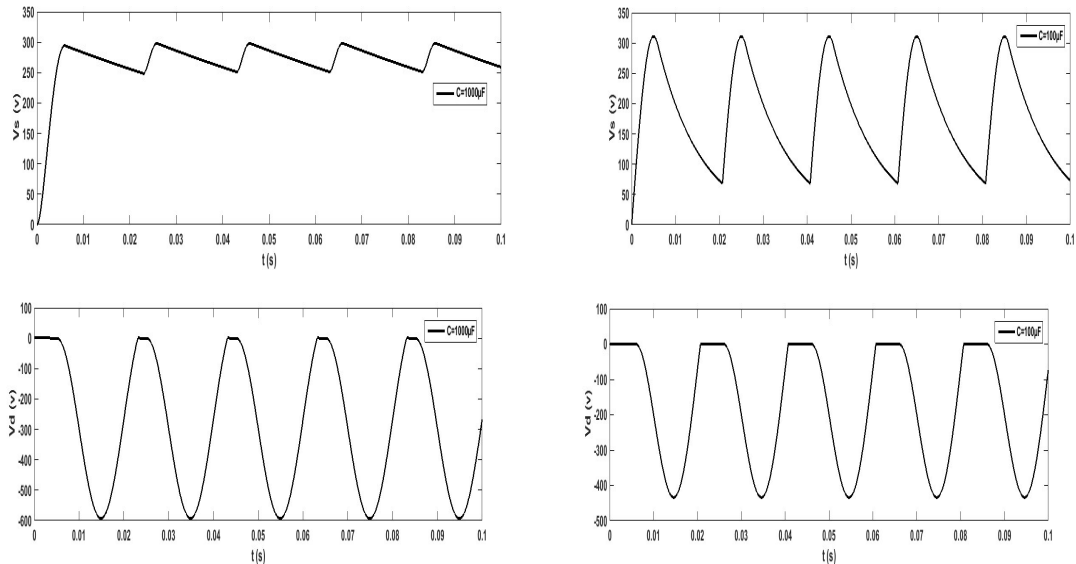


Figure 3.7 Rectifier with a capacitive load



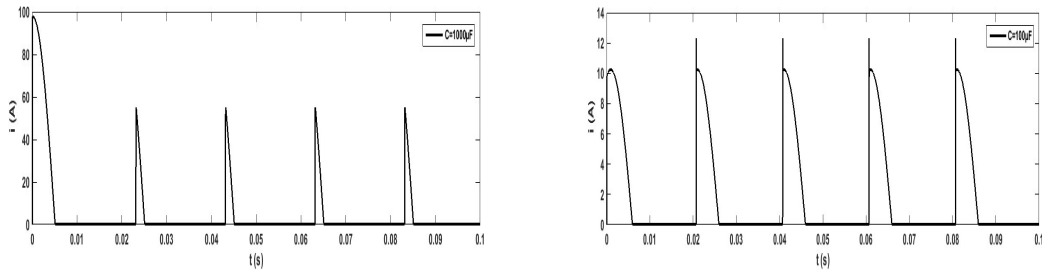


Figure 3.8 Timing diagrams of output voltages and current for two values of C for 50Hz voltage source.

3.3.1.4.a. Operational analysis.

At  $t = 0$ ,  $V_e = 0V$  and C is discharged  $V_s = 0 V$ ; the diode is perfect.

$0 < t < T/2$ :  $V_e$  increases and diode D is conducting: capacitor C charges below  $V_e(t)$

$$I_R = \frac{V_c}{R}, \quad I_c = C \frac{dV_c}{dt}$$

$$V_c(\theta) = V_s(\theta) = V_{max} \cdot \sin(\theta)$$

And the currents  $i_R$  and  $i_C$  would be:

$$I_R = \frac{V_c}{R} = \frac{V_{max}}{R} \cdot \sin(\theta) \quad \text{and} \quad I_c = C \frac{dV_c}{dt} = c\omega V_{max} \cos(\omega.t)$$

$T/2 < t < T_d$ :  $V_e$  decreases rapidly as the capacitor resists sudden voltage changes across these terminals. The potential at point K becomes higher than that at point A, and diode D is blocked. The capacitor then slowly discharges into resistor R with a time constant  $\tau = R.C$ . The voltage  $V_s$  decreases exponentially.

$$I_c = -I_R = \frac{V_{max}}{R} e^{-\frac{t}{R.C}}$$

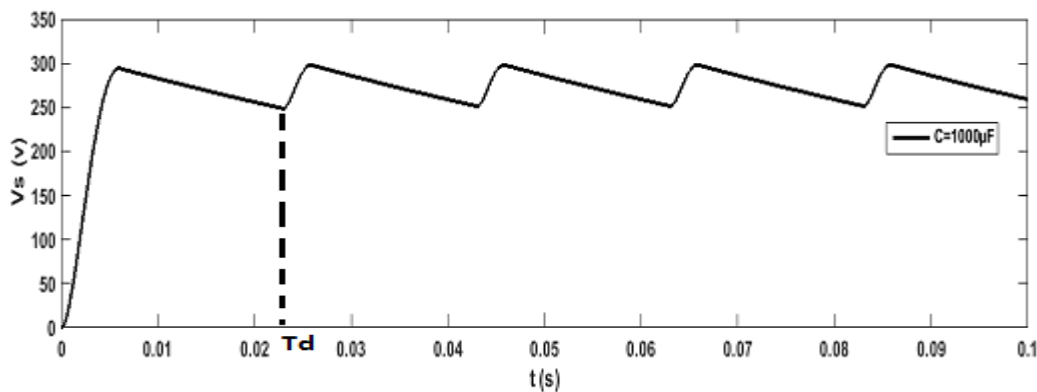


Figure 3.9 Output voltage timing diagram for C=100µF.

3.3.1.5. Form Factor

The value of the Form factor characterizes the rectified voltage. The closer this value is to unity, the closer the resulting voltage is to a continuous quantity.

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This coefficient is used to compare different rectifier assemblies.

By definition, the Form factor is the ratio:

$$F = \frac{V_{eff}}{V_{avg}}$$

$$F = \frac{V_{eff}}{V_{avg}} = \frac{V_{max} \cdot \frac{1}{2}}{V_{max} \cdot \frac{1}{\pi}} = \frac{\pi}{2} = 1.57$$

### 3.3.1.6. Ripple Rate factor

The peak ripple rate is equal to the ratio of the effective value of the alternating component of a rippled quantity to the effective value of the quantity itself and is calculated using the following relationship:

$$\tau = \sqrt{F^2 - 1} = 1.21$$

### 3.3.2. Single-phase full-wave rectification

#### 3.3.2.1. Parallel type full-wave rectifier

##### 3.3.2.1.a. Resistive load

This circuit requires a center-tapped transformer. Two voltages  $V_1$  and  $V_2$  of identical amplitude and opposite phases are obtained [3,4,5]:

$$V_1 = -V_2 = V_{max} \sin(\omega t)$$

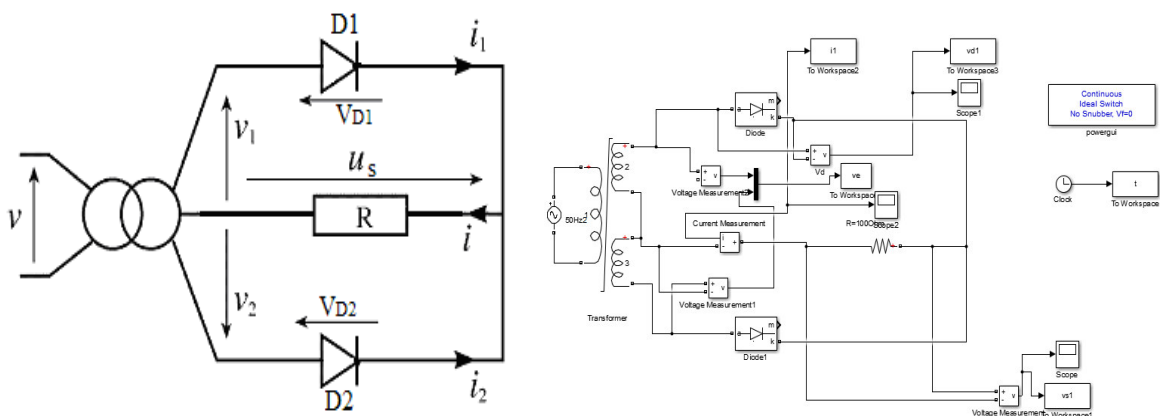


Figure 3.10 Assembly of a full-wave rectifier.

#### ► Functional analysis.

- $0 < t < T/2$

Diode D1 conducts and diode D2 remains blocked

$$V_s = V_1.$$

- $T/2 < t < T$

Diode D1 blocks and D2 conducts

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$$V_s = V_2 = -V_1$$

Diode D1 is subjected to the voltage

$$V_{D1} = V_1 - V_s = 2V_1.$$

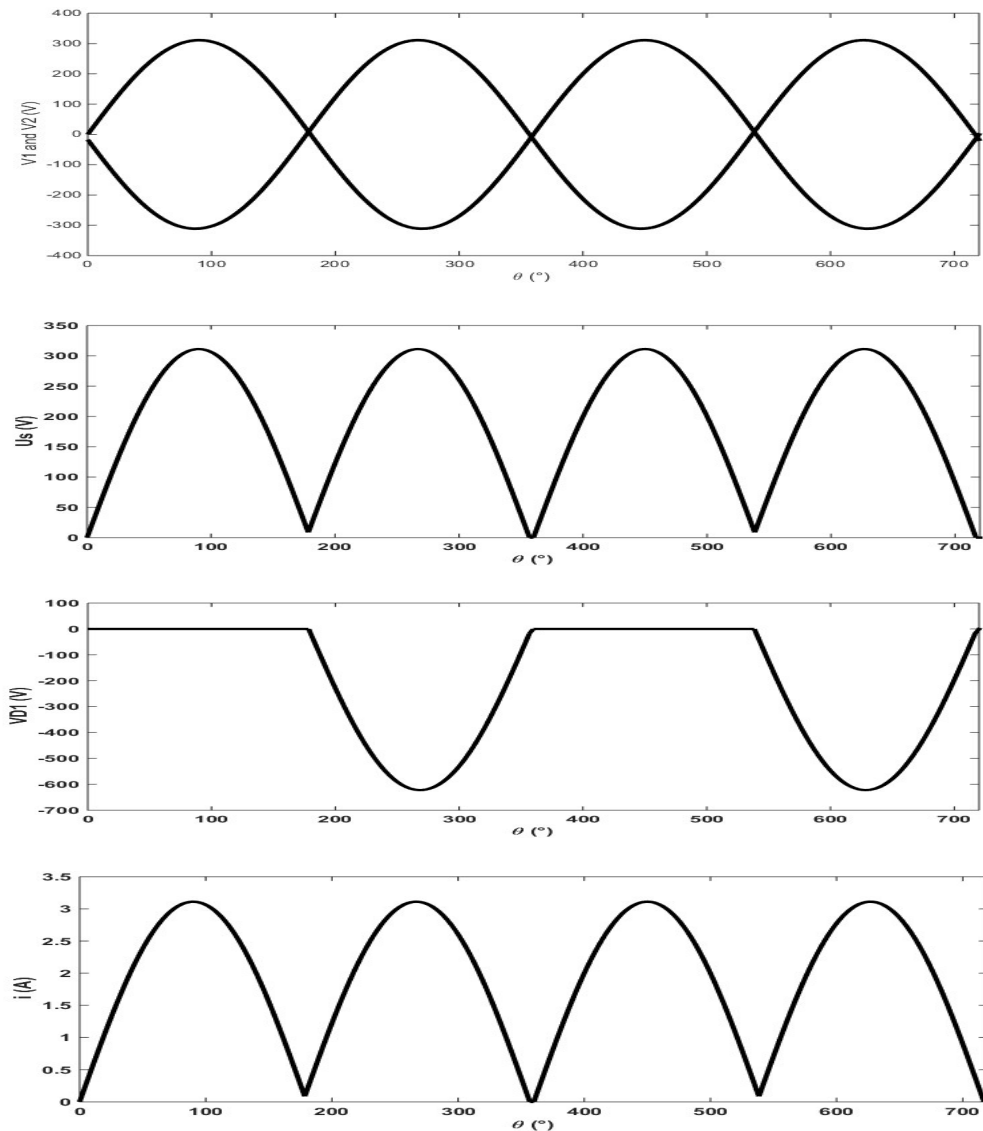


Figure 3.11 Single-phase voltage timing diagrams supplying a purely resistive load

#### ►. Calculating voltage and current

The average value of this voltage is given by:

$$V_{moy} = \frac{2}{T} \int_0^{\frac{T}{2}} v(t).dt = \frac{1}{\pi} \int_0^{\pi} V_{max} \sin(\theta).d\theta = \frac{2.V_{max}}{\pi}$$

The average current being equal to:

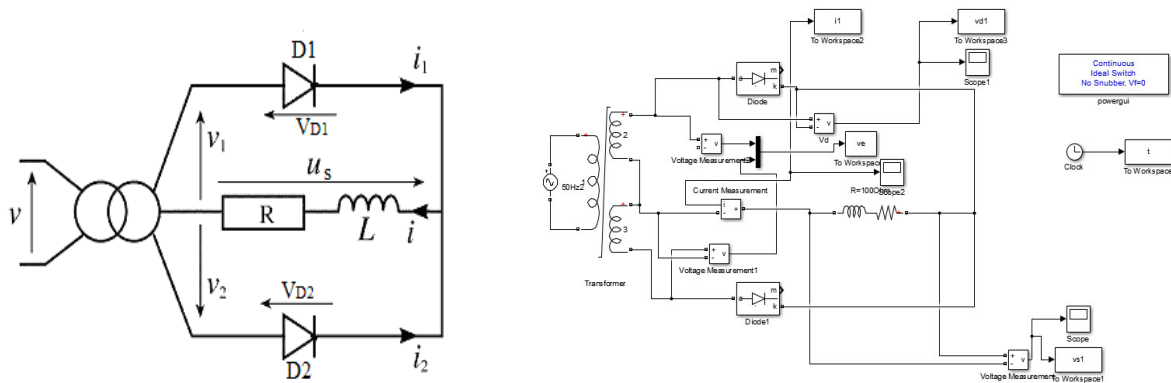
$$I_{moy} = \frac{2.V_{max}}{\pi.R}$$

And so its effective value (rms) is:

$$V_{seff} = \sqrt{\frac{V_{max}^2}{T} \int_0^T \sin^2(\theta).d\theta} = \sqrt{\frac{2.V_{max}^2}{T} \int_0^{\frac{T}{2}} \sin^2(\theta).d\theta} = \sqrt{\frac{2.V_{max}^2}{T} \int_0^{\frac{T}{2}} \frac{1-\cos(\theta)}{2}.d\theta} = \frac{V_{max}}{\sqrt{2}}$$

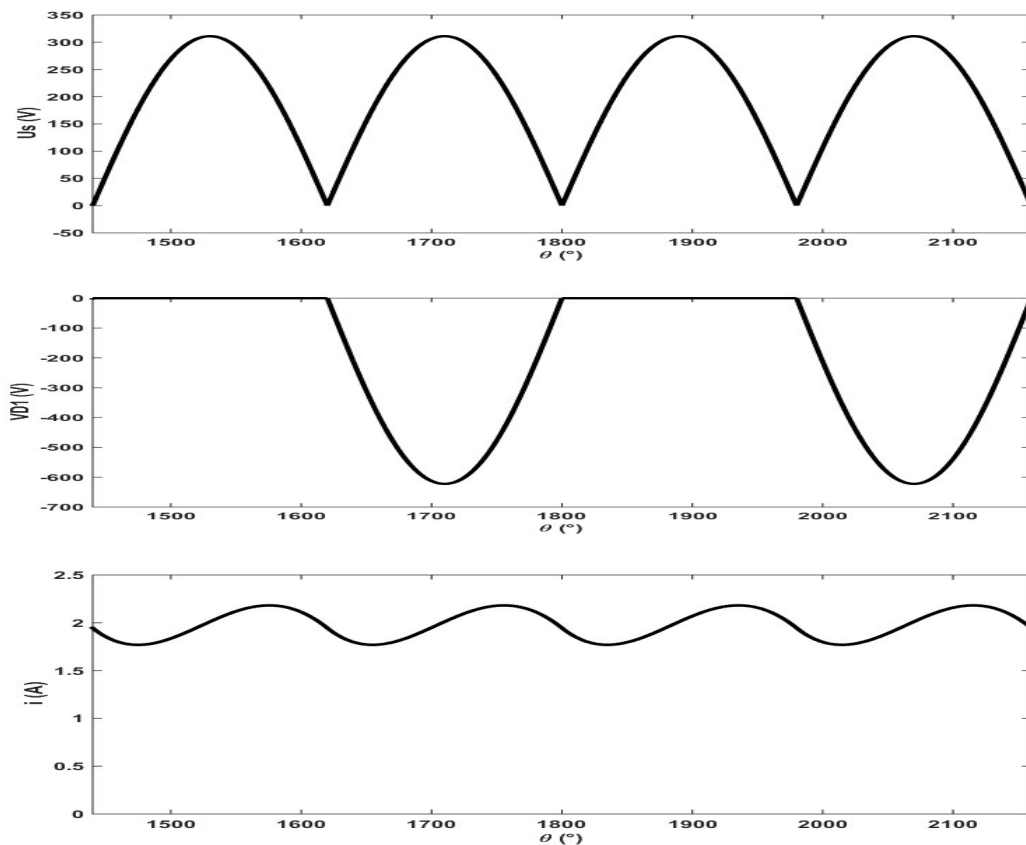
**3.3.2.1.b. Inductive Load RL:**

The load now consists of an inductor L in series with a resistor R. The inductor L opposes variations in current  $I_s$ . If L is given a sufficient value, the current in the load becomes uninterrupted [3, 4, 5].



**Figure 3.12** Diode rectifier with inductive load.

The voltage and current diagram is given in figure 3.13.



**Figure 3.13** Timing diagrams of input and output voltages and current.

### 3.3.2.2. Graëtz bridge rectification PD2:

This is the most widely used rectifier circuit due to its simplicity [3, 4, 5].

#### 3.3.2.2.a. Resistive load:

The circuit in figure 3.14, called a bridge rectifier, produces a current in the load R that always flows in the same direction regardless of the polarities of terminals 1 and 2 of the source.

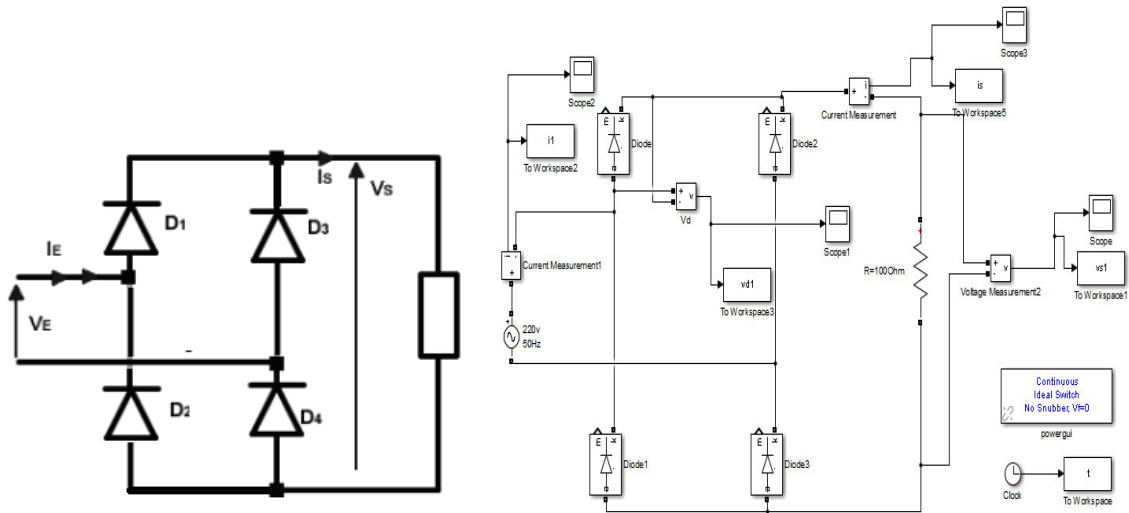


Figure 3.14 4-diode rectifier with resistive load.

#### ► Operational Analysis

- $0 < t < T/2$

D1 and D4 are forward-biased and conduct.

$$V_s = V_e.$$

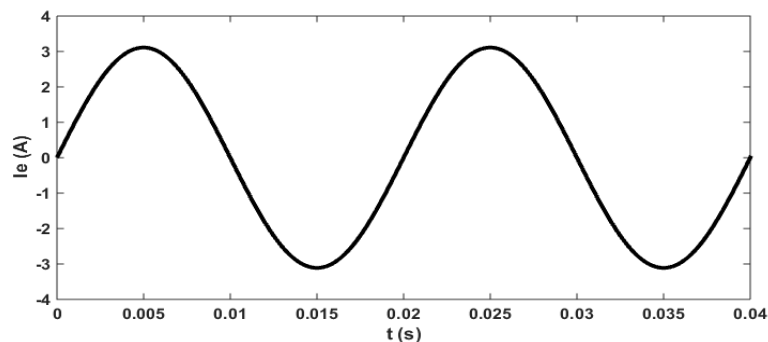
- $T/2 < t < T$

D2 and D3 are conducting.

$$V_s = -V_e.$$

$$V_{D1} = \frac{V_e - V_s}{2} = V_{\max} \sin(\omega t)$$

The voltage and current diagram is shown in figure 3.15.



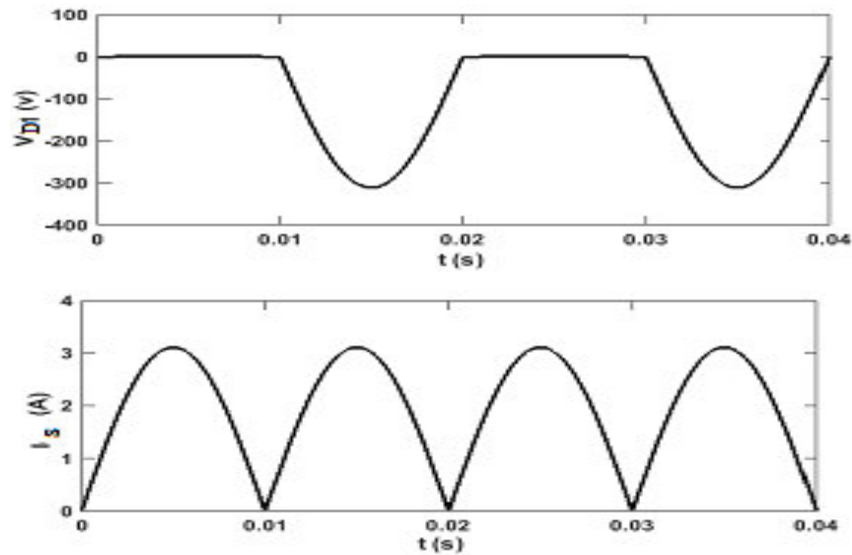


Figure 3.15 Timing diagrams of input and output voltages and current.

► **Calculating Voltage and Current**

The average value of the full-wave signal is twice the average value of the single-wave signal, since the area under the curve is doubled:

$$V_{avg} = 2 \frac{V_{max}}{\pi} = 2 \frac{\sqrt{2} V_{effs}}{\pi} \quad \text{and} \quad I_{avg} = 2 \frac{\sqrt{2} V_{effs}}{\pi R}$$

And so its effective value is:

$$V_{eff} = \sqrt{\frac{V_{max}^2}{T} \int_0^T \frac{1 - \cos(\theta)}{2} .d\theta}$$

$$V_{eff} = V_{max} \sqrt{\frac{1}{2}} = V_{max} \cdot \frac{\sqrt{2}}{2}$$

**3.3.2.2.b. Inductive load RL:**

The load now consists of an inductive load (a pure inductor L in series with a resistor R). If L is given a sufficient value, the current in the load becomes uninterrupted: this is the "continuous" conduction regime [3, 4, 5].

We can write:

$$V_s = V_R + V_L = L \frac{di}{dt} + Ri = V_e(t) = V_{max} \sin(\omega t)$$

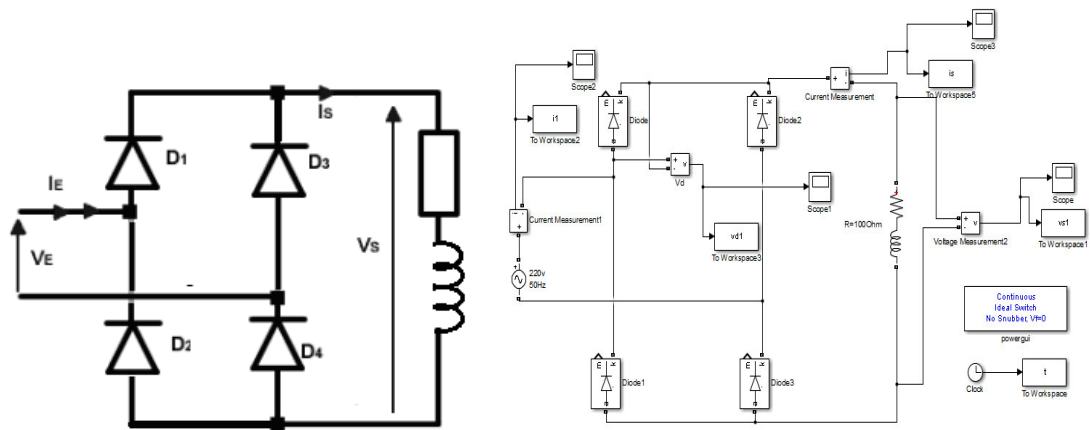


Figure 3.16 Full-wave Graetz bridge rectifier with inductive load.

The voltage and current diagram is given in figure 3.17.

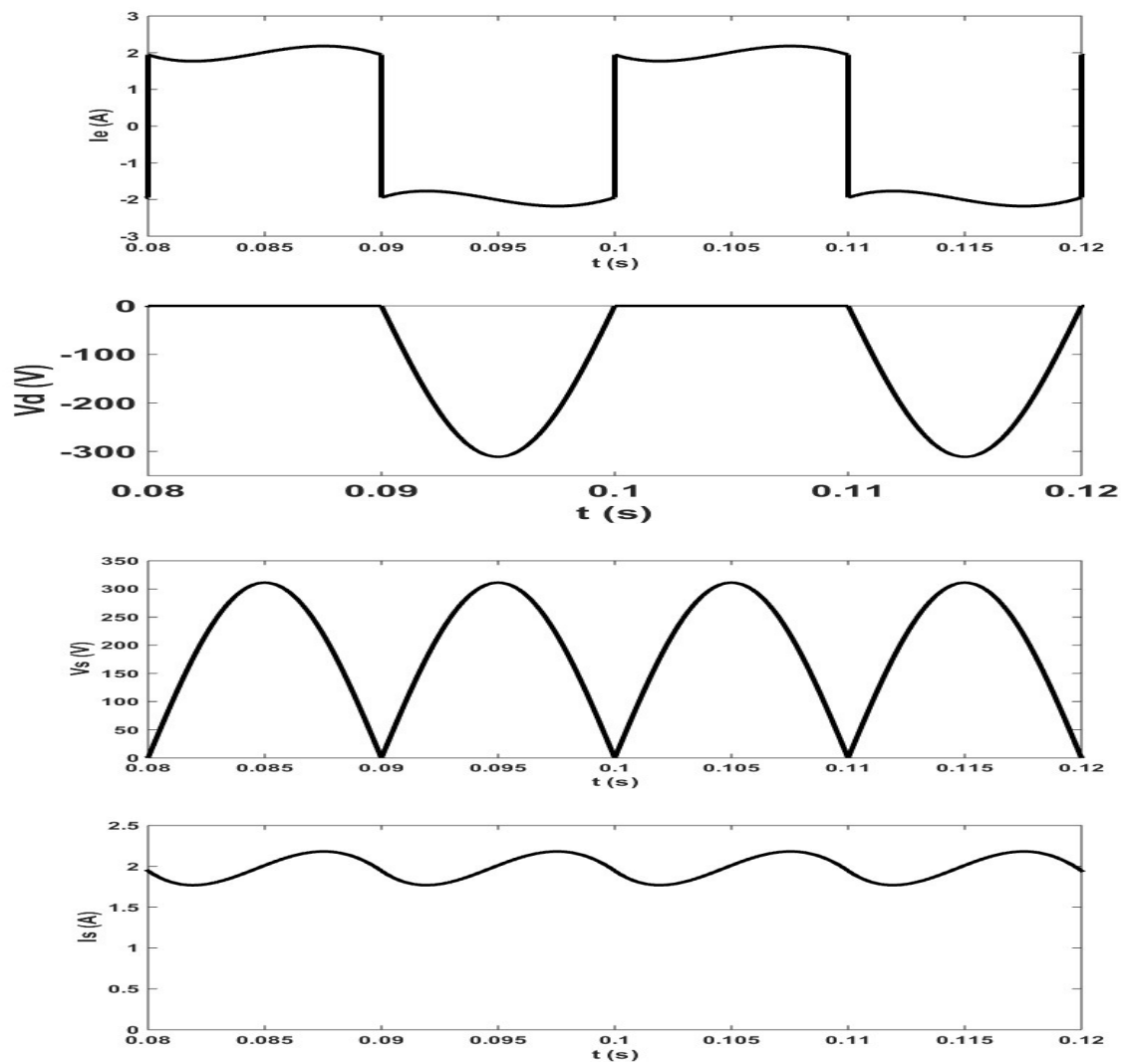


Figure 3.17 Timing diagrams of input and output voltages and current.

### 3.3.2.2.c. Form Factor

By definition, the form factor is the ratio:

$$F = \frac{V_{eff}}{V_{moy}}$$
$$F = \frac{V_{eff}}{V_{moy}} = \frac{V_{max} \cdot \frac{\sqrt{2}}{2}}{V_{max} \cdot \frac{2}{\pi}} = \frac{\sqrt{2} \cdot \pi}{4} = 1.11$$

### 3.3.2.2.d. Ripple factor

The ripple value is calculated using the following relationship:

$$\tau = \sqrt{F^2 - 1} = 0.48$$

Compared to the full-wave circuit, this circuit has the following advantages:

- better form factor,
- better hum coefficient
- better transformer utilization factor
- lower ripple value than the full-wave circuit

### 3.3.3. Three-phase rectifiers:

In three-phases, we distinguish between single-wave rectification and double-wave rectification. Three-phase circuits are used to increase output power [3, 4, 5].

#### 3.3.3.1. Three-phase P3 diode rectifiers:

##### 3.3.3.1.a. Resistive load:

The P3 diode rectifier circuit consists of three diodes, each connected to a phase of the secondary of a three-phase transformer. From the three-phase network, a balanced three-phase voltage system ( $V_1, V_2, V_3$ ) is obtained at the transformer secondary, which is denoted:

$$V_1(t) = V_{max} \cdot \sin(\omega t)$$

$$V_2(t) = V_{max} \cdot \sin(\omega t - \frac{2\pi}{3})$$

$$V_3(t) = V_{max} \cdot \sin(\omega t - \frac{4\pi}{3})$$

In this type of study, we must not lose sight of the fact that we are dealing with a balanced three-phase network and that the simple voltages are 120° out of phase with each other.

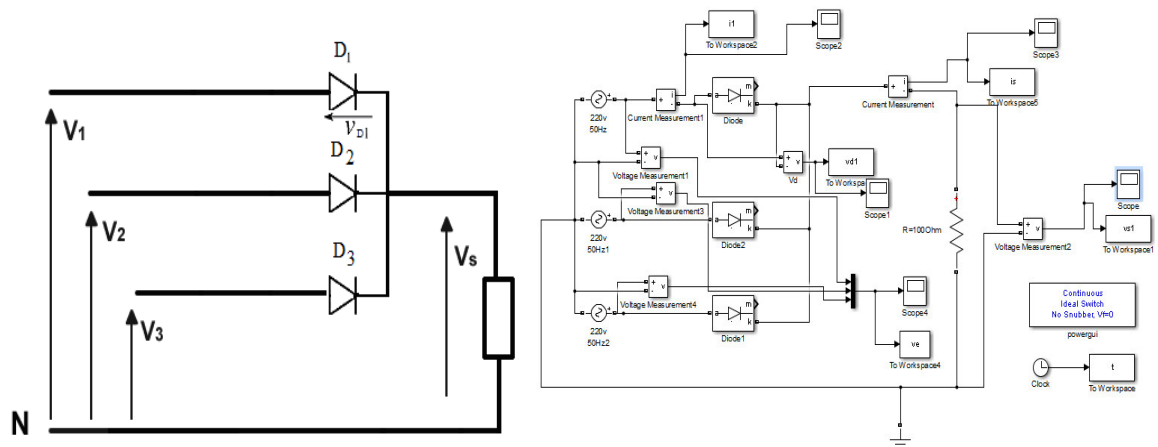


Figure 3.18 All-diode P3 circuit

3.3.3.1.b. Voltage and current patterns on the secondary circuit

At the given instant, the diode connected to the higher potential is conducting, and its anode is at the higher potential [3,4,5].

$$V_s = \sup(V_1, V_2, \text{ and } V_3)$$

We therefore have:

$$V_s = V_1 \text{ when } V_1 > V_2 \text{ and } V_3$$

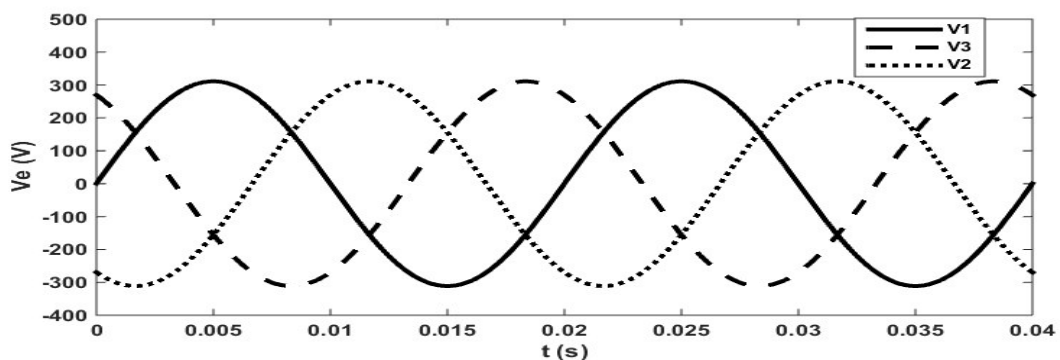
$$V_s = V_2 \text{ when } V_1 < V_2 \text{ and } V_3$$

$$V_s = V_3 \text{ when } V_1 < V_3 \text{ and } V_2$$

The different operating phases of the circuit are then described in the following table:

Intervals	Passing diode	Voltages across blocked diodes	Rectified voltage
$\pi/6 < t < 5\pi/6$	D <sub>1</sub>	$V_{D2} = V_{D1} - V_1 + V_2 = V_2 - V_1$	$V_s = V_1$
$5\pi/6 < t < 3\pi/2$	D <sub>2</sub>	$V_{D2} = 0$	$V_s = V_2$
$3\pi/2 < t < 2\pi$	D <sub>3</sub>	$V_{D2} = V_{D3} - V_3 + V_2 = V_2 - V_3$	$V_s = V_3$

The timing diagrams of the input, output, diode and current voltages are given in figure 3.19.



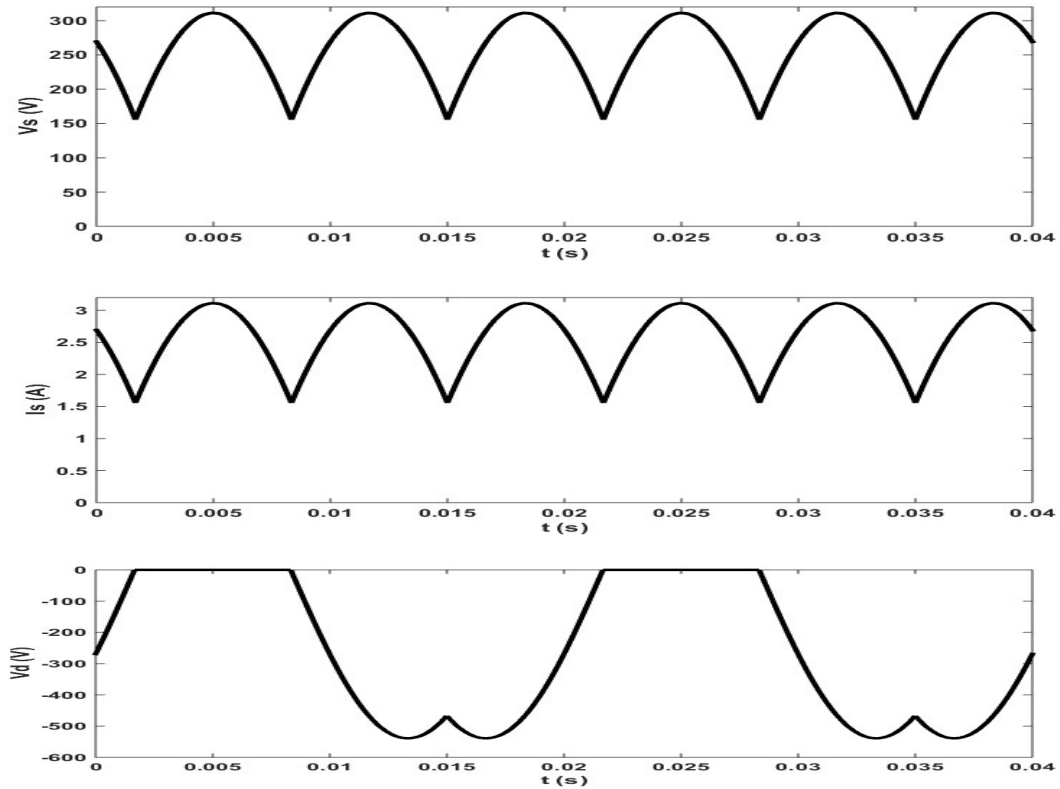


Figure 3.19 Timing diagrams of the input, output, diode, and current voltages.

### 3.3.3.1.c. Average and effective value of the output signal:

- The average value of the rectified voltage is given by:

$$V_{savg} = \frac{3}{T} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} V_s(\theta).d\theta = \frac{3V_{max}}{2\pi} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \sin(\theta).d\theta = \frac{3V_{max}}{2\pi} [-\cos\theta]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} = \frac{3\sqrt{3}V_{max}}{2\pi} = 0.826.V_{max}$$

$$I_{avg} = \frac{V_{savg}}{R} = 0.826.I_{max}$$

with  $I_{max} = \frac{V_{max}}{R}$

Effective value:

$$V_{eff} = \sqrt{\frac{3}{2\pi} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (V_{max} \sin(\theta))^2 .d\theta} = \sqrt{\frac{3V_M^2}{2\pi} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (V_{max} \sin(\theta))^2 .d\theta} + \sqrt{\frac{3V_{max}^2}{2\pi} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(\frac{1-\cos(2\theta)}{2}\right).d\theta} = 0.84.V_{max}$$

or :  $I_{eff} = 0.84.I_{max}$

### 3.3.3.1.d. Form Factor

$$F = \frac{V_{eff}}{V_{avg}} = 1.016$$

3.3.3.1.e. Ripple Rate factor

$$\tau = \sqrt{F^2 - 1} = 0.18$$

The maximum reverse voltage that the diodes can withstand is:

$$V_{DM} = V_{D2}(\theta = \frac{\pi}{3}) = -\sqrt{3}V_{\max}$$

3.3.3.1.f. Inductive load RL

the voltage shapes remain unchanged but the current follows the charge

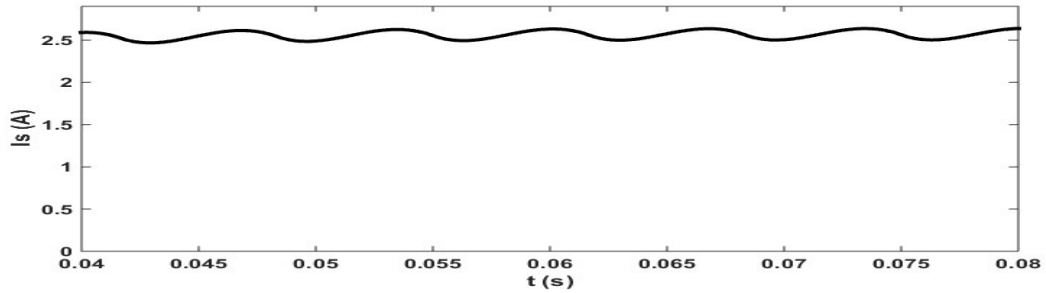


Figure 3.20 Timing diagram of the output current.

3.3.3.2. Double parallel or bridge rectification: PD3 circuit

3.3.3.2.a. Resistive load R:

The three-phase bridge rectifier is identical to the single-phase bridge rectifier except that it has three additional diodes connected to each phase (Fig. 3.21). This rectifier is also known as a Graetz bridge [3, 4, 5].

This rectifier is the prototype of the industrial rectifier. The transformer secondary is star-connected and connected to two groups of diodes: a common-cathode switch (D1, D2, D3) and a common-anode switch (D1', D2', D3').

The existence of a direct current in the load requires the conduction of two diodes at all times, one from each switch.

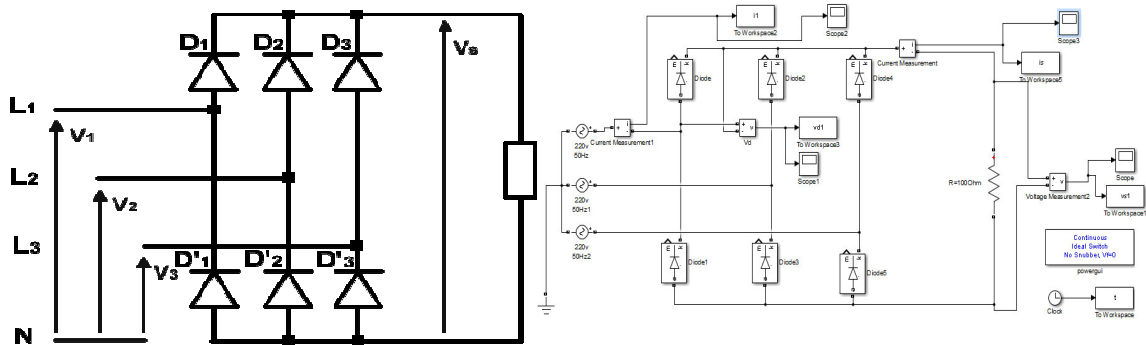


Figure 3.21 All-diode PD3 circuit

When  $V_1 > V_2 > V_3$ , D1, D'3 conduct:  $V_s = V_1 - V_3$

### Chapter 3: Conversion of AC-DC electrical energy

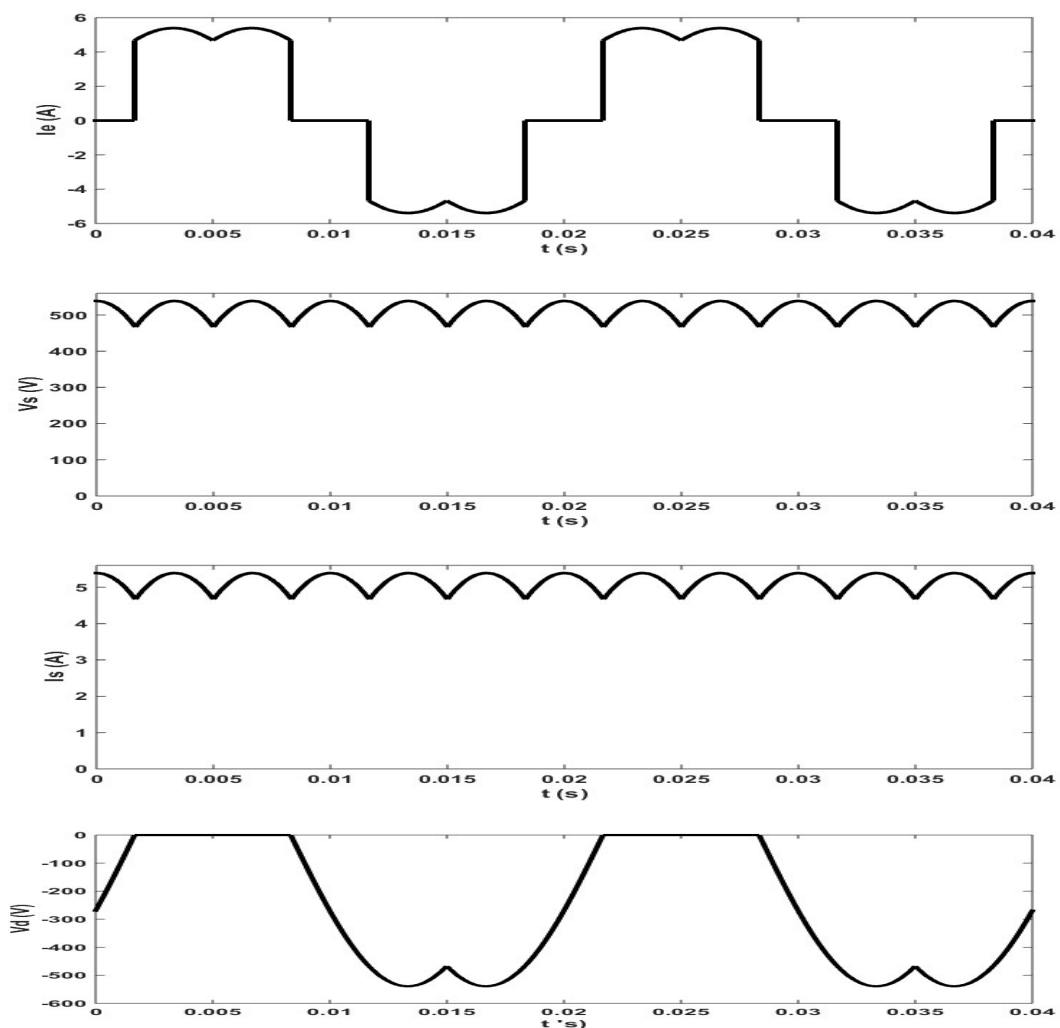
When  $V_1 > V_3 > V_2$ ,  $D_1, D'_2$  conduct:  $V_s = V_1 - V_2$

When  $V_2 > V_1 > V_3$ ,  $D_2, D'_3$  conduct:  $V_s = V_2 - V_3$

Intervals	Passing diode	Voltages across diode D1	Rectified voltage
$0 < t < \pi/6$	$D_3, D'_2$	$V_{D1} = V_1 - V_3$	$V_s = V_3 - V_2$
$\pi/6 < t < \pi/2$	$D_1$ et $D'_2$	0	$V_s = V_1 - V_2$
$\pi/2 < t < 5\pi/6$	$D_1, D'_3$	0	$V_s = V_1 - V_3$
$5\pi/6 < t < 7\pi/2$	$D_2, D'_3$	$V_{D1} = V_1 - V_2$	$V_s = V_2 - V_3$
$7\pi/2 < t < 3\pi/2$	$D_2, D'_1$	$V_{D1} = V_1 - V_2$	$V_s = V_2 - V_1$
$3\pi/2 < t < 11\pi/6$	$D_3, D'_1$	$V_{D1} = V_1 - V_3$	$V_s = V_3 - V_1$
$11\pi/6 < t < 2\pi$	$D_3, D'_2$	$V_{D1} = V_1 - V_3$	$V_s = V_3 - V_2$

The three diodes  $D_1, D_2$ , and  $D_3$  form a more positive switch, which allows the more positive voltage to pass through at all times, and diodes  $D'_1, D'_2$ , and  $D'_3$  form a more negative switch, which allows the more negative voltage to pass through.

The rectified voltage at any time is the difference between these two voltages, i.e.:



**Figure 3.22** Timing diagrams of input, output, diode, and current voltages.

### ► Average value :

Average value of a three-phase Graetz bridge:

$$V_{avg} = \frac{6}{T} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} V_{s2}(\theta) d\theta = \frac{6}{T} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} V_{max} \left( \sin\left(\theta - \frac{4\pi}{3}\right) - \sin\left(\theta - \frac{2\pi}{3}\right) \right) d\theta = \frac{6}{2\pi} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} V_{max} \sqrt{3} \cos(\theta) d\theta = \frac{3\sqrt{3}}{\pi} V_{max} = 1.654 V_{max}$$

### ► Effective value

$$V_{eff} = \sqrt{\frac{6}{2\pi} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (V_{max} \sqrt{3} \cos(\theta))^2 d\theta} = \sqrt{\frac{9V_{max}^2}{\pi} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1 + \cos(2\theta)}{2} d\theta} = \sqrt{V_{max}^2 \left( \frac{3}{2} + \frac{9}{2\pi} \sin\left(\frac{\pi}{3}\right) \right)} = 1.655 V_{max}$$

### ► Form factor

$$F = \frac{V_{eff}}{V_{avg}} = 1.001$$

### ► Ripple rate factor:

$$\tau = \sqrt{F^2 - 1} = 0.044$$

#### 3.3.3.2.b. Inductive load RL:

Figure 3.23 shows a three-phase full-wave rectifier circuit where no neutral is necessary and it will be seen that two series diodes are always conducting [3, 4, 5].

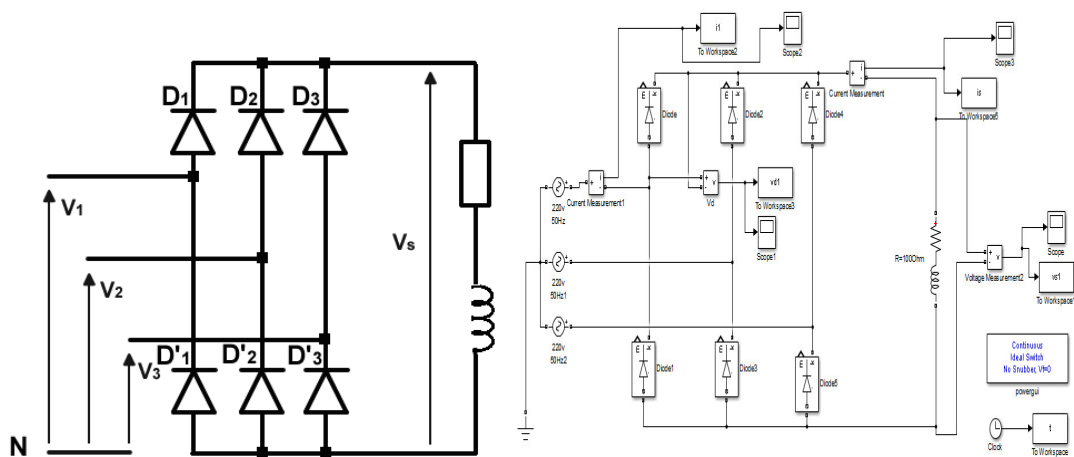


Figure 3.23 All-diode PD3 circuit on an inductive load

To draw the current across the load and source, simply consider that any conducting diode (known from the conduction diagram) is equivalent to a closed switch.

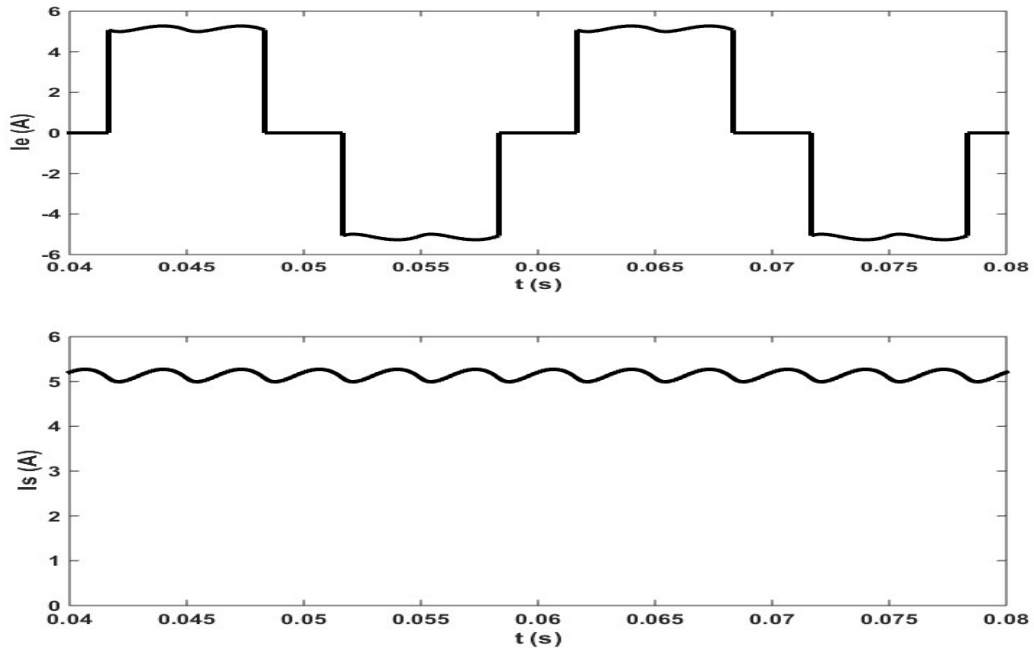


Figure 3.24 Timing diagrams of input, output current.

### 3.3.3.2.c. General case of rectifier circuits::

To obtain a DC voltage, a set of  $q$  AC voltages is rectified, usually assumed to be sinusoidal and form a balanced polyphase system (number of phases  $q$ ). These voltages can be the terminal voltages of an alternator. Generally, they are supplied by the single-phase network or, more often, by the three-phase network, usually via a transformer [3,4,5].

There are three types of circuits:

1. **Pq**: circuits with a star connected source and a single switch or half-wave rectifier;
2. **PDq**: circuits with a star connected source and two switches or bridge rectifiers;
3. **Sq**: circuits with a delta connected source and two switches or bridge rectifiers with a polygon source.

#### ► Form Factor

The value of the form factor  $F$  calculated for some values of  $q$ , i.e [3, 4, 5].:

$$V_{savg} = \frac{q}{T} \int_{\frac{T}{4} - \frac{T}{2q}}^{\frac{T}{4} + \frac{T}{2q}} v(t).dt = \frac{q}{\pi} V_{max} \cdot \sin\left(\frac{\pi}{q}\right)$$

The effective value is:

$$V_{seff} = \sqrt{\frac{q}{T} \int_{\frac{T}{4} - \frac{T}{2q}}^{\frac{T}{4} + \frac{T}{2q}} v^2(t).dt} = \frac{V_{max}}{\sqrt{2}} \sqrt{1 + \frac{q}{2\pi} \cdot \sin\left(\frac{2\pi}{q}\right)}$$

Thus the form factor:

### Chapter 3: Conversion of AC-DC electrical energy

$$F = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{1 + \frac{q}{2\pi} \cdot \sin\left(\frac{2\pi}{q}\right)}}{\frac{q}{\pi} \cdot \sin\left(\frac{\pi}{q}\right)}$$

<b>q</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>6</b>	<b>12</b>	<b>18</b>
<b>F</b>	<b>1.2716</b>	<b>1.02</b>	<b>1.01</b>	<b>1.001</b>	<b>1.00005</b>	<b>1</b>

As q values increase, the shape value improves, meaning the rectified voltage is almost continuous.

#### ► Power Factor

By definition, in sinusoidal conditions, the secondary power factor is the ratio of the active power available at the output of the transformer to the apparent power developed in the transformer windings [3, 4, 5]:

$$F_s = \frac{P_s}{S} = \frac{P_{red}}{S}$$

with

$$S = q \cdot v_{seff} \cdot I_{seff}$$

We have seen that the effective current is given by:

$$I_{seff} = \frac{I_{red}}{\sqrt{q}}$$

So

$$S = \sqrt{q} \cdot v_{seff} \cdot I_{red}$$

and

$$F_s = \frac{P_{red}}{S} = \frac{U_{red} \cdot I_{red}}{S} = \frac{\frac{q}{\pi} \cdot V_{max} \cdot \sin\left(\frac{\pi}{q}\right) \cdot I_{red}}{\sqrt{q} \cdot \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{red}}{\sqrt{q}}} = \frac{\sqrt{2} \cdot q}{\pi} \cdot \sin\left(\frac{\pi}{q}\right)$$

<b>q</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>6</b>	<b>12</b>	<b>18</b>
<b>F<sub>s</sub></b>	<b>0.637</b>	<b>0.675</b>	<b>0.637</b>	<b>0.55</b>	<b>0.4</b>	<b>0.332</b>

We note that increasing the number of phases improves the form factor, but the power factor is low and decreases.

3.3.4. Controlled rectifiers (Thyristors)

3.3.4.1. Controlled single-phase rectification

A controlled rectifier circuit allows for an adjustable DC voltage to be obtained from a sinusoidal AC voltage. The use of components such as thyristors allows the construction of rectifiers whose average output voltage can vary depending on the delay angle [3,4,5].

3.3.4.2. Single-phase half-wave rectification:

3.3.4.2.a. Pure resistive load R:

The thyristor is considered perfect;  $\alpha$  is called the delay angle [3,4,5].

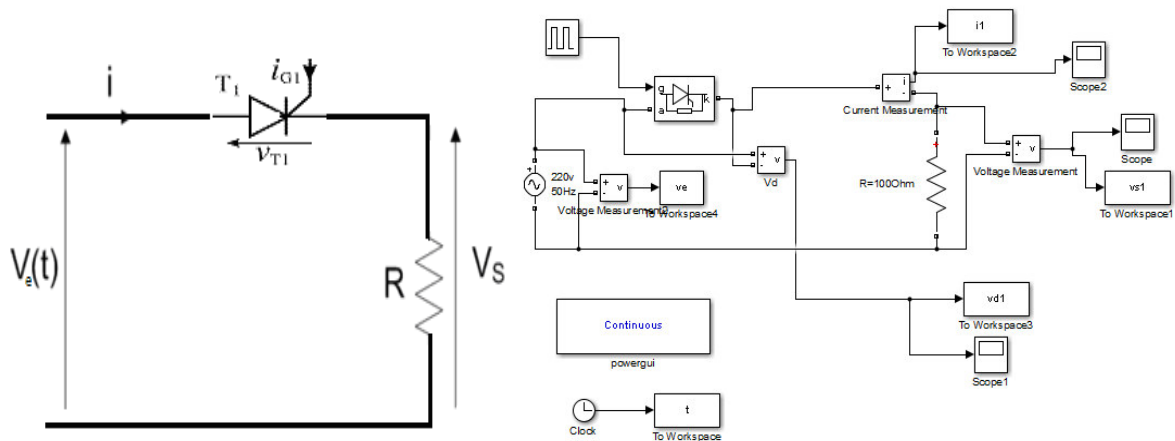


Figure 3.25 Assembly of a single-phase rectifier supplying a purely resistive load

The delay pulses are sent with a delay relative to the mains zero by an angle  $\alpha$ . Thus, the thyristor fires at times  $\alpha, 2\pi + \alpha, 4\pi + \alpha$ , etc.

► Operational Analysis

- $0 < t < \alpha$

The thyristor is off.

$$V_s = 0 \text{ and } V_{T1} = V_e - V_s = V_{\max} \sin(\omega t)$$

- $\alpha < t < T/2$

T1 is conducting.

$$V_s = V_e \text{ and } V_{T1} = V_e - V_s = 0$$

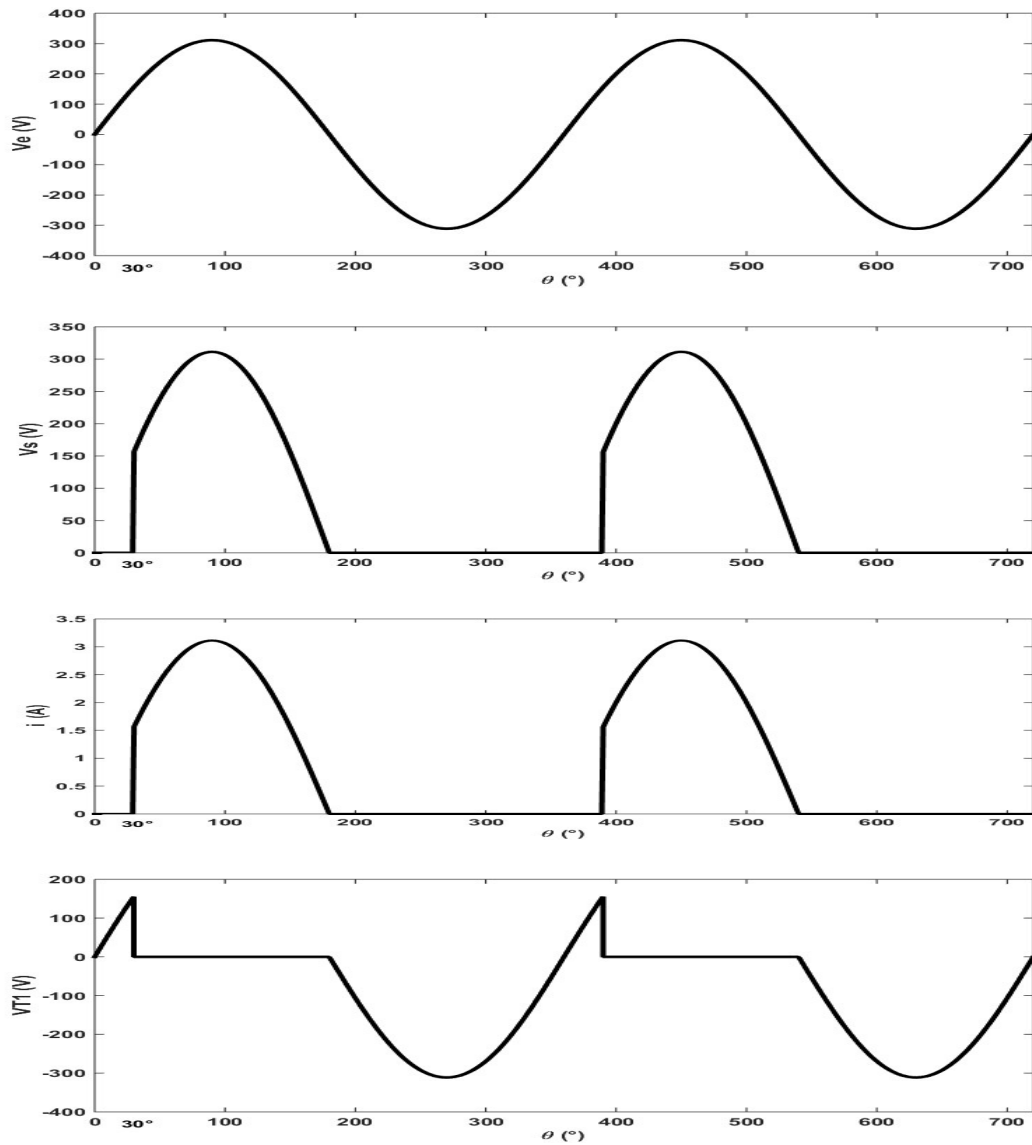
- $T/2 < t < T$

The thyristor is off.

$$V_s = 0.$$

$$V_{T1} = V_e - V_s = V_{\max} \sin(\omega t)$$

The voltage timing diagram for a half-wave controlled rectification on a resistive load for  $\alpha = 30^\circ$  is given in figure 3.26.



**Figure 3.26** Voltage timing diagram of a single-wave controlled rectification on a resistive load for  $\alpha = 30^\circ$

► **Voltage and current calculation**

- The average value of the rectified voltage is given by:

$$V_{savg} = \frac{1}{T} \int_0^T V_s(\theta) \cdot d\theta = \frac{1}{2\pi} \int_{\alpha}^{\pi} V_{max} \sin(\theta) \cdot d\theta = \frac{V_{max}}{\pi} [-\cos\theta]_{\alpha}^{\pi} = \frac{V_{max}}{2\pi} [-\cos(\pi) + \cos\alpha] = \frac{V_{max}}{2\pi} [1 + \cos\alpha]$$

The average value of the output voltage  $V_s$  varies depending on the value of the delay angle  $\alpha$ .. The average value of the current  $I_{avg}$  is therefore:

$$I_{savg} = \frac{V_{max}}{2 \cdot \pi \cdot R} [1 + \cos\alpha]$$

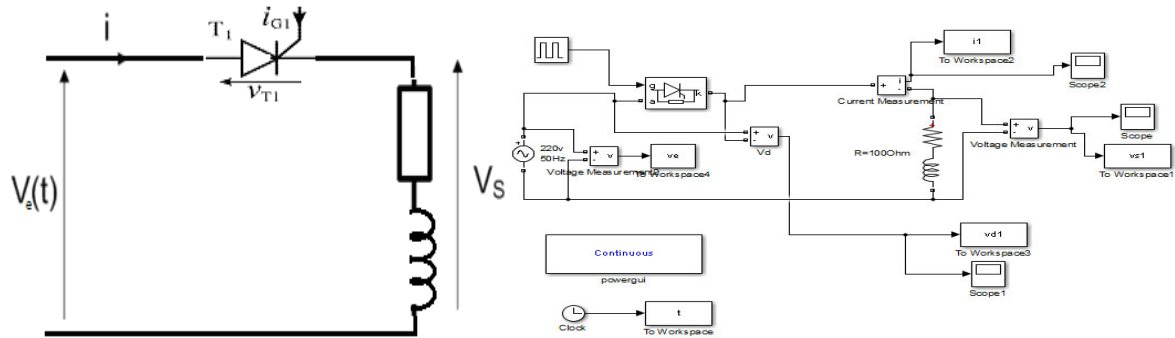
### Chapter 3: Conversion of AC-DC electrical energy

-The effective voltage at the terminals of the load:

$$V_{eff} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi} (V_{max} \sin(\theta))^2 d\theta} = \sqrt{\frac{V_{max}^2}{2\pi} \int_{\alpha}^{\pi} \sin^2(\theta) d\theta} = \sqrt{\frac{V_{max}^2}{2\pi} \left[ \frac{\theta}{2} - \frac{\sin(2\theta)}{4} \right]_{\alpha}^{\pi}} = V_{max} \sqrt{\frac{1}{2\pi} \left[ \frac{\pi}{2} - \frac{\alpha}{2} + \frac{\sin(2\alpha)}{4} \right]}$$

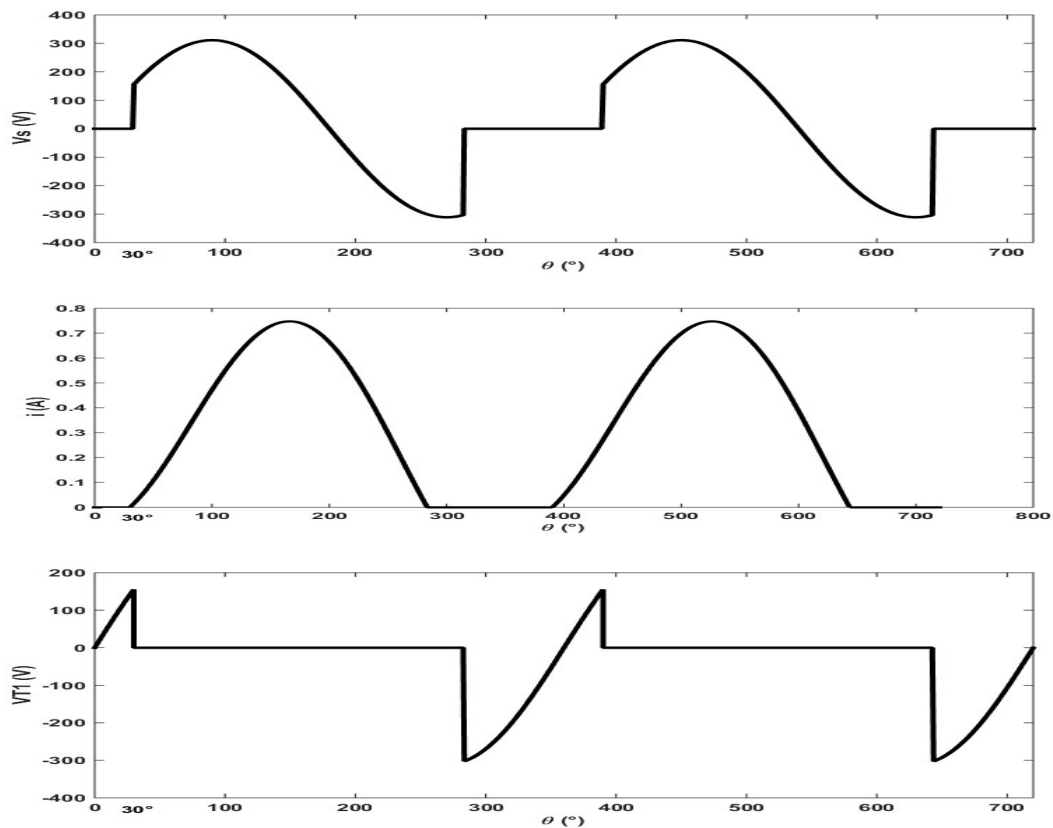
#### 3.3.4.2.b. Inductive Load:

In electrical engineering, loads are often combined: inductive and resistive. The diagram enabling this new study is shown in figure 3.27 [3, 4, 5].



**Figure 3.27** Assembly of a single-phase rectifier supplying an inductive load

The voltage and current shapes are given in figure 3.28.



**Figure 3.28** Voltage timing diagram of a single-wave controlled rectification on an inductive load

### Chapter 3: Conversion of AC-DC electrical energy

Before the instant of application of the control pulse at  $t = t_0$ , the thyristor is blocked  $i(t_0) = 0$ , hence:  $V_s = 0$ ; but when the pulse is applied: the thyristor is closed  $V_{TH} = 0$

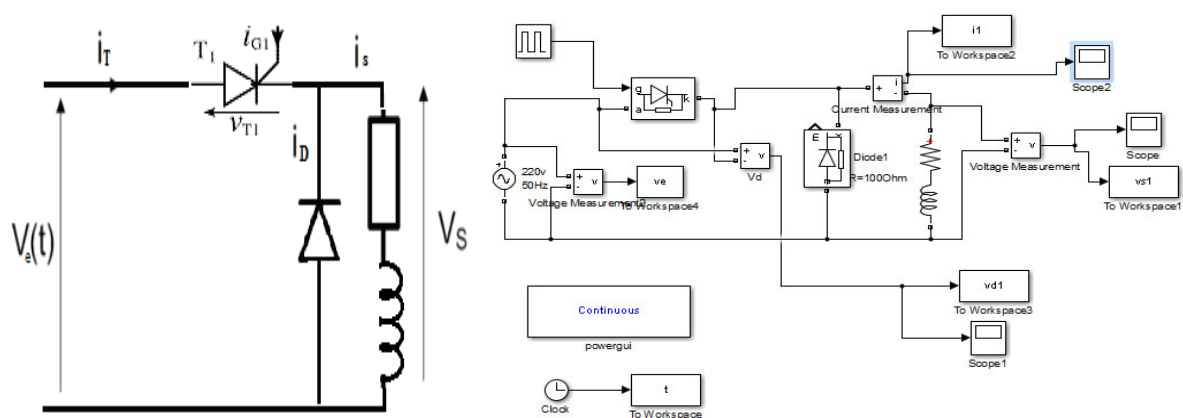
$$L \frac{di}{dt} + Ri = V_{\max} \sin(\omega t)$$

if  $i(t_0)=0$  so :  $\Rightarrow$

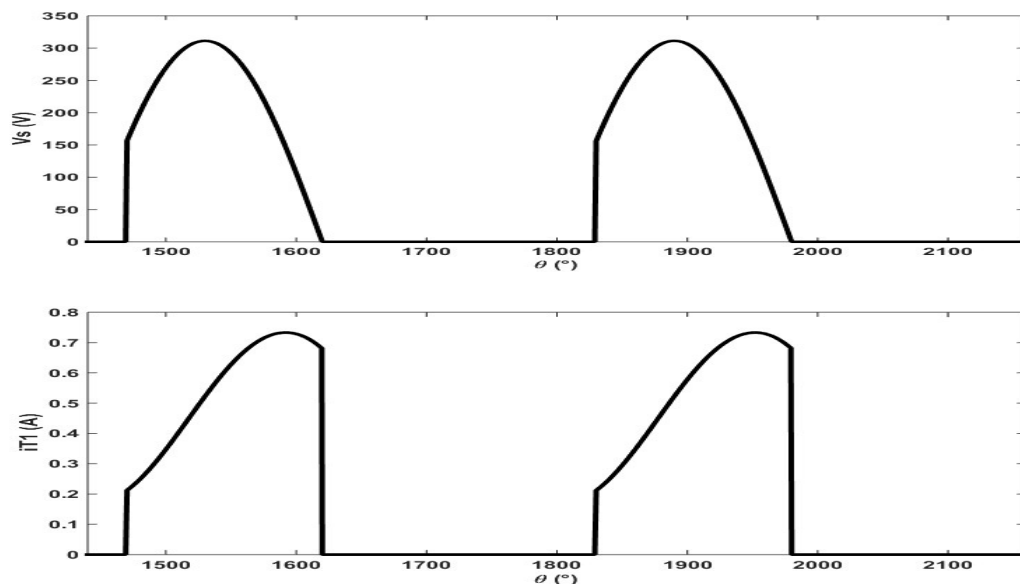
$$i(t) = \frac{V_{\max}}{\sqrt{(L\omega)^2 + R^2}} \left[ -\sin(\omega t_0 - \varphi) e^{-\frac{R(t-t_0)}{L}} + \sin(\omega t - \varphi) \right]$$

Therefore, the thyristor will only turn off when the current is zero.

To prevent the appearance of a negative part in the input voltage, a freewheeling diode is used, connected in parallel with the inductive load, as shown in figure 3.29 [3, 4, 5].



**Figure 3.29** Assembly of a single-phase input, output and current rectifier.



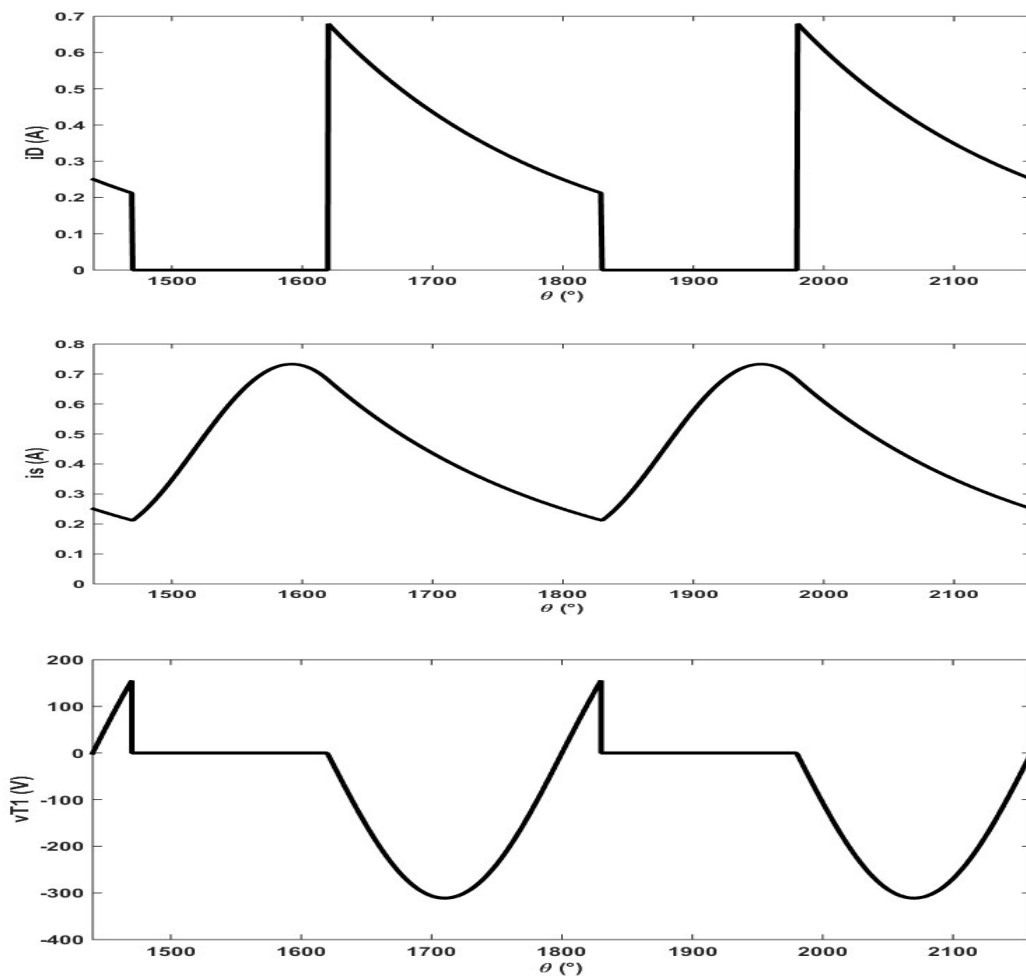


Figure 3.30 Voltage timing diagrams supplying an inductive load with a freewheeling diode

### 3.3.4.3. Single-phase full-wave rectification

#### 3.3.4.3.a. Resistive load R:

The two voltages in phase opposition are obtained using a center-tapped transformer [3, 4, 5].

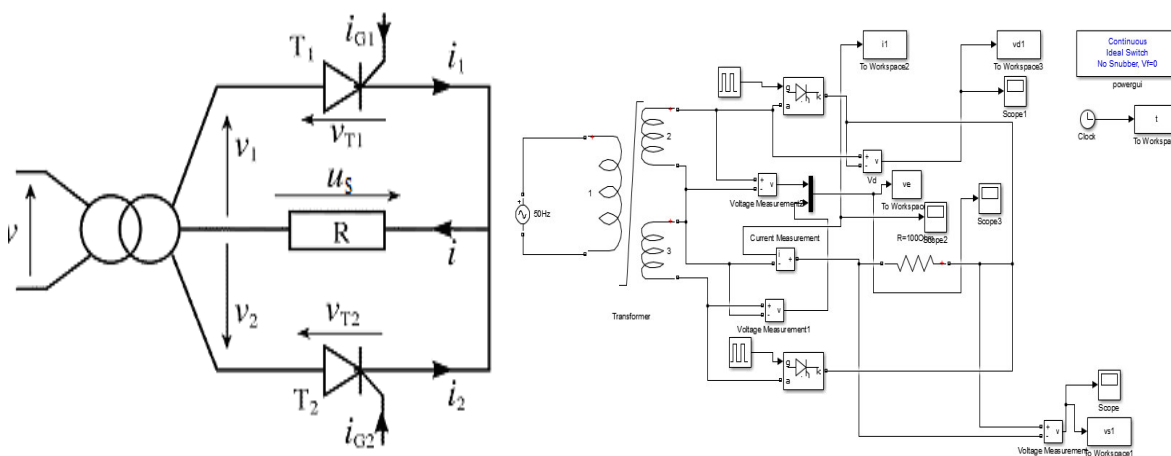


Figure 3.31 Assembly of a single-phase full-wave rectifier supplying a resistive load

### ► Operational Analysis

- $0 < \theta < \alpha$

Thyristors Th1 and TH2 are off.

$$V_s=0 \text{ and } V_{T1} = V_e - V_s = V_{\max} \sin(\omega t)$$

- $\alpha < \theta < \pi$

T1 is conducting and T2 is off.

$$V_s=V_e \text{ and } V_{T1} = V_e - V_s = 0$$

- $\pi < \theta < \pi + \alpha$

Thyristors Th1 and TH2 are off.

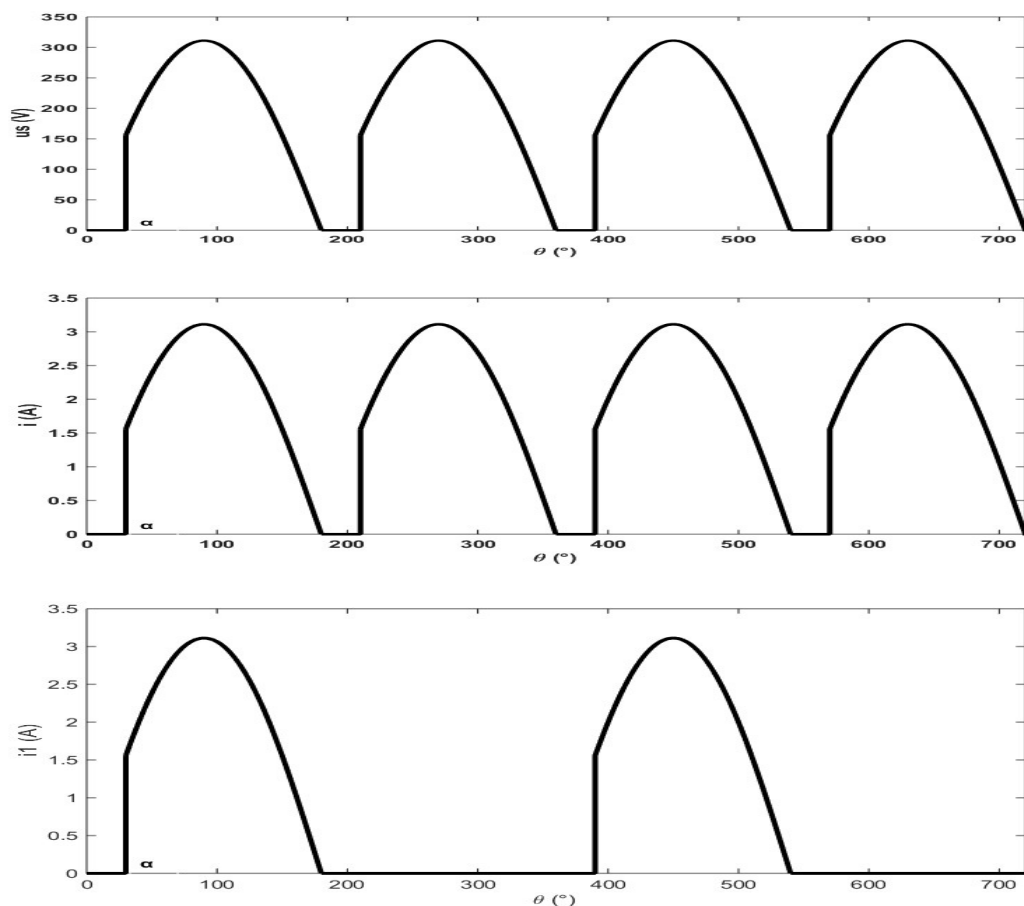
$$V_s=0 \text{ and } V_{T1} = V_e - V_s = V_{\max} \sin(\omega t)$$

- $\pi + \alpha < \theta < 2\pi$

T2 is conducting and T1 is off.

$$V_s=V_e \text{ and } V_{T1} = V_e - V_s = 2V_{\max} \sin(\omega t)$$

The voltage timing diagram for a full-wave controlled rectification on a resistive load for  $\alpha=30^\circ$  is given in figure 3.32.



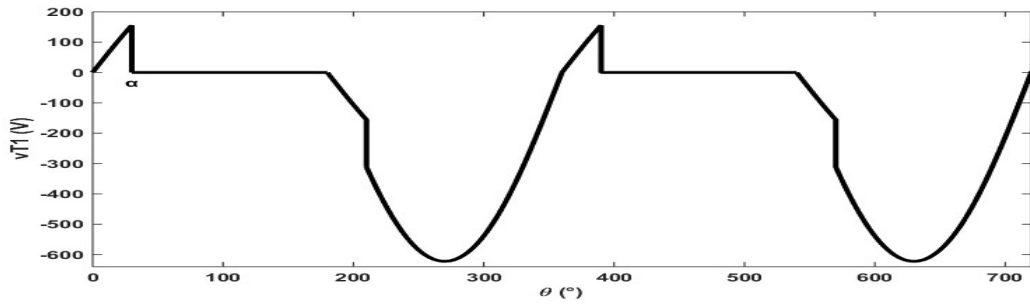


Figure 3.32 Voltage and current timing diagram for a full-wave rectifier on a resistive load

► Voltage and current calculation:

$$V_{savg} = \frac{1}{T} \int_0^T V_s(\theta).d\theta = \frac{2}{T} \int_0^{\pi} V_s(\theta).d\theta = \frac{V_{max}}{\pi} \int_{\alpha}^{\pi} \sin(\theta).d\theta = \frac{V_{max}}{\pi} (1 + \cos(\alpha))$$

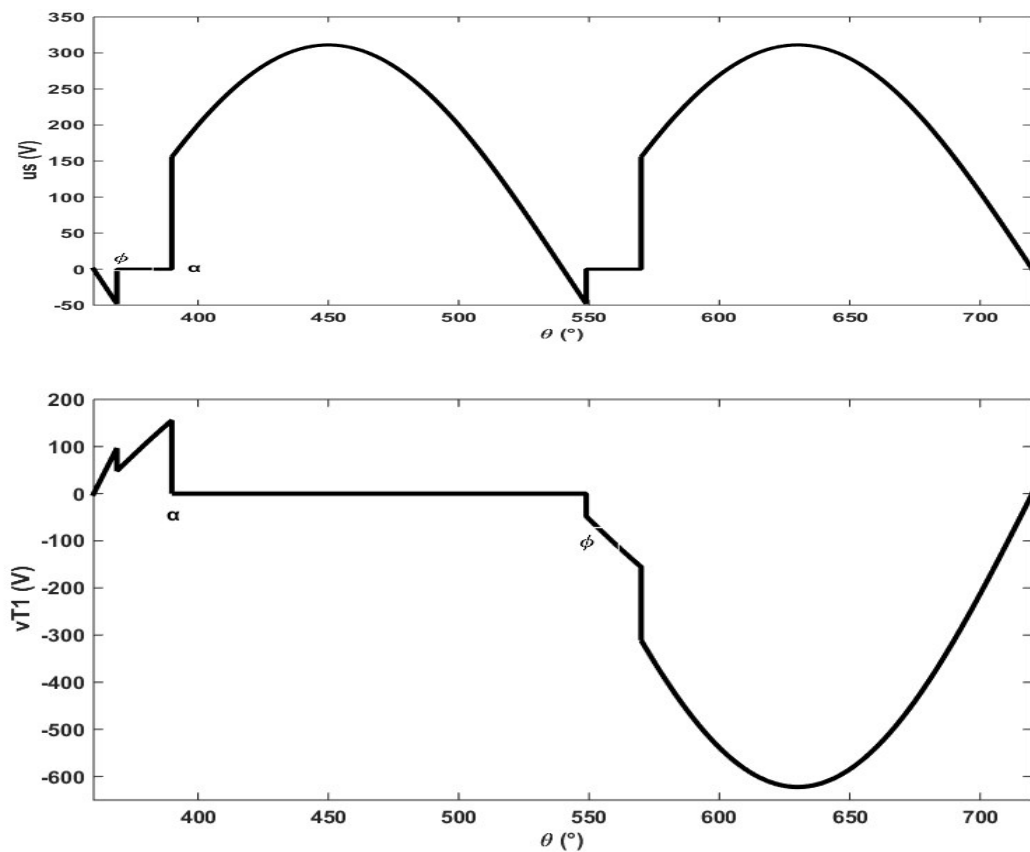
$$I_{savg} = \frac{V_{max}}{\pi.R} (1 + \cos(\alpha))$$

3.3.4.3.b. Inductive load:

There are two cases:

1 case where the extinction angle is less than the delay angle  $\alpha > \phi$ .

The voltage and current curves are as follows.



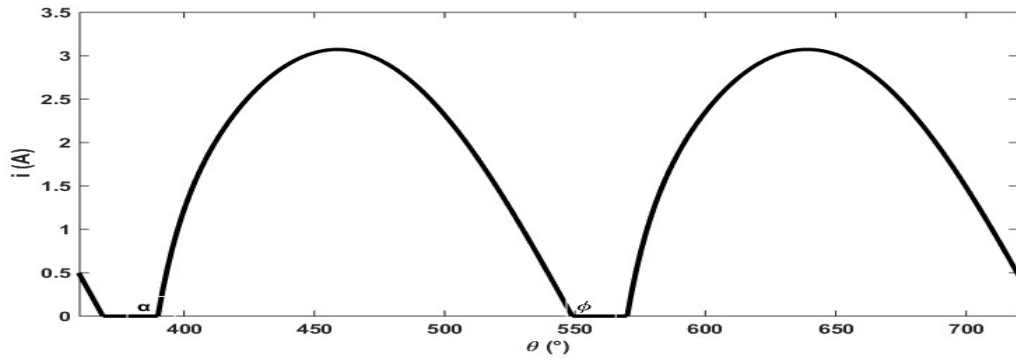


Figure 3.33 Voltage and current timing diagram for a full-wave rectifier on an inductive load

**2st Case where the extinction angle is equal to the delay angle  $\alpha \leq \phi$ .**

The current flowing through the thyristor does not cancel out; it remains conductive even when  $V_1$  becomes negative. The thyristor turns off when the second thyristor fires.

If the inductance is large enough, the ripple  $i_L$  (representing the difference between the maximum and minimum currents) becomes negligible, and the current is then considered constant [3, 4, 5].

For the secondary circuit, the general circuit equations are as follows:

- Semiconductor properties:  $i_{TH} > 0 \rightarrow V_{TH} = 0$
- Mesh law:  $V_1 = V_{TH1} + V_S$ ,  $V_2 = V_{TH2} + V_S$
- Node law:  $i_{TH1} + i_{TH2} = i_S$
- Ohm's law across the load terminals:

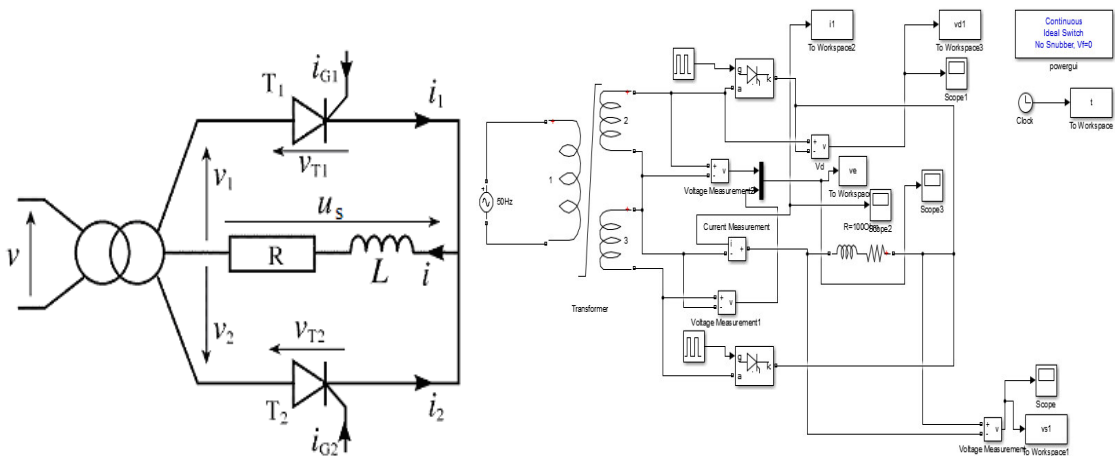


Figure 3.34 Assembly of a single-phase full-wave rectifier supplying an inductive load.

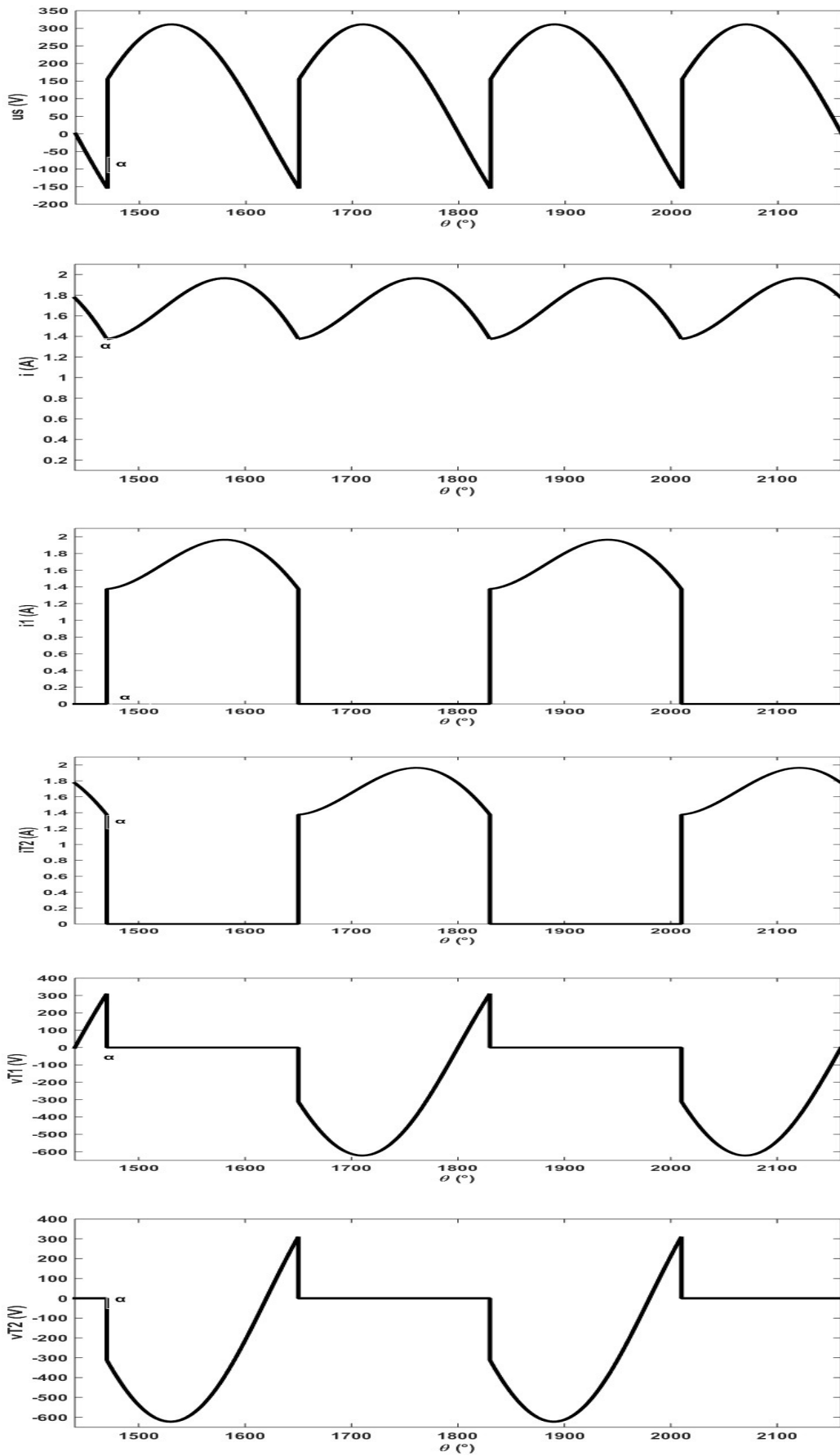


Figure 3.35 Voltage timing diagram for a full-wave rectifier on an inductive load

3.3.4.4. PD2 full-wave rectifier with thyristors:

3.3.4.4.a. Resistive load:

The PD2 thyristor rectifier circuits consists of four thyristors [3, 4, 5]:

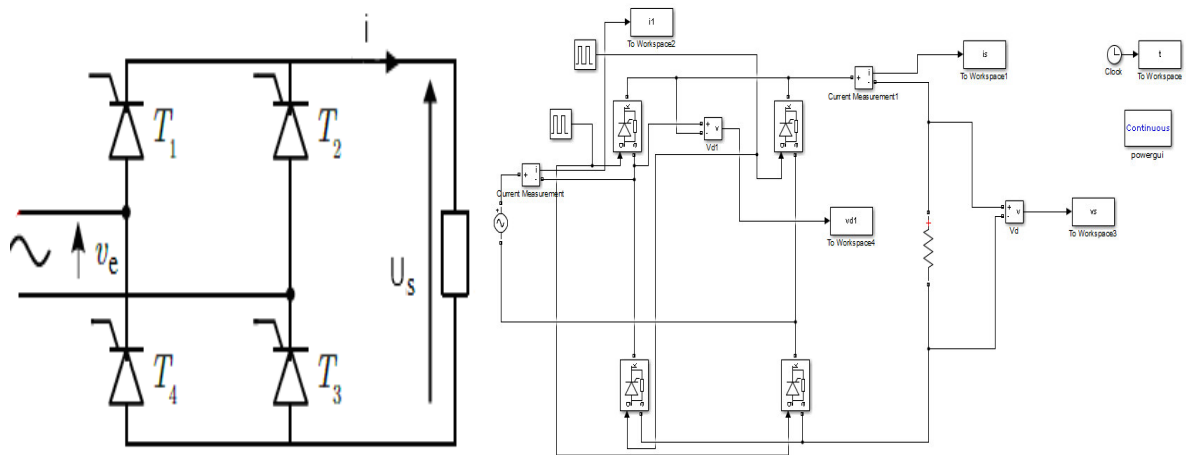
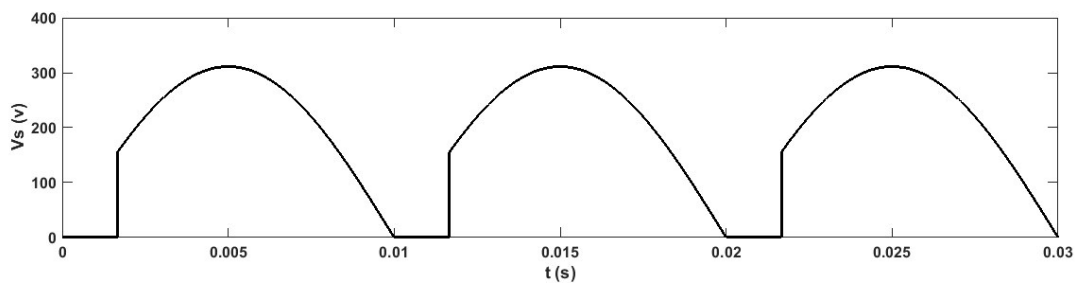
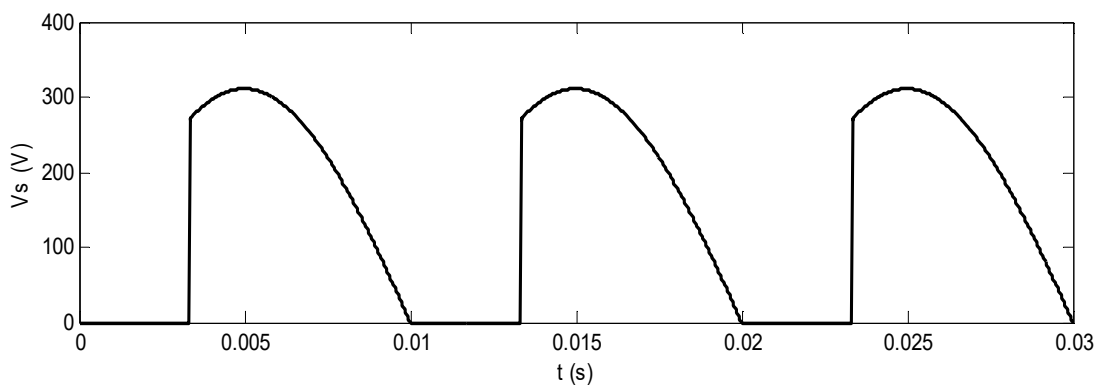


Figure 3.36 Assembly of a single-phase full-wave rectifier PD2 supplying a resistive load.

$\alpha = \pi/6$  :



$\alpha = \pi/3$  :



$\alpha = \pi/2 :$

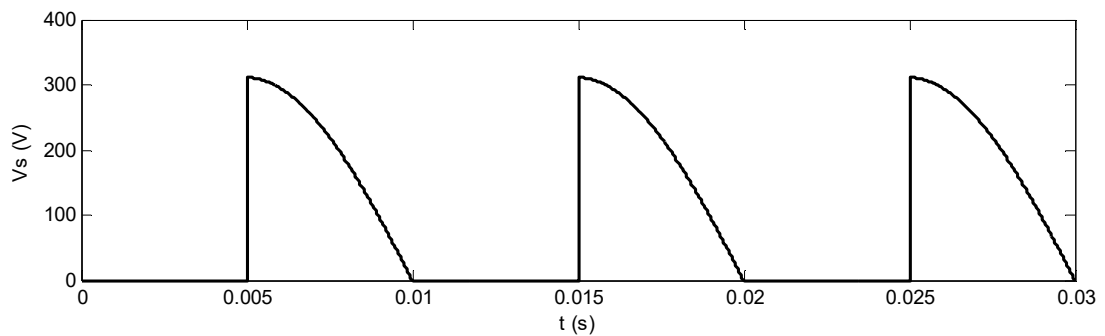


Figure 3.37 Voltage timing diagram of a PD2 full-wave rectifier on a resistive load

**3.3.4.4.b. Inductive load:**

We will assume that the load connected to the rectifier is such that the current never equals zero during the period, so there are always thyristors conducting (continuous conduction assumption) [3, 4, 5].

During the positive half-wave, thyristors Th1 and Th3 are triggered at time  $\alpha$ , thus  $V_s = V_e$ . Thyristors Th1 and Th3 continue to conduct even after the mains voltage reverses since the current is not interrupted.

At time  $\alpha + \pi$ , a trigger pulse is sent to the gates of Th2 and Th4. These trigger since the applied voltage  $V_{AK}$  is positive. Their triggering causes the anode current to be extracted from Th1 and Th3 and their voltage  $V_{AK}$  to reverse, thus blocking them. Under these conditions,  $V_s = -V_e$ .

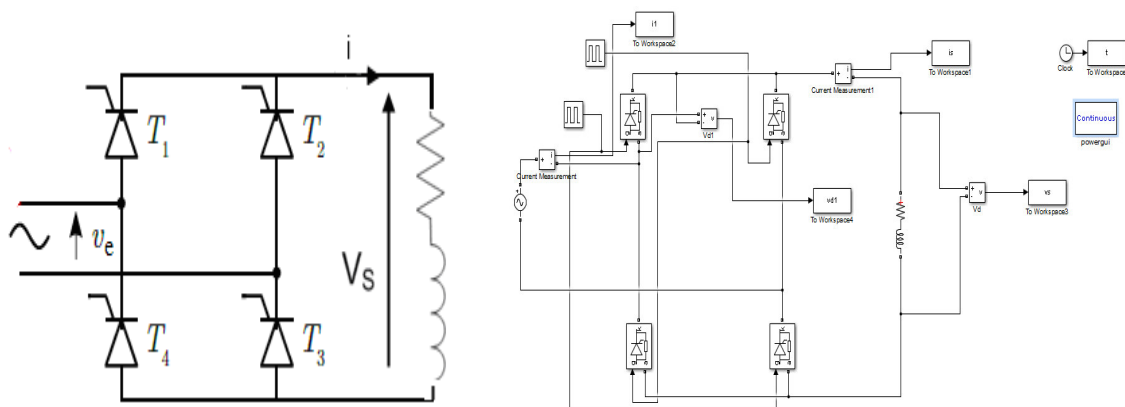
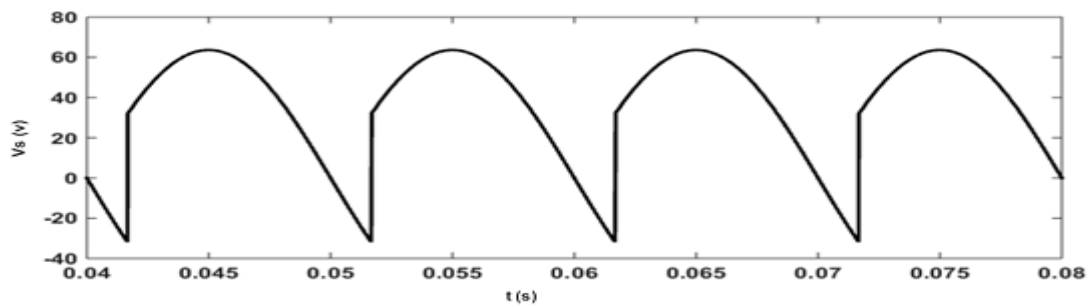


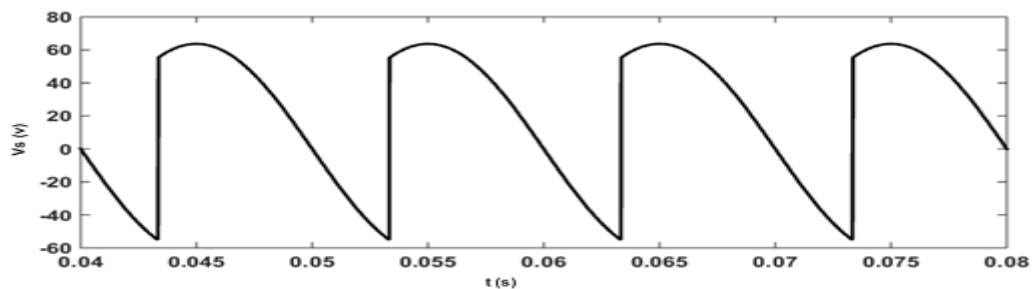
Figure 3.38: Assembly of a single-phase full-wave rectifier supplying an inductive load.

The following figures show the voltage timing diagrams for different firing angles.

$\alpha = \pi/6$  :



$\alpha = \pi/3$  :



$\alpha = \pi/2$  :

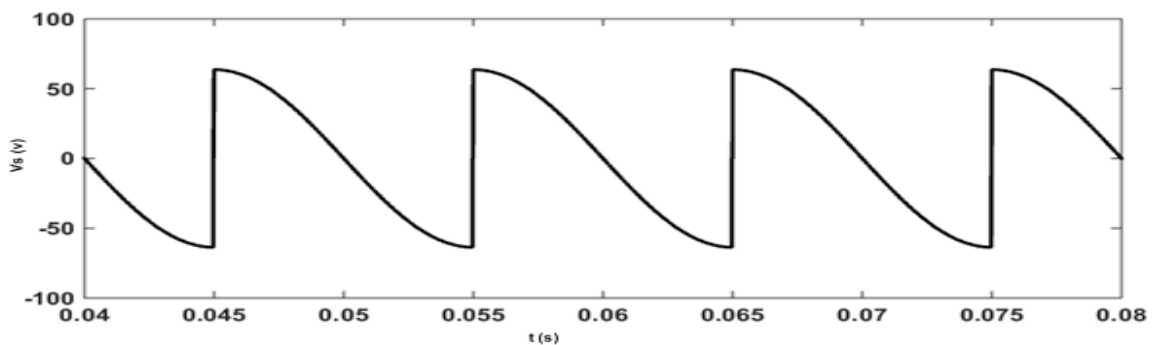


Figure 3.39 Voltage timing diagram for a full-wave rectification on an inductive load

► **Average value recovered:**

We can demonstrate that this average value, which obviously depends on the angle  $\alpha$ , can be written as:

$$V_{avg} = \frac{1}{T} \int_0^T V_s(\theta).d\theta = \frac{1}{\pi} \int_{\alpha}^{\pi+\alpha} V_{max} \sin(\theta).d\theta = \frac{V_{max}}{\pi} [-\cos(\theta)]_{\alpha}^{\pi+\alpha} = \frac{V_{max}}{\pi} [-\cos(\pi + \alpha) + \cos(\alpha)] = \frac{2V_{max}}{\pi} \cos(\alpha)$$

Note that for values of the conduction delay angle  $\alpha$  less than  $\pi/2$ , the average value recovered is positive. If  $\alpha$  exceeds  $\pi/2$ , this average value becomes negative. The circuit now operates as a non-autonomous (or assisted) inverter: the energy transfers from the DC side to the AC side [3,4,5].

► **Voltage across the thyristors:**

Thyristors Th1 and Th4 switch simultaneously; we then write  $V_{Th1}=V_{Th4}=(V_e-V_s)/2$ .

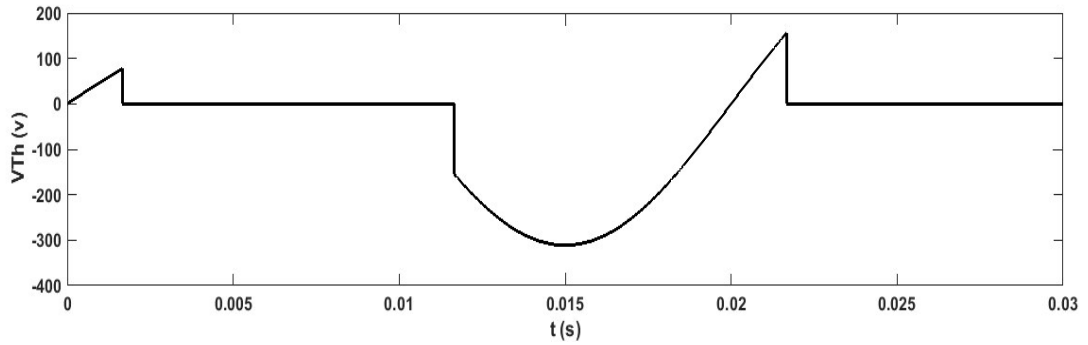


Figure 3.40 Thyristor voltage timing diagram

The thyristors must therefore withstand a reverse voltage  $V_1 = V_{\max}$ .

### 3.3.4.5. Full-wave rectifier with mixed bridge

We consider a bridge rectifier circuit for which there are three possible circuits (Figure 3.41). The delay pulses are sent with a delay relative to the mains zero by an angle  $\alpha$ . Thus, the thyristor fires at times  $\alpha, \pi + \alpha, 2\pi + \alpha, \dots$ , etc [3,4,5].

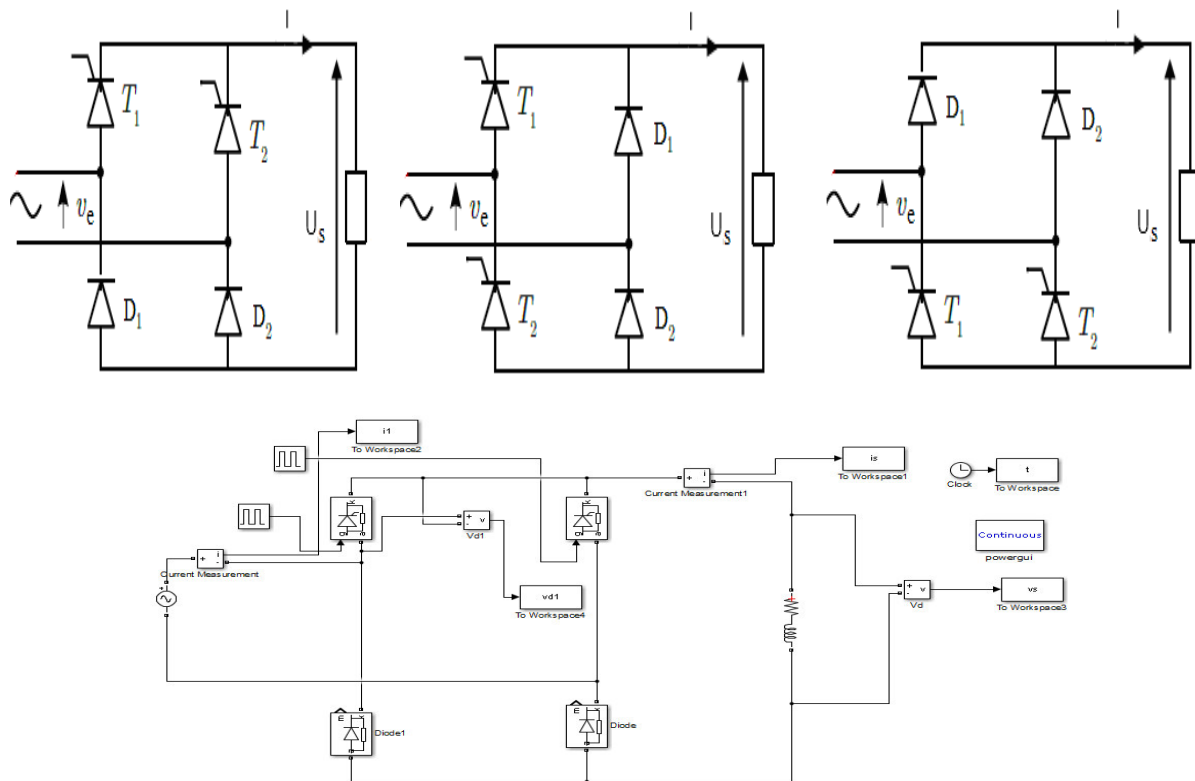


Figure 3.41 Assembly of a rectifier controlled by a mixed bridge.

#### 3.3.4.5.a. Operation:

- $0 < t < \alpha$

Thyristors Th1 and Th2 are off.

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$$V_s=0$$

- $\alpha < \theta < \pi$

Th1 and D2 are ON, and Th2 and D1 are OFF.

$$V_s=V_e.$$

- $\pi < \theta < \pi + \alpha$

Thyristors Th1 and Th2 are OFF.

$$V_s=0$$

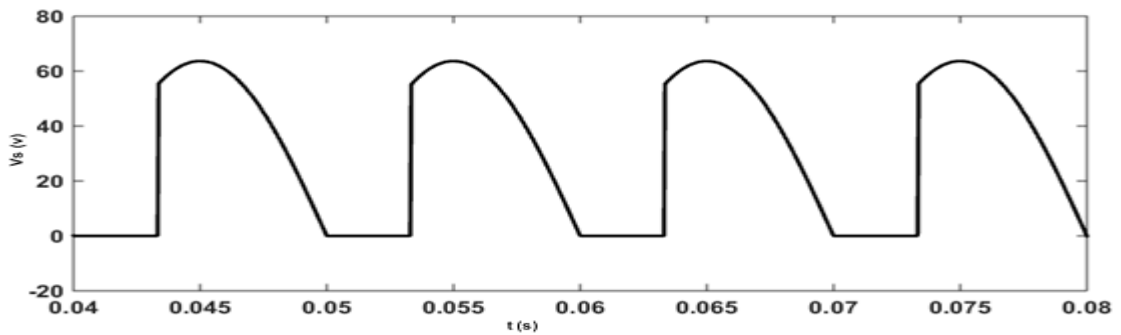
- $\pi + \alpha < \theta < 2\pi$

T2 and D1 are ON, and T1 and D2 are OFF.

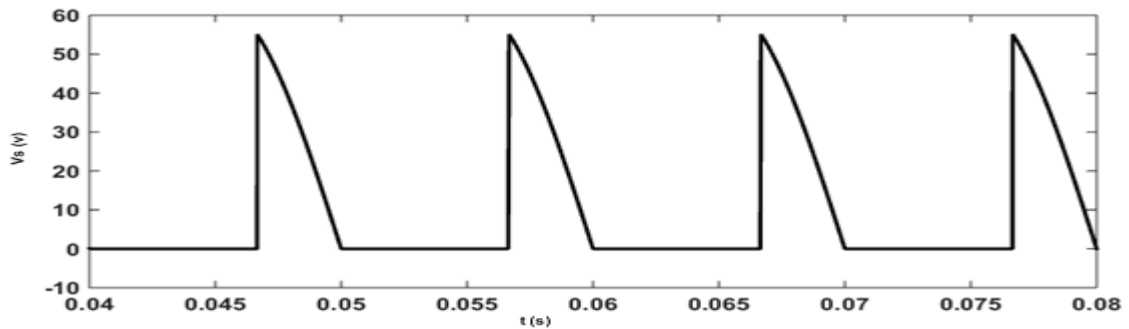
$$V_s=V_e.$$

The following figures show the voltage timing diagrams for different firing angles.

$\alpha = \pi/3$  :



$\alpha = 2\pi/3$  :



**Figure 3.42** Voltage timing diagram of a mixed full-wave rectifier on an inductive load

#### 3.3.4.5.b. Voltage calculation

- The average value of the rectified voltage is given by:

$$V_{avg} = \frac{1}{T} \int_0^T V_s(\theta).d\theta = \frac{1}{\pi} \int_{\alpha}^{\pi} V_{max} \sin \theta.d\theta = \frac{V_{max}}{\pi} [-\cos \theta]_{\alpha}^{\pi} = \frac{V_{max}}{2\pi} \left[ -\cos \frac{T=2\pi}{2} + \cos \alpha \right] = \frac{V_{max}}{\pi} [1 + \cos \alpha]$$

### Chapter 3: Conversion of AC-DC electrical energy

The average value of the output voltage  $V_s$  varies depending on the value of the delay angle  $\alpha$ .

The effective value of the output voltage is given by:

$$V_{seff} = \sqrt{\frac{1}{\pi} \int_{\alpha}^{\pi} (V_{max} \sin(\theta))^2 .d\theta} = \frac{V_{e\max}}{\sqrt{2}} \sqrt{1 - \frac{\alpha}{\pi} + \frac{1}{2\pi} \sin(2\alpha)}$$

#### 3.3.4.6. Three-phase thyristor-controlled rectifiers:

Unlike single-phase control, where the thyristor delay angle is referenced to the zero of the mains sinusoid, in three-phase control, the reference point is the instant when two voltages comprising the balanced three-phase system become equal (the instant of diode conduction in an uncontrolled rectifier) [3, 4, 5].

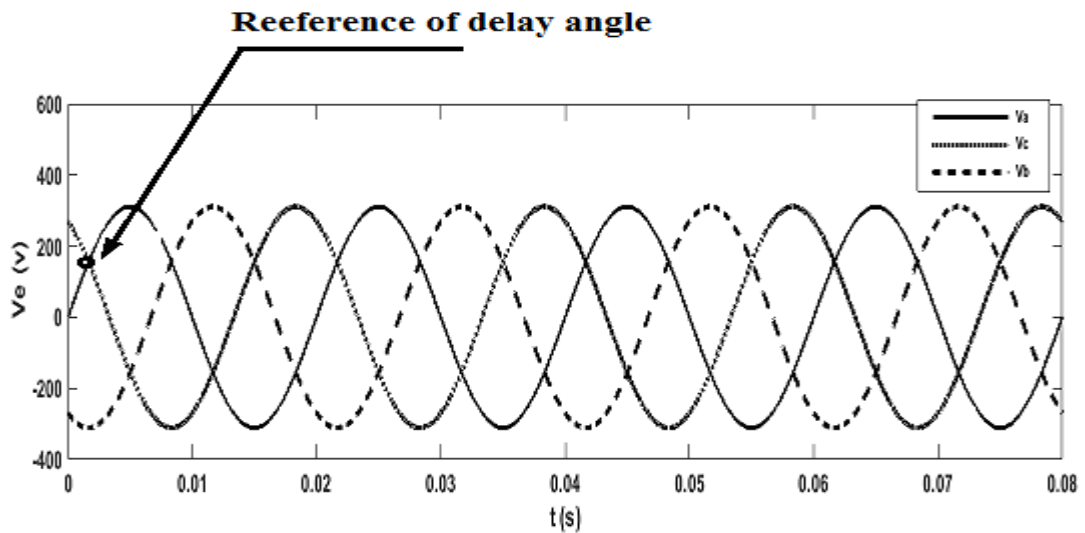


Figure 3.43: Three-phase voltage timing diagram

#### 3.3.4.6.a. Parallel rectifier type P3

Figure 3.44 shows a half-wave (single way), three-phase diode rectifier circuit [3, 4, 5].

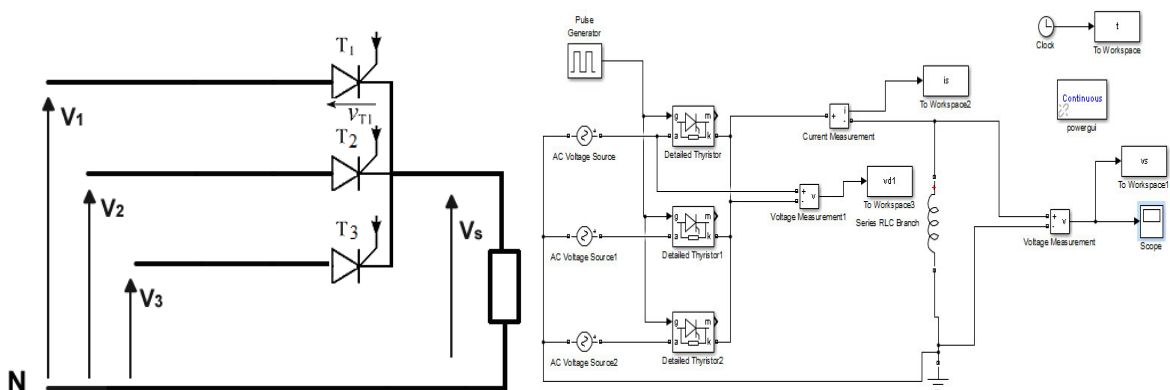


Figure 3.44 Assembly of a three-phase P3 rectifier supplying a resistive load.

### Chapter 3: Conversion of AC-DC electrical energy

The different operating phases of the assembly for an inductive load are described in the following table:

Intervals	Passing thyristors	Voltages across blocked thyristors	Rectified voltage
$\pi/6+\alpha < \theta < 5\pi/6+\alpha$	T <sub>1</sub>	$V_{T2}=V_{T1}-V_1+V_2=V_2-V_1$	$V_s = V_1$
$5\pi/6+\alpha < \theta < 3\pi/2+\alpha$	T <sub>2</sub>	$V_{T2} = 0$	$V_s = V_2$
$3\pi/2+\alpha < \theta < 2\pi+\alpha$	T <sub>3</sub>	$V_{T2}=V_{T3}-V_3+V_2=V_2 - V_3$	$V_s = V_3$

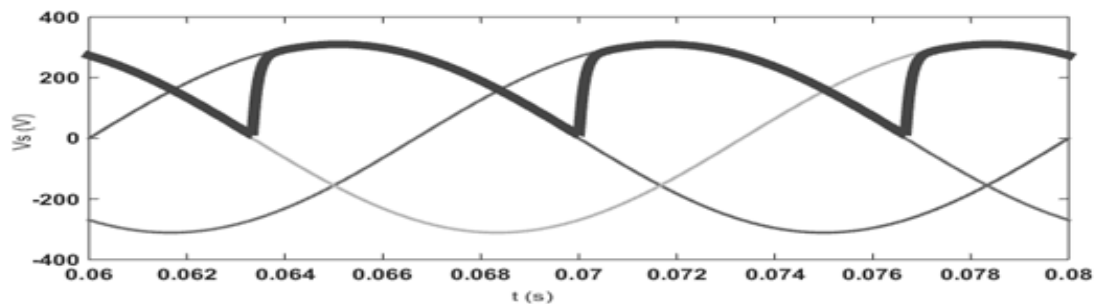
#### a/ Voltage recovery curves for a resistive load:

The different operating phases of the system for a resistive load are then described in the following table [3, 4, 5]:

Intervals	Passing thyristors	Voltages across blocked thyristors	Rectified voltage
$\pi/6+\alpha < \theta < 5\pi/6$	T <sub>1</sub>	$V_{T2}=V_{T1}-V_1+V_2=V_2-V_1$	$V_s = V_1$
$5\pi/6+\alpha < \theta < 3\pi/2$	T <sub>2</sub>	$V_{T2} = 0$	$V_s = V_2$
$3\pi/2+\alpha < \theta < 2\pi$	T <sub>3</sub>	$V_{T2}=V_{T3}-V_3+V_2=V_2 - V_3$	$V_s = V_3$

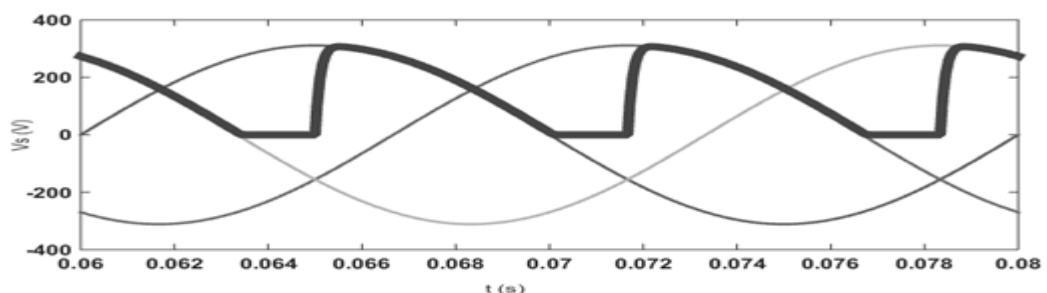
The difference with an inductive load is that when the load voltage  $V_s$  passes through zero, the current flowing through the thyristor is canceled and the thyristor is blocked; thus,  $V_s=0$ . For a resistive load, the voltage  $V_s$  cannot become negative under any circumstances.

for  $\alpha=30^\circ$ :



The resulting shape is identical to that obtained in the case of an inductive load.

for  $\alpha=60^\circ$ :



for  $\alpha=150^\circ$ :

We note the presence of zero levels due to the spontaneous blocking of the thyristors.

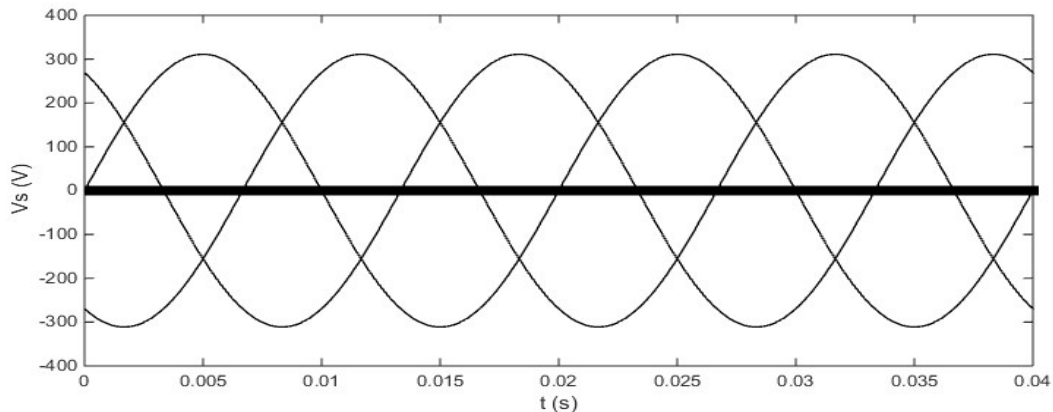


Figure 3.45 Voltage timing diagram of a three-phase controlled rectifier on a resistive load for different firing angles

We demonstrate that the average recovered voltage is given by the relationship:

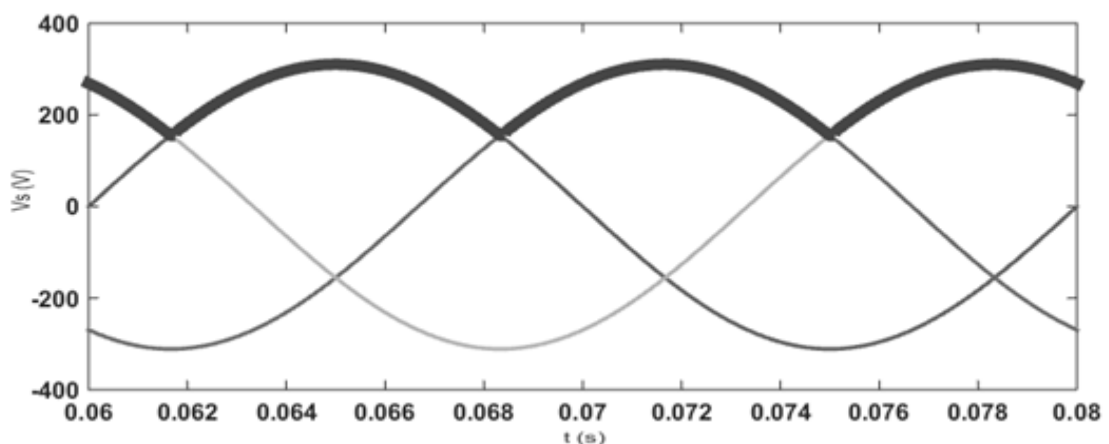
$$\text{for } \alpha < \pi/6 : \quad V_{savg} = \frac{1}{T} \int_0^T V_s(\theta).d\theta = \frac{3V_M}{2\pi} \int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} V_{\max} \sin(\theta).d\theta = \frac{3\sqrt{3}V_{\max}}{2\pi} \cos(\alpha).$$

$$\text{for } \alpha > \pi/6 : \quad V_{savg} = \frac{1}{T} \int_0^T V_s(\theta).d\theta = \frac{3V_{\max}}{2\pi} \int_{\alpha}^{\pi} V_{\max} \sin(\theta).d\theta = \frac{3V_{\max}}{2\pi} (1 + \cos(\alpha + \frac{\pi}{6}))$$

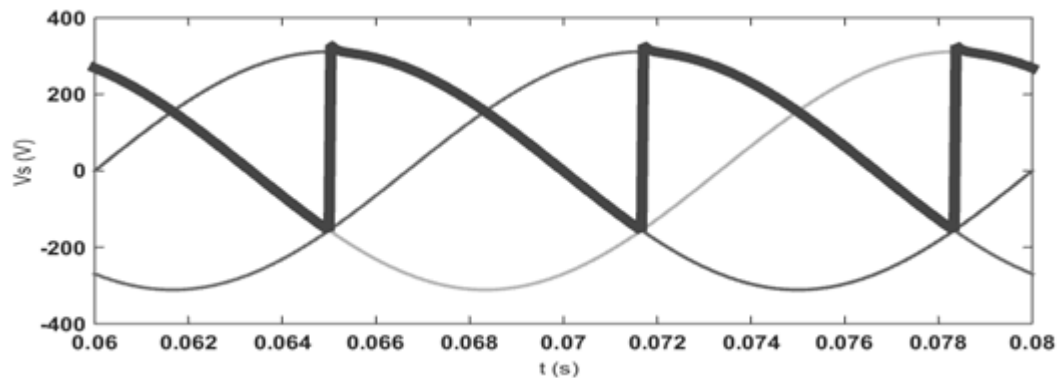
**b/ Voltage curves recovered for an inductive load (continuous conduction):**

The following figures show the voltage timing diagrams for different firing angles [3,4,5].

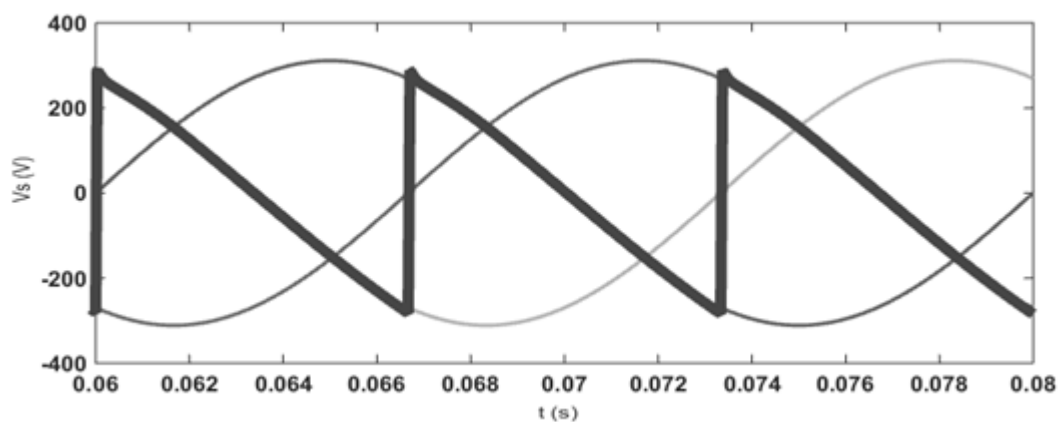
**Voltage curves for  $\alpha=0^\circ$ :**



Voltage curves for  $\alpha=60^\circ$ :



Voltage curves for  $\alpha=90^\circ$ :



**Figure 3.46** Voltage timing diagram of a three-phase controlled rectifier on an inductive load for different firing angles

It has been shown that the average output voltage value is given by:

$$V_{avg} = \frac{1}{T} \int_0^T V_s(\theta) d\theta = \frac{3\sqrt{3}V_{max}}{2\pi} \cos(\alpha)$$

We also note that for  $\alpha < 90^\circ$ , the circuit operates as a rectifier ( $V_{avg} > 0$ ), while the operation is that of an assisted inverter for  $\alpha > 90^\circ$ .

### 3.3.4.6.b. PD3 Dual Parallel Rectifier

As with a three-phase dual parallel diode rectifier, the load sees a voltage equal to the difference between the voltage delivered by the "more positive" switch and that delivered by the "more negative" switch [3, 4, 5].

Thyristor Th1 is likely to conduct when voltage  $V_1$  is the most positive of the components  $V_1$ ,  $V_2$ , and  $V_3$ . It is controlled at firing after a delay angle  $\alpha$  (delay relative to the natural conduction of the diodes). Thyristor Th'2 is in turn likely to conduct when  $V_2$  becomes the

## Chapter 3: Conversion of AC-DC electrical energy

most negative. It is controlled at firing after a delay angle  $\alpha$ . If these two thyristors conduct simultaneously, the output will be  $V_s = V_1 - V_2$  [3, 4, 5].

### 1/ Shape of $V_s$ for a resistive load

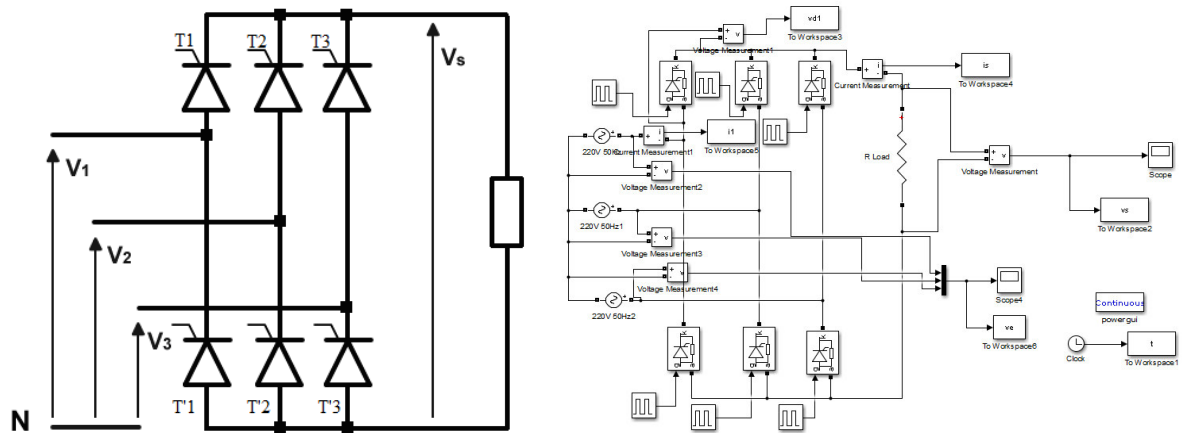
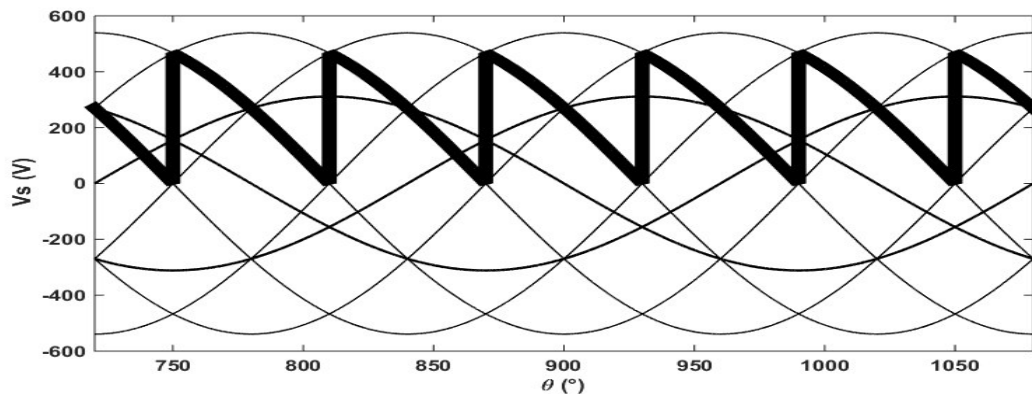


Figure 3.47 Assembly of a three-phase PD3 rectifier supplying a resistive load.

The voltage curves are then obtained using the usual reasoning (Figure 3.48).

- for  $\alpha=60^\circ$ :



- $\alpha = 90^\circ$  :

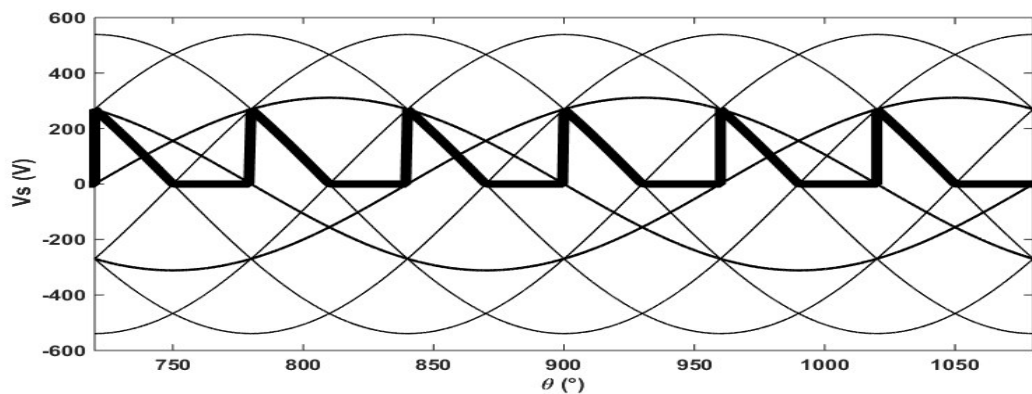


Figure 3.48 Voltage timing diagrams for a three-phase PD3 controlled rectifier on a resistive load.

## Chapter 3: Conversion of AC-DC electrical energy

The mean and effective values are calculated:

$$V_{avg} = \frac{6}{T} \int_{\alpha+\frac{\pi}{6}}^{\alpha+\frac{\pi}{2}} V_{s12}(\theta).d\theta = \frac{6}{T} \int_{\alpha+\frac{\pi}{6}}^{\alpha+\frac{\pi}{2}} (V_{max} (\sin(\theta) - \sin(\theta - \frac{2\pi}{3}))).d\theta = \frac{3}{\pi} V_{max} \cdot \sin(\frac{\pi}{3}) \cdot \cos(\alpha)$$

and effective voltage value:

$$V_{eff} = \sqrt{\frac{6 \cdot V_{max}^2}{T} \int_{\alpha+\frac{\pi}{6}}^{\alpha+\frac{\pi}{2}} (\sin(\theta) - \sin(\theta - \frac{2\pi}{3}))^2 dt} = V_{max} \sqrt{\frac{3}{2} + \frac{9}{2\pi} \sin(\frac{\pi}{3}) \cos(2\alpha)}$$

### 2/ $V_s$ curve for an inductive load:

In this case, the current  $I_s$  is never interrupted and is switched by a pair of thyristors.

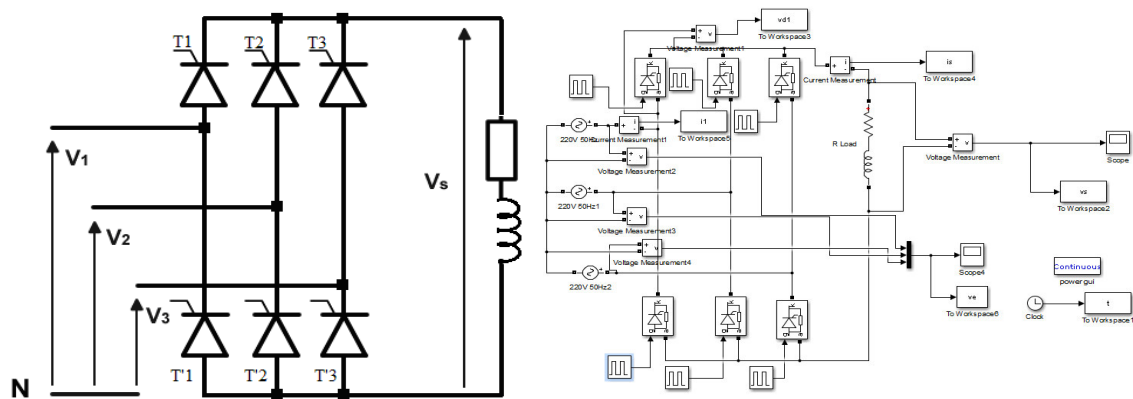
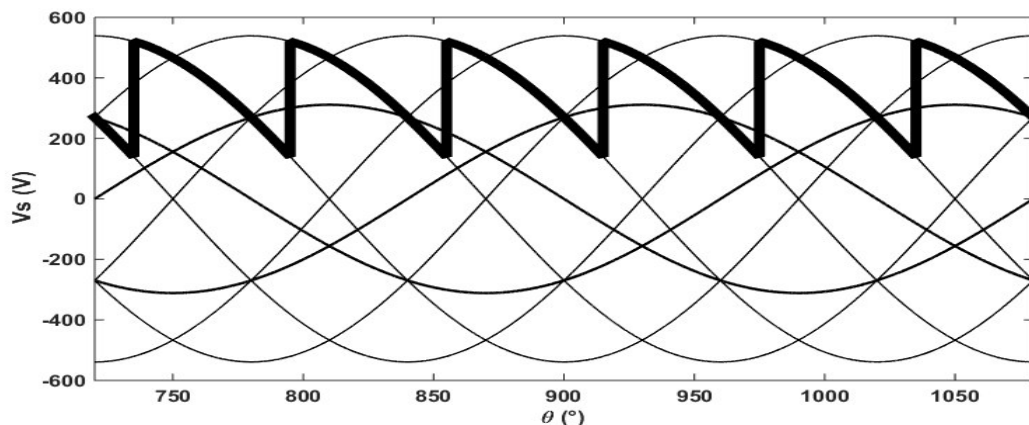


Figure 3.49 Assembly of a three-phase PD3 rectifier supplying a resistive load.

The voltage curves are then obtained using the usual reasoning (Figure 3.50).

- for  $\alpha = 45^\circ$ :



- $\alpha = 90^\circ$  :

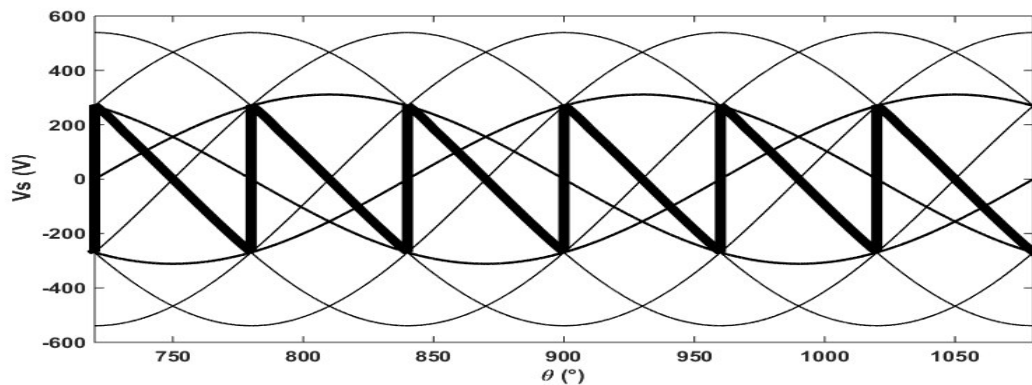


Figure 3.50 Voltage timing diagram of a three-phase controlled rectifier on an inductive load for different firing angles

► voltage average value

The rectified voltage  $V_s$  is periodic with a period of  $T/3$ .

Between  $\frac{T}{12} + \alpha$  and  $\frac{5T}{12} + \alpha$ , this voltage is expressed as:

$$V_{savg} = \frac{6}{T} \int_{\frac{T}{12} + \alpha}^{\frac{5T}{12} + \alpha} V_{12}(\theta).d\theta = \frac{6V_{max}}{T} \int_{\frac{T}{12} + \alpha}^{\frac{5T}{12} + \alpha} (\sin(\theta) - \sin(\theta - \frac{2\pi}{3})).d\theta = \frac{3\sqrt{3}V_{max}}{\pi} \cos(\alpha)$$

3.3.4.7.. AC-dc converter generalised equations

Alternating sinusoidal voltages

$$V_1(t) = V_{max} \cdot \sin(\omega.t)$$

$$V_2(t) = V_{max} \cdot \sin(\omega.t - \frac{2\pi}{q})$$

·  
·  
·

$$V_3(t) = V_{max} \cdot \sin(\omega.t - (q-1)\frac{2\pi}{q})$$

where  $q$  is the number of phases (number of voltage sources) [3,4,5].

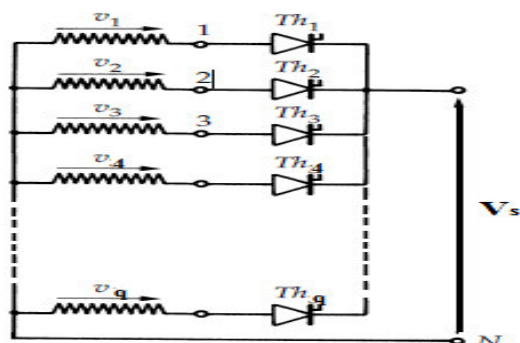


Figure 3.51 Polyphase half-wave controlled converter:

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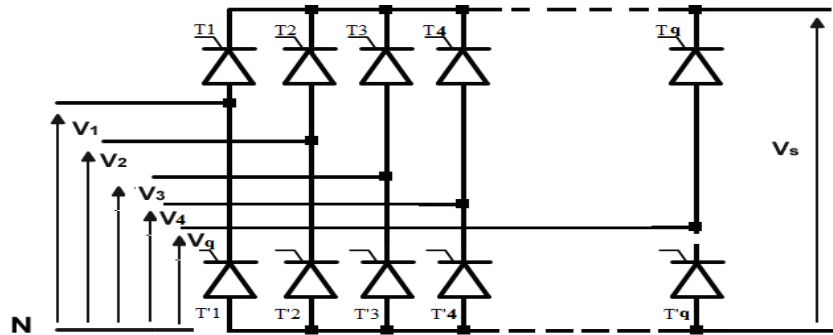
The mean and effective values are calculated for several phase values  $q$ , namely:

$$V_{savg} = \frac{1}{T} \int_0^T v(t).dt = \frac{q}{T} \int_{\frac{\pi}{2} - \frac{\pi}{q} + \alpha}^{\frac{\pi}{2} + \frac{\pi}{q} + \alpha} V_{max} \cdot \sin(\omega t).dt = \frac{q}{\pi} V_{max} \cdot \sin\left(\frac{\pi}{q}\right) \cdot \cos(\alpha)$$

The effective value:

$$V_{seff} = \sqrt{\frac{1}{T} \int_0^T v^2(t).dt} = \sqrt{\frac{q}{T} \int_{\frac{\pi}{2} - \frac{\pi}{q} + \alpha}^{\frac{\pi}{2} + \frac{\pi}{q} + \alpha} (V_{max} \cdot \sin(\omega t))^2 .dt} = V_{max} \cdot \sqrt{\frac{1}{2} + \frac{q}{4 \cdot \pi} \cdot \sin\left(\frac{2\pi}{q}\right) \cdot \cos(\alpha)}$$

Figure 3.52 shows the schematic of a polyphase fully rectifier which consists of several thyristors connected in a bridge (Graetz bridge) [3, 4, 5].



**Figure 3.52** Polyphase fully controlled converter:

The mean and effective values are calculated for several  $q$  phases values and  $2q$  thyristors:

$$V_{savg} = \frac{1}{T} \int_0^T v(t).dt = \frac{2q}{T} \int_{-\frac{\pi}{2q} + \alpha}^{\frac{\pi}{2q} + \alpha} V_{max} \cdot (\sin(\theta) - \sin(\theta - \frac{2 \cdot \pi}{3})) .dt = \frac{2q}{\pi} V_{max} \cdot \sin\left(\frac{\pi}{q}\right) \cdot \cos(\alpha)$$

The effective value:

$$V_{seff} = \sqrt{\frac{1}{T} \int_0^T v^2(t).dt} = \sqrt{\frac{2q}{T} \int_{-\frac{\pi}{2q} + \alpha}^{\frac{\pi}{2q} + \alpha} (V_{max} \cdot (\sin(\omega t) - \sin(\theta - \frac{2 \cdot \pi}{3})))^2 .dt} = V_{max} \cdot \sqrt{\frac{1}{2} + \frac{q}{2 \cdot \pi} \cdot \sin\left(\frac{\pi}{q}\right) \cdot \cos(\alpha)}$$

### 3.3.4.8. Three-phase controlled rectifier with a mixed bridge

We repeat the PD3 circuit (full thyristor bridge) and replace thyristors Th'1, Th'2, and Th'3 with diodes. The thyristors operate with a delay angle  $\alpha$  of compared to natural firing (conduction is assumed to be continuous) [3, 4, 5].

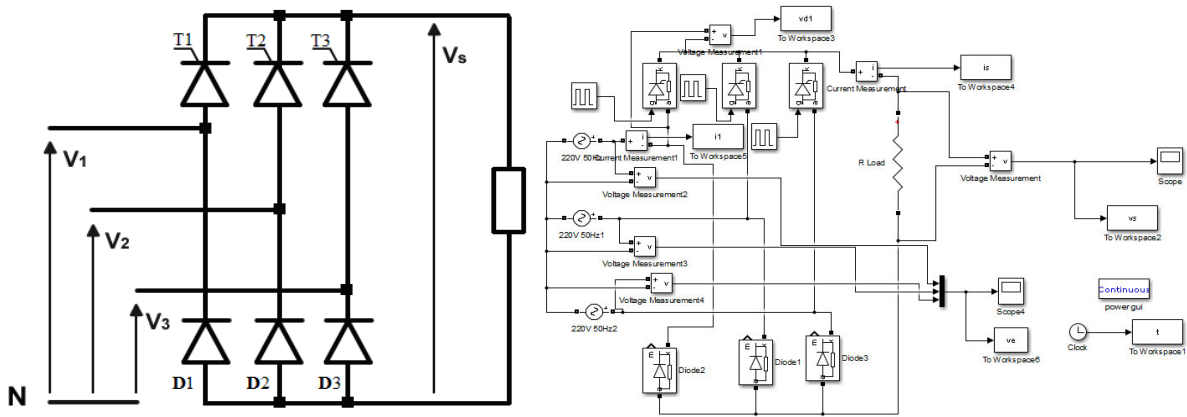


Figure 3.53 Assembly of a mixed PD3 three-phase rectifier supplying a resistive load

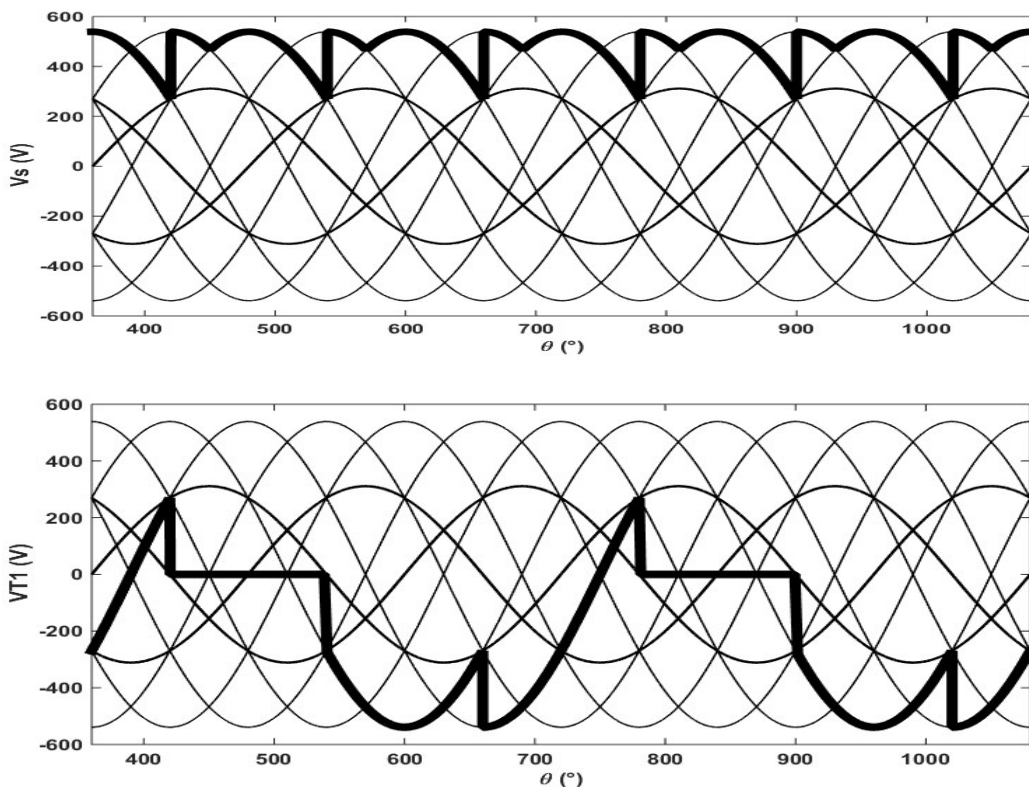


Figure 3.54 Timing diagrams of the load and thyristor voltage of a mixed PD3 three-phase controlled rectifier on an inductive load for  $\alpha = 30^\circ$ .

### 3.3.4.9. Conclusion

AC-DC converters, also known as rectifiers, play a vital role in modern electronic and electrical systems. They transform alternating current into direct current, suitable for a wide range of industrial, domestic, and on-board applications. Depending on performance, efficiency, and signal quality requirements, several rectifier designs are available (single-phase or three-phase, diode or thyristor). Their design and mastery are therefore essential for designing efficient power supply systems that are adapted to the constraints of the loads being powered..

# **Chapter 4: DC-DC Electrical Energy Conversion**

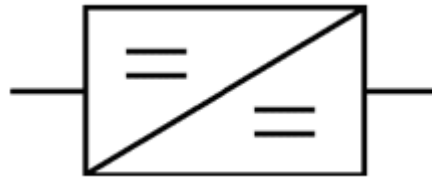
## Chapter 4: DC-DC Electrical Energy Conversion

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### 4.1. Introduction

A DC/DC converter is a power device designed to convert fixed-voltage direct current into variable current. It regulates the energy transfer between a source and a load, depending on whether the load is capacitive (similar to a voltage source) or inductive (similar to a current source) [3, 4, 5].

### 4.2. Symbol

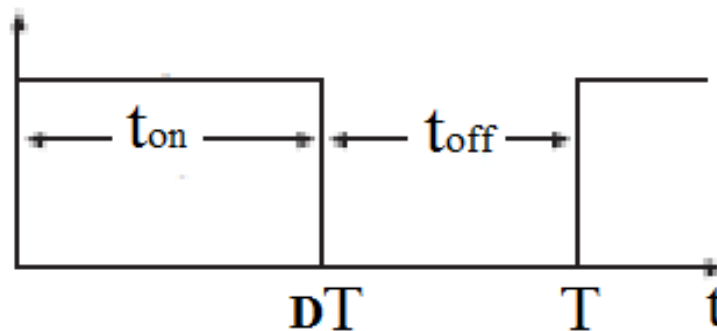


### 4.3. Definition of the duty cycle:

The duty cycle is defined as the time  $t_{ON}$  during which the switch is closed divided by the operating period of the circuit  $T$ , i.e.:

$$D = \frac{t_{ON}}{T}$$

The time the switch is open is also defined by:  $t_{OFF} = T - t_{ON}$



This leads to the study of the simplest DC/DC converters. In this context, three main families of static converters (also called choppers) are distinguished:

- the step-down (buck) chopper (series),
- the step-up (boost) chopper (parallel),
- the buck-boost chopper.

4.4. Step-down Chopper (Buck)

4.4.1. Principle

To vary the average value of the voltage  $U_M$  at the receiver terminals, we create the equivalent of the following simplified circuit [3, 4, 5].

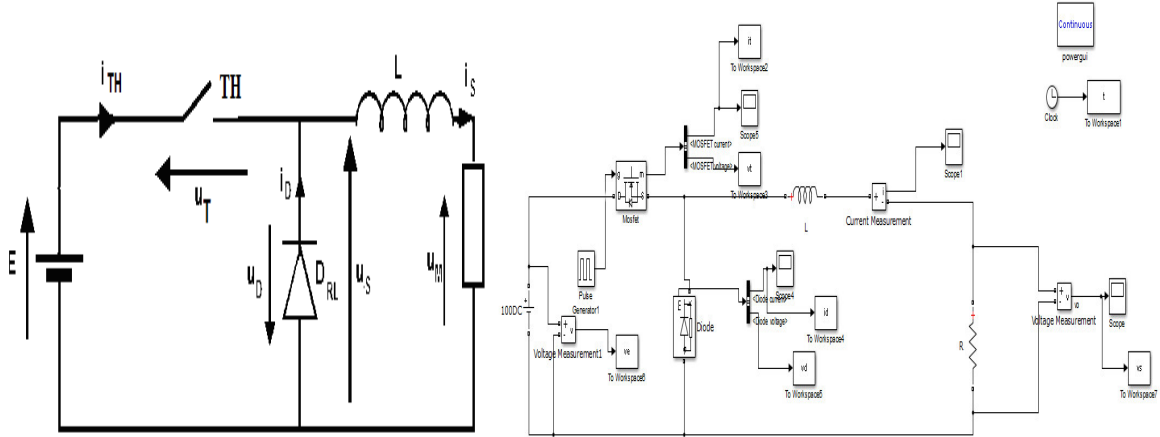


Figure 4.1 Electric circuit of a buck chopper

4.4.2. Continuous Conduction

4.4.2.1. Operational Analysis

The switches TH and  $D_{RL}$  are assumed to be perfect: zero voltage in the on state, zero current in the off state. We can break this analysis into two parts [3, 4, 5]:

1-  $0 < t < DT$  (TH closed,  $D_{RL}$  open).

When TH is closed and  $D_{RL}$  is open during the interval  $[0, DT]$ , the converter's electrical circuit is shown in figure 4.2:

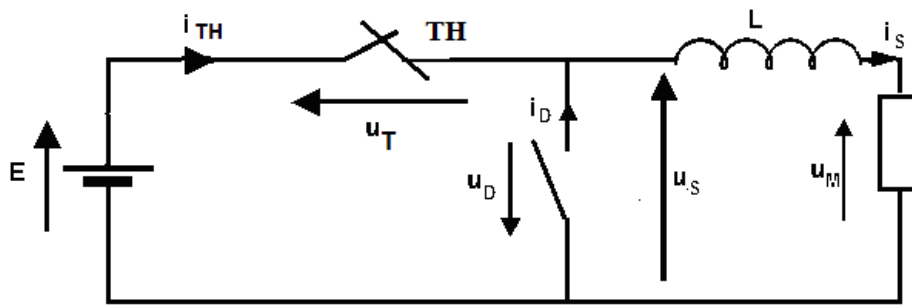


Figure 4.2 Equivalent diagram of a series chopper for  $t \in [0, DT]$

Applying the mesh laws, we have

$$E = R.i_s(t) + L \frac{di_s}{dt}(t) \text{ with } i_s(0) = i_{\min} \text{ and } i_s(D.T) = i_{\max}$$

This equation has two solutions:

Solution without right-hand side

## Chapter 4: DC-DC Electrical Energy Conversion

$$0 = R.i_s(t) + L \frac{di_c}{dt}(t) \Rightarrow i_{sh}(t) = A.e^{\frac{-R.t}{L}}$$

Special solution:  $i_{sp}(t) = cst$

$$E = R.i_{sp}(t) + L \frac{di_{sp}}{dt}(t) \Rightarrow E = R.i_{sp}(t)$$

$$\Rightarrow i_{sp}(t) = \frac{E}{R}$$

General solution:

$$i_s(t) = \frac{E}{R} + A.e^{\frac{-R.t}{L}}$$

at  $t = 0$  we have  $i_s(0) = i_{\min}$

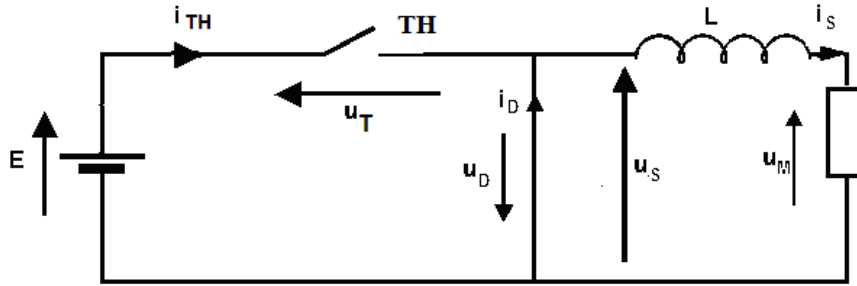
$$i_{\min} = \frac{E}{R} + A.e^{\frac{-R.0}{L}} \Rightarrow A = i_{\min} - \frac{E}{R}$$

so

$$i_s(t) = \frac{E}{R} + (i_{\min} - \frac{E}{R}).e^{\frac{-R.t}{L}}$$

### 2- $DT < t < T$ ( TH open, $D_{RL}$ closed ).

When TH is open and  $D_{RL}$  is closed during the interval  $[DT, T]$ , the electrical circuit of the converter is shown in figure 4.3:



**Figure 4.3** Equivalent diagram of a series chopper for  $t \in [DT, T]$

Therefore the current  $i_{TH}$  is zero and the currents  $i_D$  and  $i_s$  are identical and solutions of the differential equation

$$0 = R.i_s(t) + L \frac{di_s}{dt}(t) \Rightarrow i_s(t) = A.e^{\frac{-R.t}{L}} \text{ with } i_s(D.T) = i_{\max}$$

$$i_s(D.T) = A.e^{\frac{-R.D.T}{L}} = i_{\max} \Rightarrow A = i_{\max} \cdot e^{\frac{R.D.T}{L}}$$

so

$$i_s(t) = i_{\max} \cdot e^{\frac{-R \cdot (t - \alpha \cdot T)}{L}}$$

### 4.4.2.2. Relationship between input and output voltages

In steady state, the average voltage across the inductor is zero.

$$V_{Lavg} = \frac{1}{T} \int L \cdot di_s = 0 \Rightarrow \frac{1}{T} \int_0^{DT} (E - U_M) dt + \frac{1}{T} \int_{DT}^T -U_M dt = 0$$

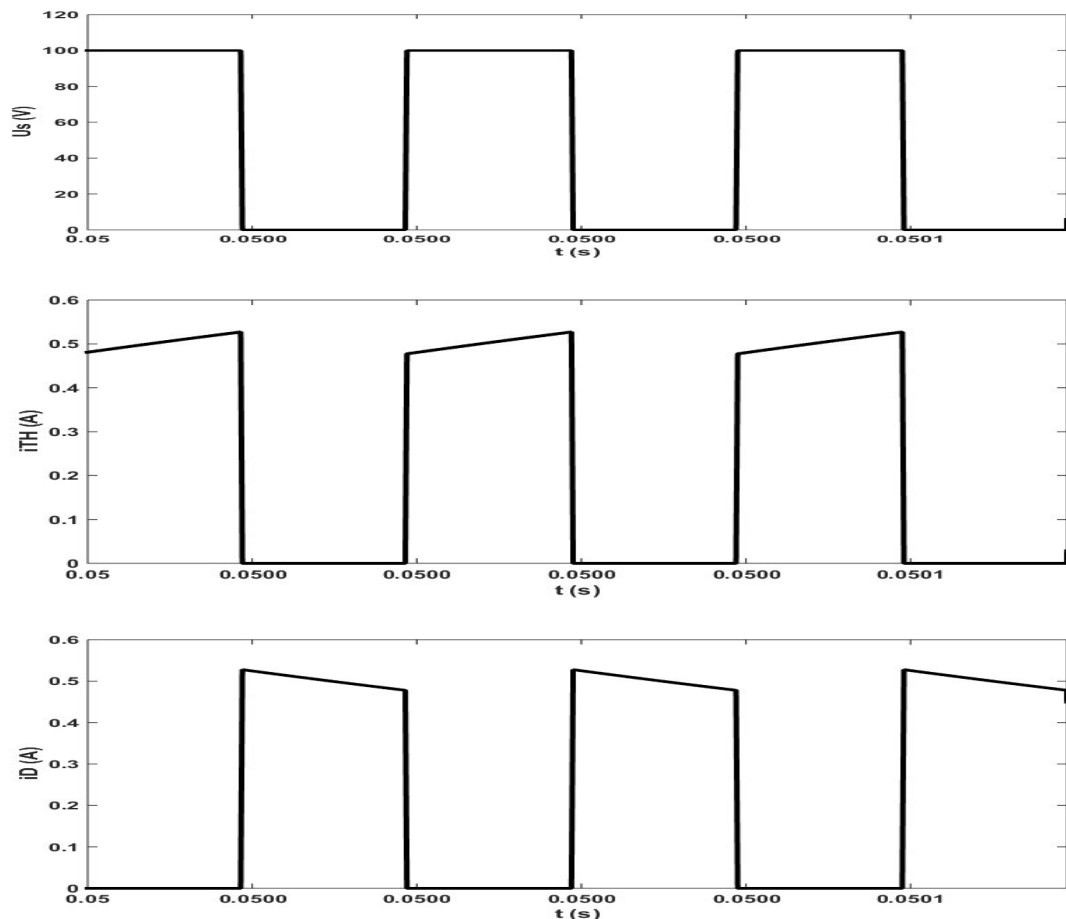
So

$$U_{Mavg} = D \cdot E$$

and  $i_{savg} = \frac{D \cdot E}{R}$

The series chopper can be likened to a non-reversible DC transformer, whose transformation ratio is denoted  $\alpha$ , with  $D \leq 1$ .

The voltages and currents are simulated using Matlab (Figure 4.4).



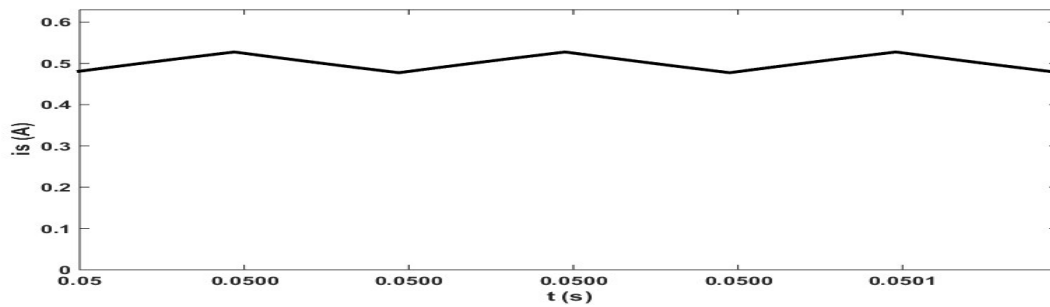


Figure 4.4 Waveforms of the main parameters of a series chopper for an R-L load for 50kHz switching frequency case

4.4.3. Discontinuous conduction:

In this case, the energy stored in the inductor is insufficient to maintain current flow until the end of the period; there are then three phases over the period [3,4,5].

4.4.3.1. Operational analysis

1-  $0 < t < D_1 T$  (TH closed, DRL open).

we have :  $U_{DR} = -u$ ,  $U_T = 0$  and  $i_{DR} = 0$ .

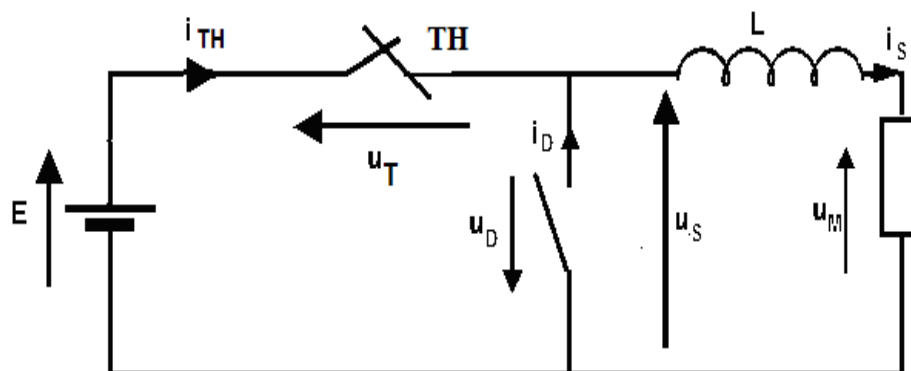


Figure 4.5 Equivalent diagram of a series chopper for  $t \in [0, DT]$

$$i_s(t) = \frac{E}{R} \cdot (1 - e^{-\frac{R \cdot t}{L}})$$

2-  $D_1 T < t < D_2 T$  (TH open, DRL closed).

we have :  $U_{DR} = 0$  -  $i_Q = 0$  and  $U_T = E$ .

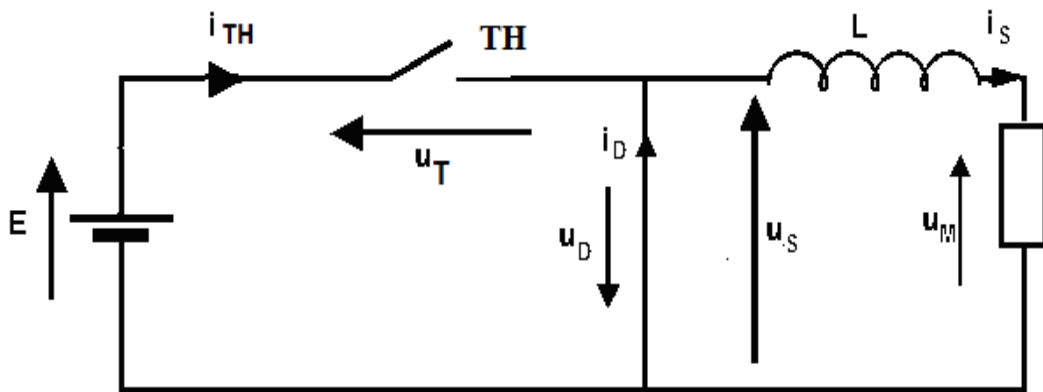


Figure 4.6 Equivalent diagram of a series chopper for  $t \in [D_1T, D_2T]$

$$i_s(t) = i_{\max} \cdot e^{\frac{-R_s(t-D_1T)}{L}}$$

3-  $D_2T < t < T$  (TH open, DRL open).

we have :  $U_{DR} = -u$ ,  $i_Q = 0$ ,  $i_{DR} = 0$  and  $i = 0$

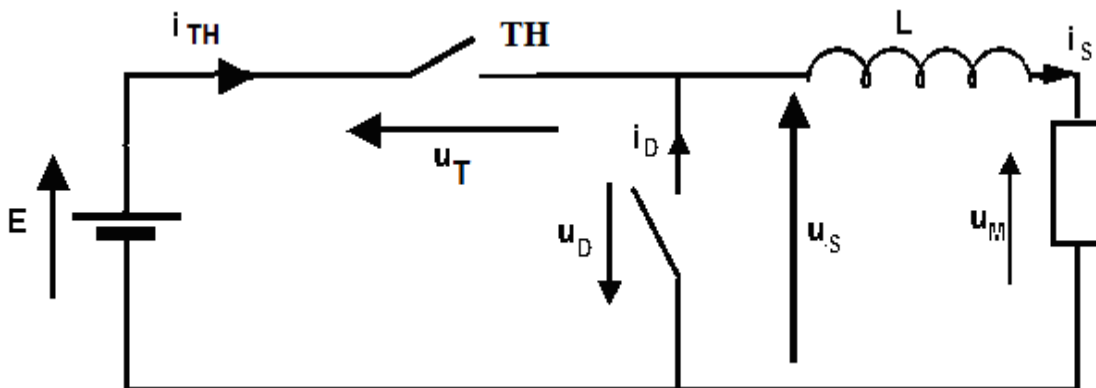


Figure 4.7 : Equivalent diagram of a series chopper for  $t \in [D_2T, T]$

#### 4.4.3.2. Relationship between input and output voltages

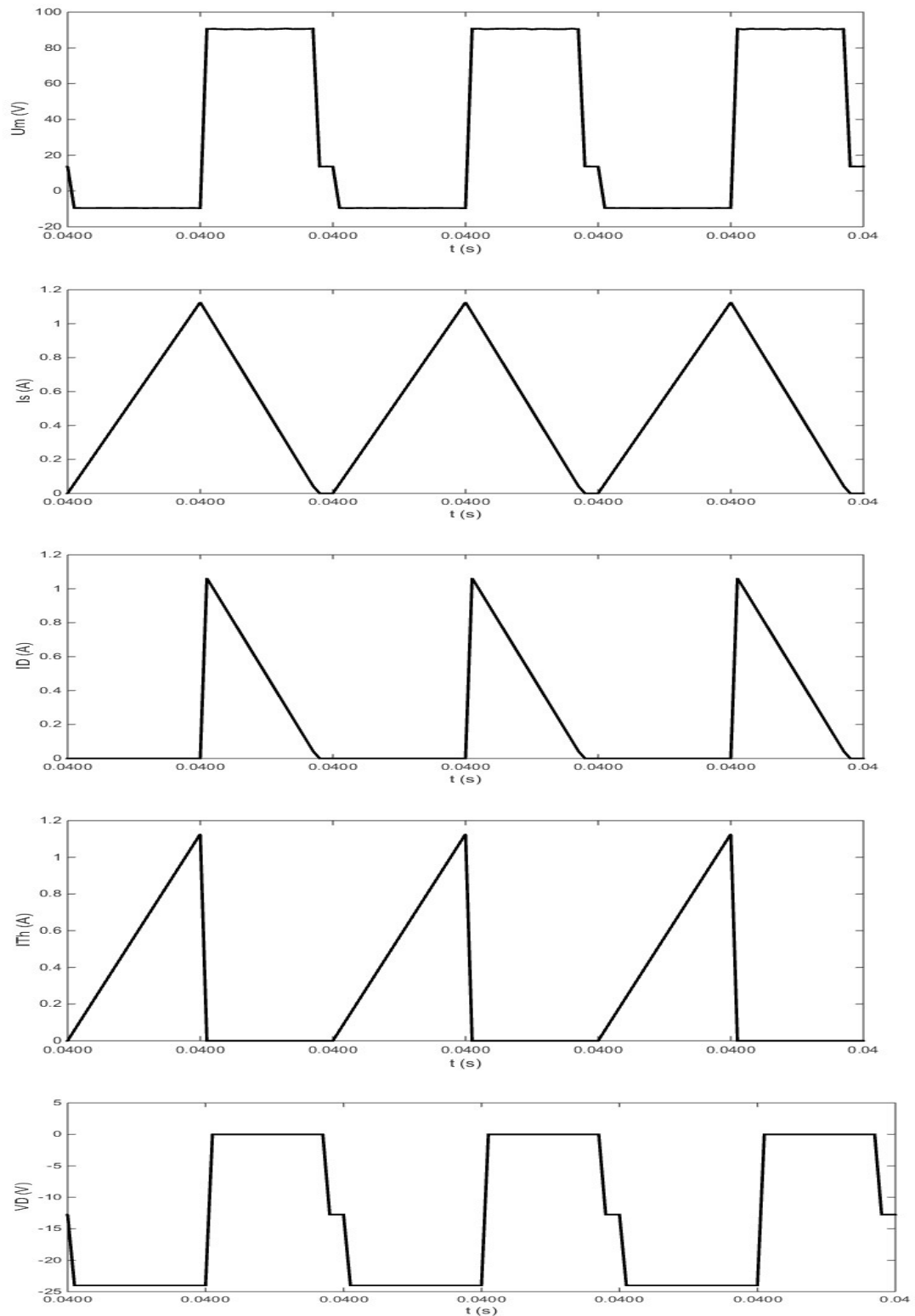
In steady state, the average voltage across the inductor is zero.

$$V_{Lavg} = \frac{1}{T} \int L \cdot \frac{di_s}{dt} dt = 0$$

so

$$V_{Lavg} = \frac{1}{T} \int_0^{D_1T} (E - U_M) \cdot dt + \frac{1}{T} \int_{D_1T}^{D_2T} -U_M \cdot dt = D_1(E - U_M) - (D_2 - D_1)U_M = 0$$

$$U_M = \frac{D_1}{D_2} \cdot E$$



**Figure 4.8** Waveforms of the main parameters of a series chopper for discontinuous conduction.

### 4.4.4. Current ripple.

The graph below represents the change in current in the load over time.

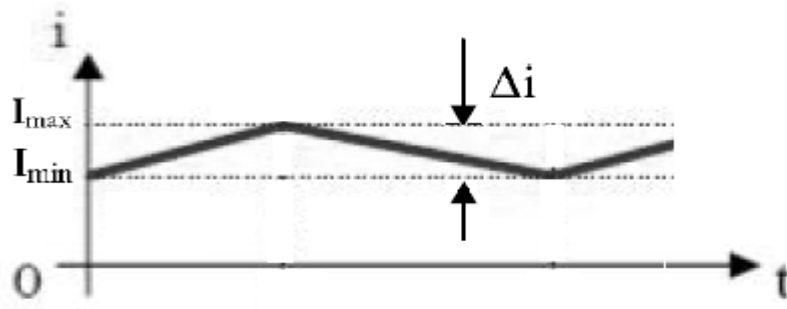


Figure 4.9 Evolution of the current

Ripple is the difference between the minimum and maximum values of the intensity:

$$\Delta i = I_{\max} - I_{\min}$$

#### 4.4.5. Ripple Expression

To determine it, we use the expression for the current between 0 and DT, writing that the current is maximum at time DT, which gives [6,7,8]:

$$i_s(DT) = I_{\max} = \frac{E}{R} + (i_{\min} - \frac{E}{R}) \cdot e^{-\frac{R \cdot DT}{L}}$$

and

$$i_s(T) = I_{\min} = i_{\max} \cdot e^{-\frac{R \cdot (1-D)T}{L}} \Rightarrow I_{\max} = I_{\min} \cdot e^{\frac{R \cdot (1-D)T}{L}}$$

The inductance L is considered very large, which implies that the time constant  $\tau = L/R$  is much greater than the period T (i.e.,  $\tau \gg T$ ). In this case, the exponential portions of the current curves can be treated as straight line segments, which simplifies the calculation of the maximum currents  $I_{\max}$  and minimum currents  $I_{\min}$ .

The transformed equation above gives

$$\Delta i = i_{\max} - i_{\min} = \frac{E - U_M}{L} \cdot (DT) = \frac{E - DE}{L} \cdot (DT) = \frac{(1-D) \cdot E}{L} \cdot (DT)$$

#### 4.4.6. Factors Affecting Ripple

The current in the load should be as close to DC as possible: its ripple should therefore tend toward the smallest possible value.

If  $D=0$ , then the ripple is zero; the same is true if  $D=1$ .

#### 4.4.7. Value of the duty cycle $\alpha$ at which the ripple is maximum.

To find this value, we derive the relationship with  $\Delta i = \frac{(1-D) \cdot E}{L} \cdot (DT)$  respect to D. The

curve representing the evolution of  $\Delta i$  as a function of  $\alpha$  is an inverted parabola. The ripple will be maximum when the derivative is zero, which  $\frac{d\Delta i}{d\alpha} = \frac{(1-2D) \cdot E}{L} \cdot T = 0$  gives  $D=0,5$ .

## Chapter 4: DC-DC Electrical Energy Conversion

The ripple is maximum for  $D=0,5$  and its maximum value is given by  $\Delta i = \frac{E}{4.L}.T$ .

The maximum ripple value is lower when:

- The voltage  $E$  is low,
- the frequency  $f$  is high,
- the inductance  $L$  is high.

Since the value of voltage  $E$  is determined by the chopper load, it is often impossible to adjust it to reduce the ripple.

All that remains is the product  $L.f$ . Once its value is fixed, the higher  $f$ , the lower the inductance. However, the frequency must be limited to a value compatible with the switches used [6,7,8].

### 4.5. Parallel chopper or voltage booster

A boost converter, or parallel chopper, is a power supply that converts a DC voltage into a higher DC voltage.

This chopper consists of a controlled-opening switch in parallel with the receiver and a spontaneously closing and opening switch between the source and the receiver [3,4,5].

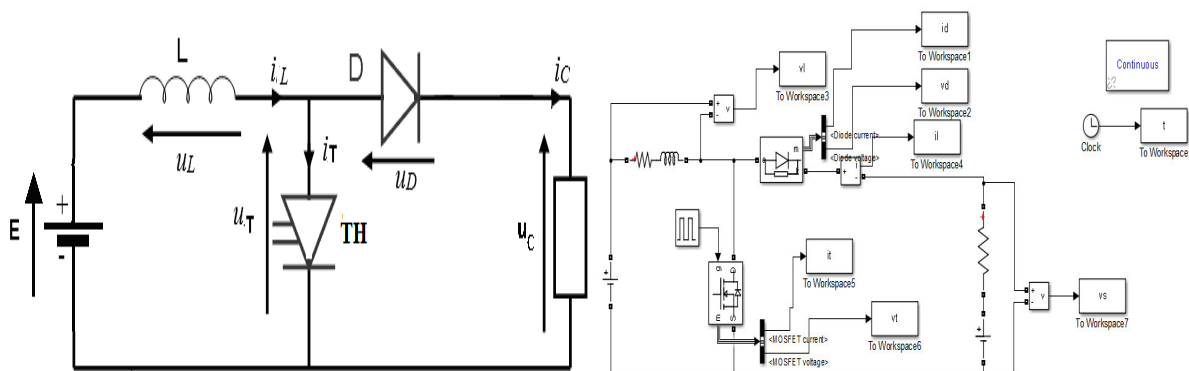


Figure 4.10 Diagram of a Parallel Chopper

#### 4.5.1. Continuous Conduction Study of a Parallel Chopper

Generally, the inductance  $L$  of the current source is high enough so that the average value  $I_L$  of the current  $i_L(t)$ , below which conduction becomes discontinuous, is such that it makes  $RI_L$  negligible compared to  $E$  [6,7,8].

##### 4.5.1.1. Operational Analysis

We can break this analysis down into two parts:

##### 1- $0 < t < DT$ (TH closed, D open).

Let's determine the current  $i(t)$ : we have  $E \gg R.i_L(t)$ , so

$$E = R.i_L + L \frac{di_L}{dt}(t) \text{ with } i_L(0) = i_{\min} \text{ and } i_L(D.T) = i_{\max}$$

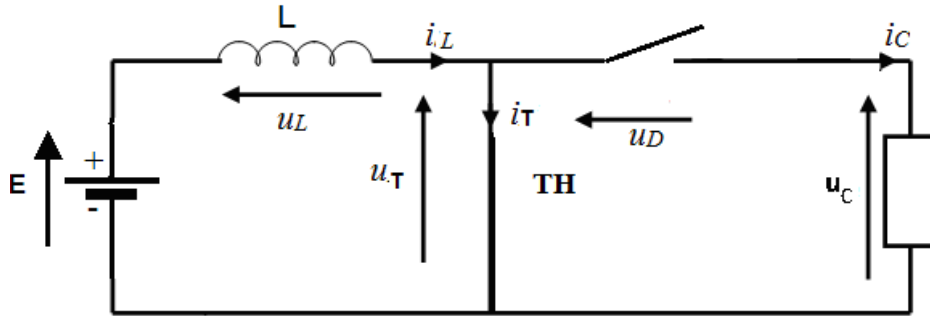
so

$$E = L \frac{di_L}{dt}(t) \Rightarrow i_L(t) = \frac{E}{L}t + A$$

at  $t = 0$  we have  $i_L(0) = i_{\min}$

$$i_L(0) = \frac{E}{L} \cdot 0 + A = i_{\min} \Rightarrow A = i_{\min}$$

$$i_L(t) = \frac{E}{L}t + i_{\min}$$



**Figure 4.11** Equivalent diagram of a parallel chopper for  $t \in [0, DT]$

### 2- $DT < t < T$ (TH open, D closed).

Assume that the source discharges into a load voltage source.

Let us determine the current  $i_L(t)$ : we have

$$E = U_c + L \frac{di_L}{dt}(t) \text{ with } i_L(T) = i_{\min} \text{ and } i_c(D.T) = i_{\max}$$

So

$$E = U_c + L \frac{di_L}{dt}(t) \Rightarrow E - U_c = L \frac{di_L}{dt} \Rightarrow i_L(t) = \frac{E - U_c}{L}t + A$$

at  $t = DT$  we have  $i_c(D.T) = i_{\max}$

$$i_L(D.T) = \frac{E - U_c}{L} \cdot D.T + A = i_{\max} \Rightarrow A = i_{\max} - \frac{E - U_c}{L} \cdot D.T$$

$$i_L(t) = \frac{E - U_c}{L} \cdot (t - D.T) + i_{\max}$$

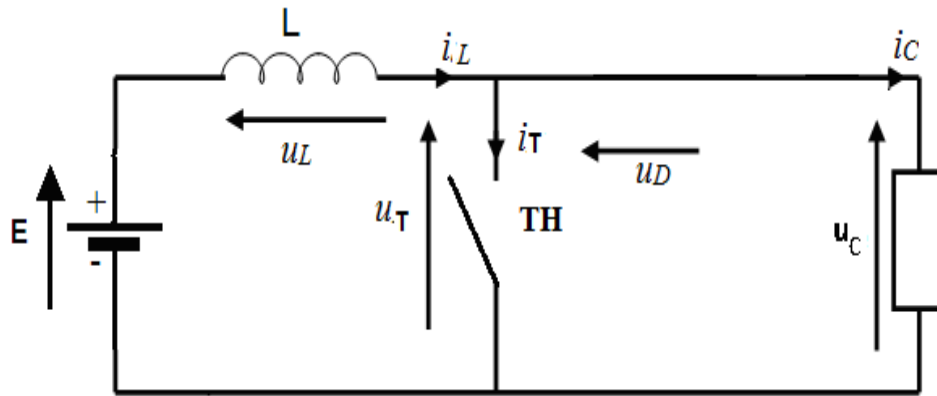
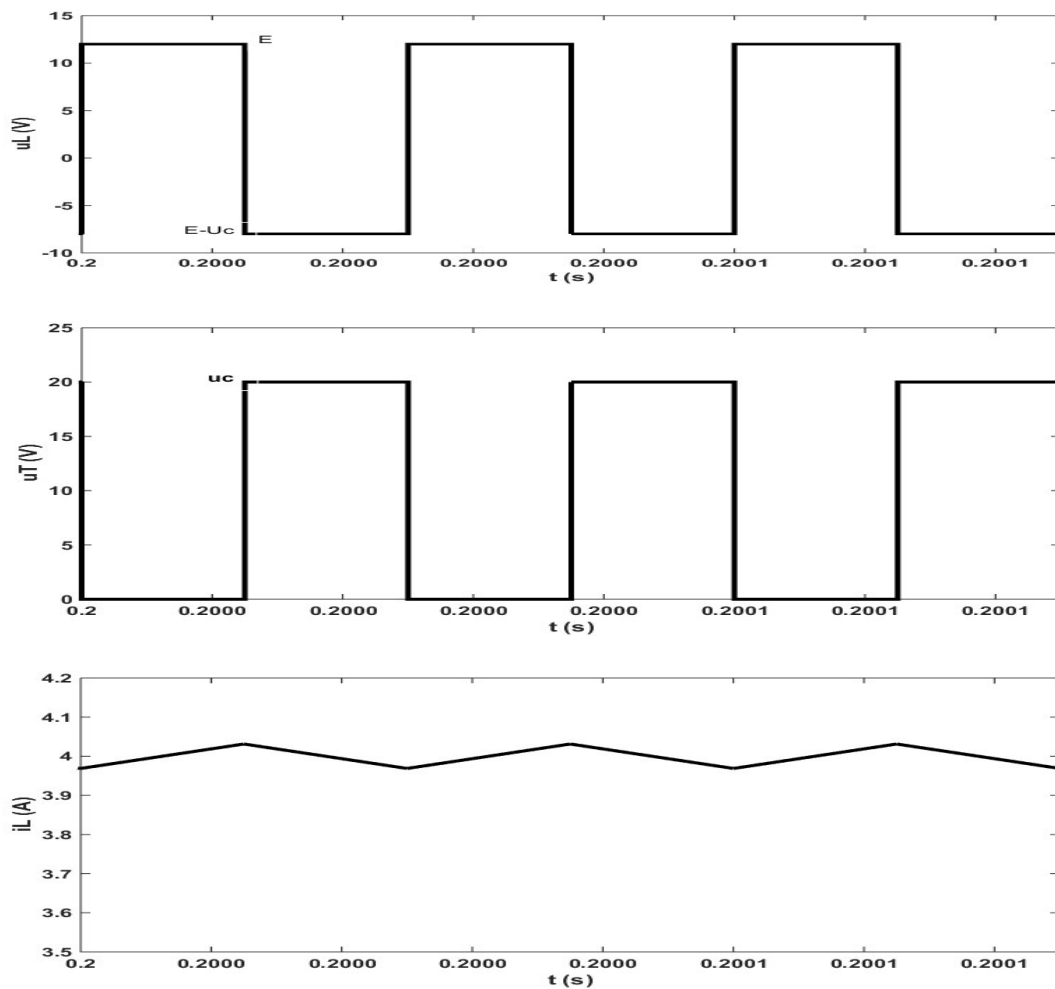


Figure 4.12 Equivalent diagram of a parallel chopper for  $t \in [DT, T]$

#### 4.5.1.2. Waveforms of the Main Quantities

The waveforms of the currents and voltages in the converter and its load are shown for illustration purposes in figure 4.13.



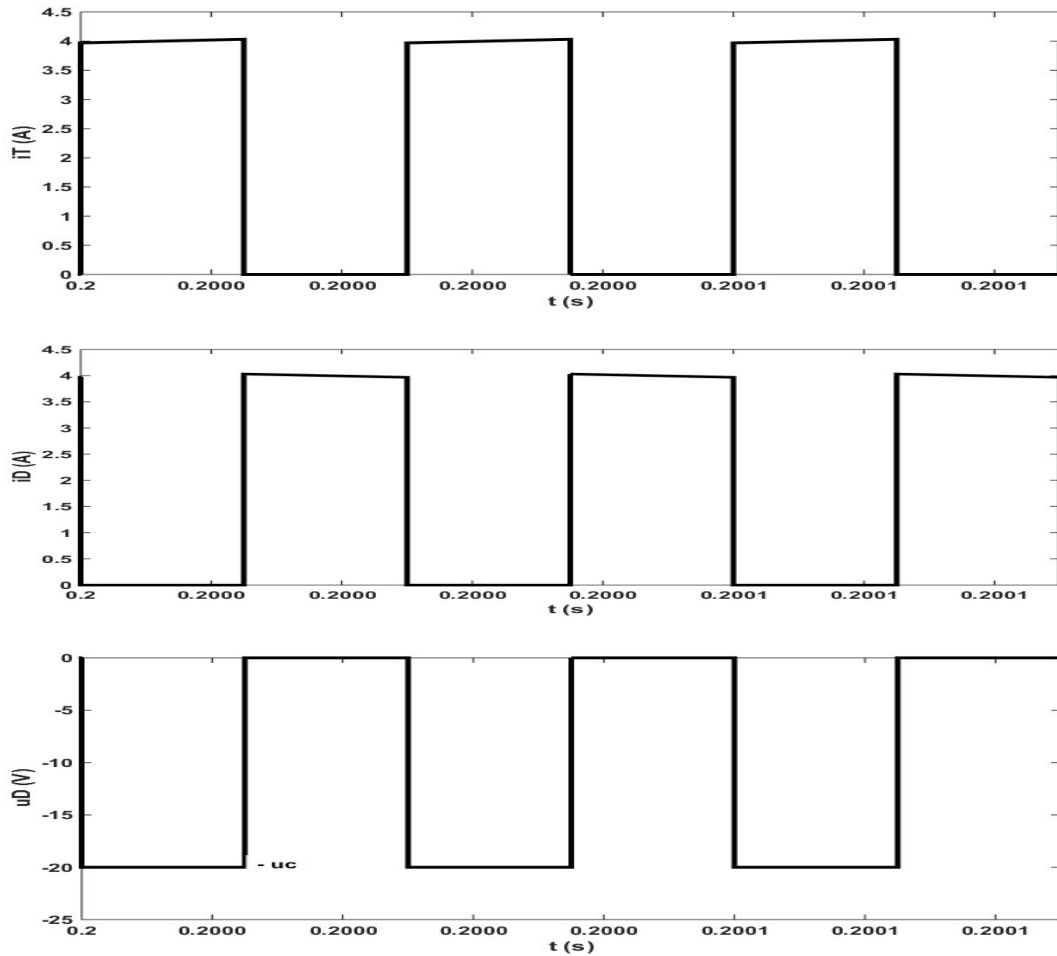


Figure 4.13 Waveform of the main quantities of a parallel chopper for 40kHz switching frequency case

In steady state, the average voltage across the inductance is zero. Therefore:

$$V_{Lavg} = \frac{1}{T} \int L.di_L = 0 \Rightarrow \frac{1}{T} \int_0^{D.T} E.dt + \frac{1}{T} \int_{D.T}^T (U_c - E).dt = 0 \Rightarrow D.E + (1-D).(U_{cmoy} - E) = 0$$

$$\Rightarrow E = (1-D).U_{cmoy}$$

So

$$U_{cavg} = \frac{E}{1-D}$$

#### 4.5.2. Discontinuous conduction:

Conduction is discontinuous if the minimum current value  $I_{LMIN}$  is zero at each period at  $t=D_2T$  for  $D_2T \in [D_1T, T]$ ; i.e.  $i(D_2T) = 0$ .

##### 4.5.2.1. Functional Analysis

We can break this analysis down into three distinct parts [6,7,8]:

1-  $0 < t < D_1 T$  (TH closed, D open).

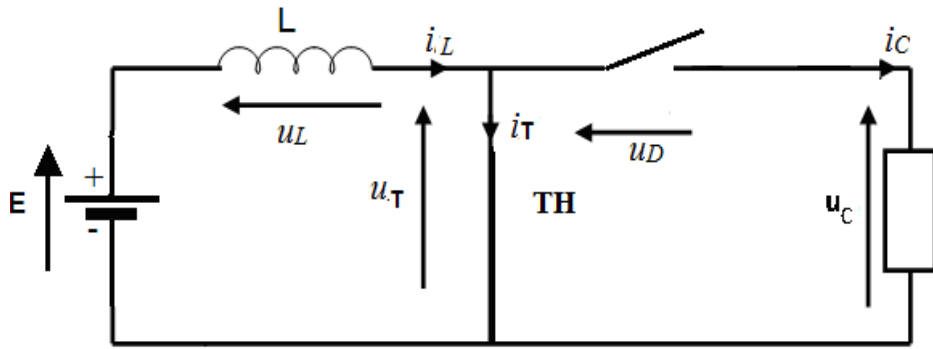


Figure 4.14 Equivalent diagram of a parallel chopper for  $t \in [0, D_1 T]$

Let us determine the current  $i(t)$ : we have  $E \gg R i_c(t)$  therefore

$$E = R i_L + L \frac{di_L}{dt}(t) \text{ with } i_L(0) = 0 \text{ and } i_L(D_1 T) = i_{\max}$$

So

$$E = L \frac{di_L}{dt}(t) \Rightarrow i_L(t) = \frac{E}{L} t + A$$

at  $t = 0$  we have  $i_L(0) = 0$

$$i_L(0) = \frac{E}{L} \cdot 0 + A = 0 \Rightarrow A = 0$$

$$i_L(t) = \frac{E}{L} t$$

2-  $D_1 T < t < D_2 T$  (TH open, D closed).

we have :  $U_{TH} = U_C$ ,  $v_D = 0$ ,  $i_{TH} = 0$ ,  $i_D = i_L$

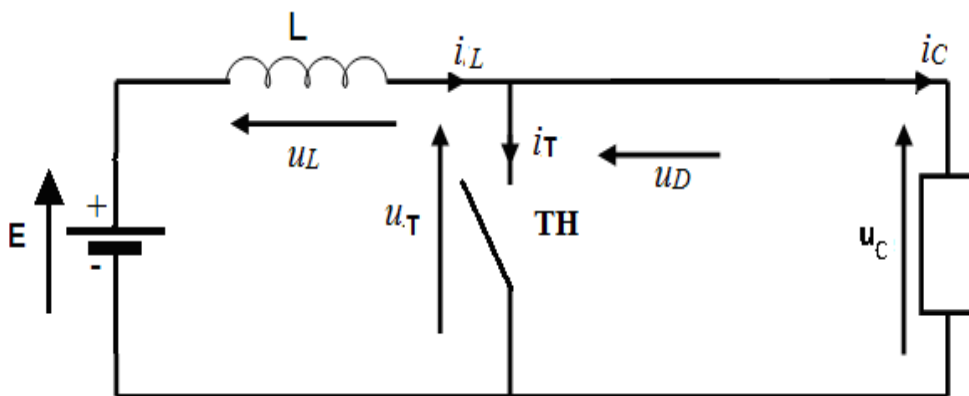


Figure 4.15 Equivalent diagram of a parallel chopper for  $t \in [D_1 T, D_2 T]$

Let us determine the current  $i_L(t)$ : we have

$$E = E_c + L \frac{di_L}{dt}(t) \text{ with } i_L(T) = i_{\min} \text{ and } i_c(D_1 T) = i_{\max}$$

So

$$E = E_c + L \frac{di_L}{dt}(t) \Rightarrow E - E_c = L \frac{di_L}{dt} \Rightarrow i_L(t) = \frac{E - E_c}{L} \cdot t + A$$

at  $t = D_2 T$  we have  $i_c(D_2 T) = 0$

$$i_L(D_1 T) = \frac{E - E_c}{L} \cdot D_1 T + A = i_{\max} \Rightarrow A = i_{\max} - \frac{E - E_c}{L} \cdot D_1 T$$

$$i_L(t) = \frac{E - E_c}{L} \cdot (t - D_1 T) + i_{\max}$$

### 3- $D_2 T < t < T$ (TH open, D open).

We have:

$$U_T = E, v_D = E - E_c, i_T = 0, i_D = 0, i_L = 0 \text{ et } v_L = 0$$

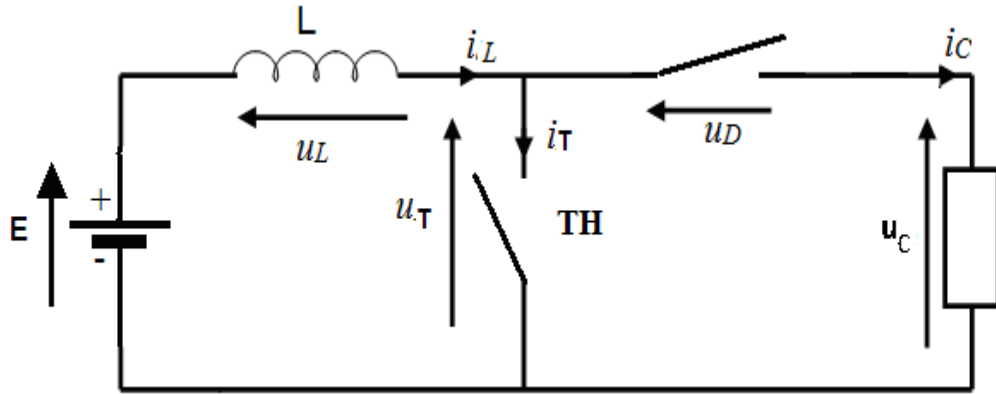


Figure 4.16 Equivalent diagram of a parallel chopper for  $t \in [D_2 T, T]$

$$U_c = 0.$$

#### 4.5.2.2. Relationship between input and output voltages

We have

$$\begin{aligned} V_{Lavg} = \frac{1}{T} \int L di_L = 0 &\Rightarrow \frac{1}{T} \int_0^{D_1 T} E dt + \frac{1}{T} \int_{D_1 T}^{D_2 T} (U_c - E) dt = 0 \Rightarrow D_1 E + (D_2 - D_1)(U_{c moy} - E) = 0 \\ &\Rightarrow D_2 E = (D_2 - D_1) U_{c moy} \end{aligned}$$

$$U_{c avg} = U_c = \frac{D_2}{D_2 - D_1} E$$

The only difference with the operating principle described previously is that the inductance is completely discharged at the beginning of the cycle (see the waveforms in figure 4.17).

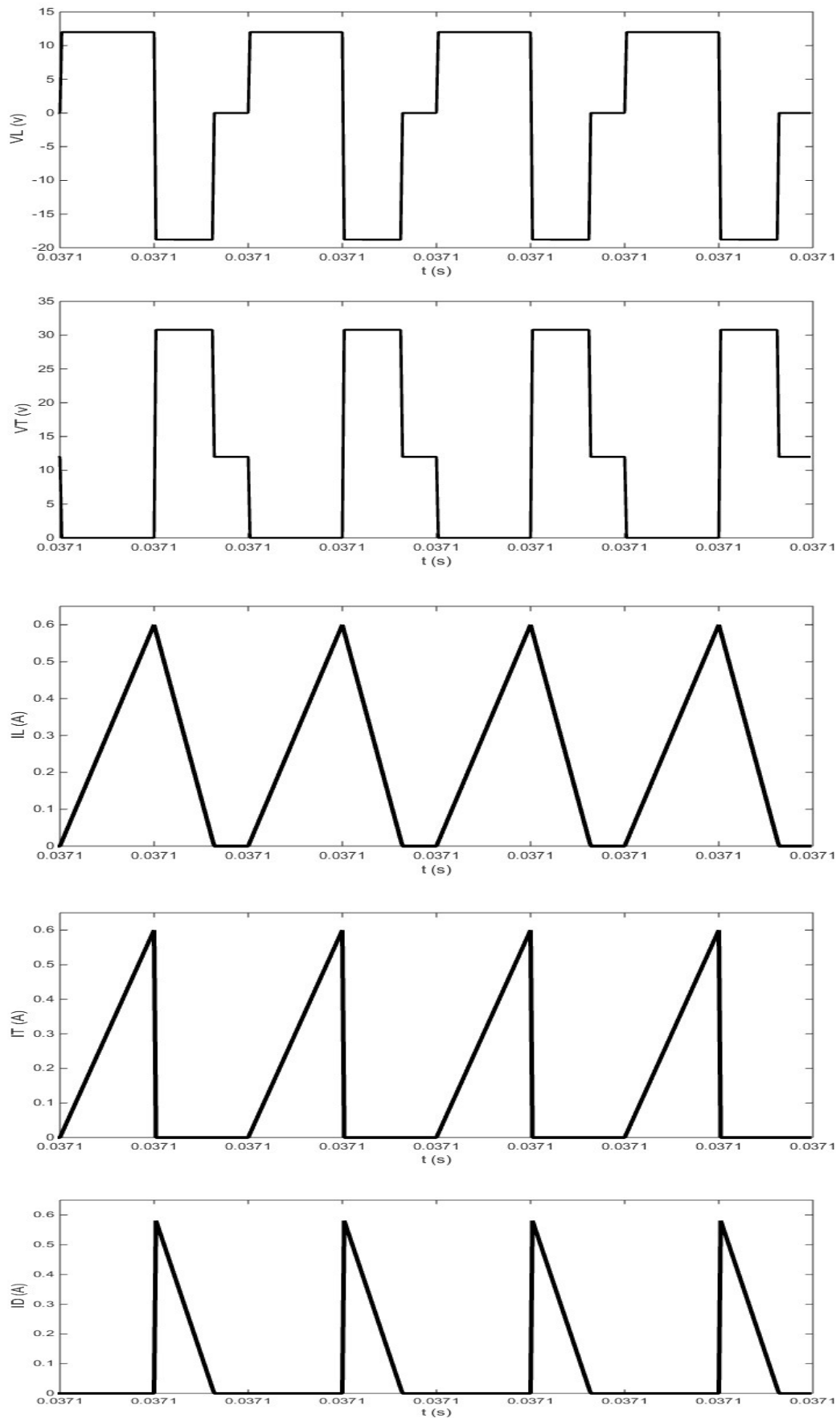


Figure 4.17 Waveform of the main quantities of a parallel chopper Discontinuous conduction

4.6. Buck-Boost Chopper

Another type of boost chopper can be obtained by modifying the classic structure. Rather than starting with a simple configuration consisting of a voltage source, a switch, and a current source, an energy storage element is inserted between the two. This results in a structure similar to: source 1 + switch + storage element + switch + source 2. This architecture allows indirect energy conversion between two generators (or sources) of the same type [6,7,8].

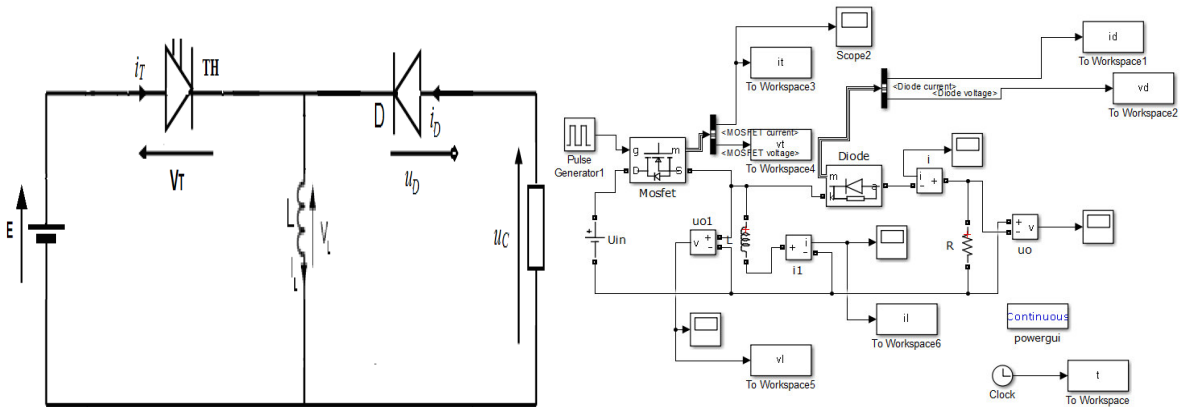


Figure 4.18 Schematic of a buck-boost chopper

4.6.1. Study of an Energy Storage Chopper [6, 7, 8]

As in the previous section, the system is studied within the framework of an approximation:

- The load is assumed to be at constant voltage  $U_{av} = U_C$ .
- The storage inductance  $L$  is resistance-free (non-dissipation of stored energy).

4.6.2. Operational Analysis

The two operating phases are:

1-  $0 < t < DT$  (TH closed, D open)

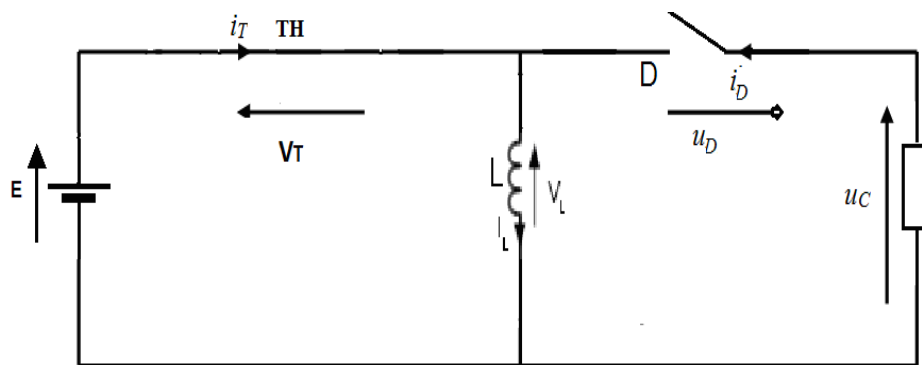


Figure 4.19: Equivalent diagram of a buck-boost chopper for  $t \in [0, DT]$

Let us determine the current  $i(t)$ : we have

$$E = Ri_L + L \frac{di_L}{dt}(t) \text{ with } i_L(0) = i_{\min} \text{ and } i_L(D.T) = i_{\max}$$

So

$$E = L \frac{di_L}{dt}(t) \Rightarrow i_L(t) = \frac{E}{L}t + A$$

at  $t = 0$  we have  $i_L(0) = i_{\min}$

$$i_L(0) = \frac{E}{L}.0 + A = i_{\min} \Rightarrow A = i_{\min}$$

$$i_L(t) = \frac{E}{L}t + i_{\min}$$

## 2- $DT < t < T$ (TH open, D closed).

Let us determine the current  $i(t)$ : we have

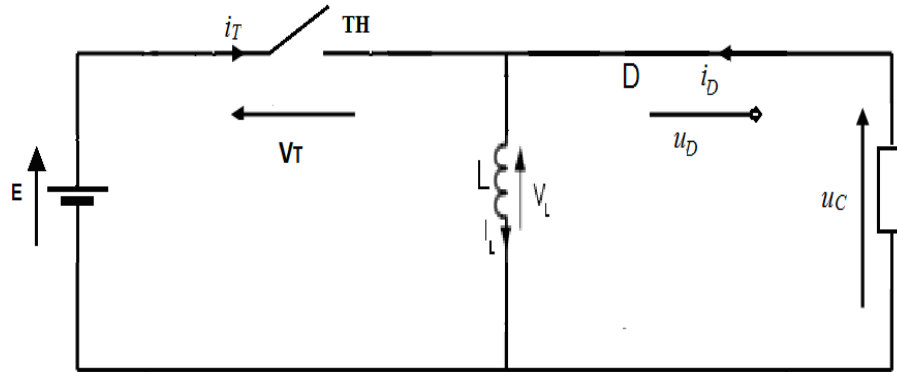


Figure 4.20 Equivalent diagram of a buck-boost chopper for  $t \in [DT, T]$

Applying the mesh laws, we have:

$$0 = U_c + L \frac{di_L}{dt}(t) \text{ with } i_L(T) = i_{\min} \text{ and } i_L(D.T) = i_{\max}$$

So

$$0 = U_c + L \frac{di_L}{dt}(t) \Rightarrow -U_c = L \frac{di_L}{dt}(t) \Rightarrow i_L(t) = \frac{-U_c}{L}t + A$$

at  $t = D.T$  we have  $i_L(D.T) = i_{\max}$

$$i_L(D.T) = \frac{-U_c}{L}.D.T + A = i_{\max} \Rightarrow A = i_{\max} + \frac{U_c}{L}.D.T$$

$$i_L(t) = \frac{-U_c}{L}.(t - D.T) + i_{\max}$$

### 4.6.3. Relationship between input and output voltages

In steady state, the average voltage across the inductor is zero.

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$$V_{Lavg} = \frac{1}{T} \int L di_L = 0 \Rightarrow \frac{1}{T} \int_0^{D\alpha T} E dt + \frac{1}{T} \int_{D\alpha T}^T (-U_c) dt = 0 \Rightarrow D.E + (1-D)(-U_c) = 0$$

$$\Rightarrow D.E = (1-D)U_c$$

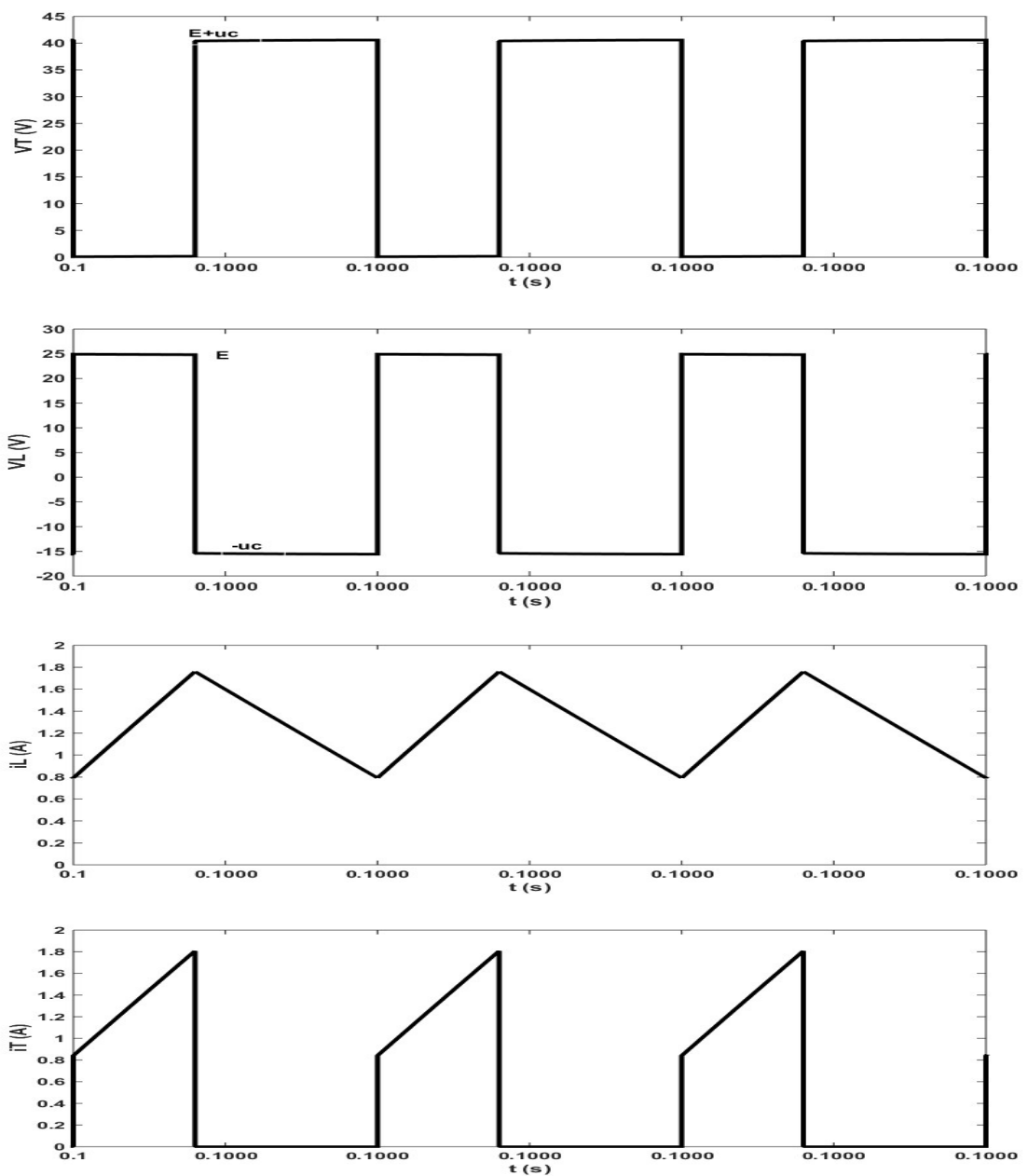
So

$$U_{cavg} = U_c = \frac{D}{1-D} . E$$

If the duty cycle is less than 0.5: the chopper operates as a step-down chopper.

If the duty cycle is greater than 0.5: the chopper operates as a step-up chopper.

### 4.6.4. Waveforms of the main variables



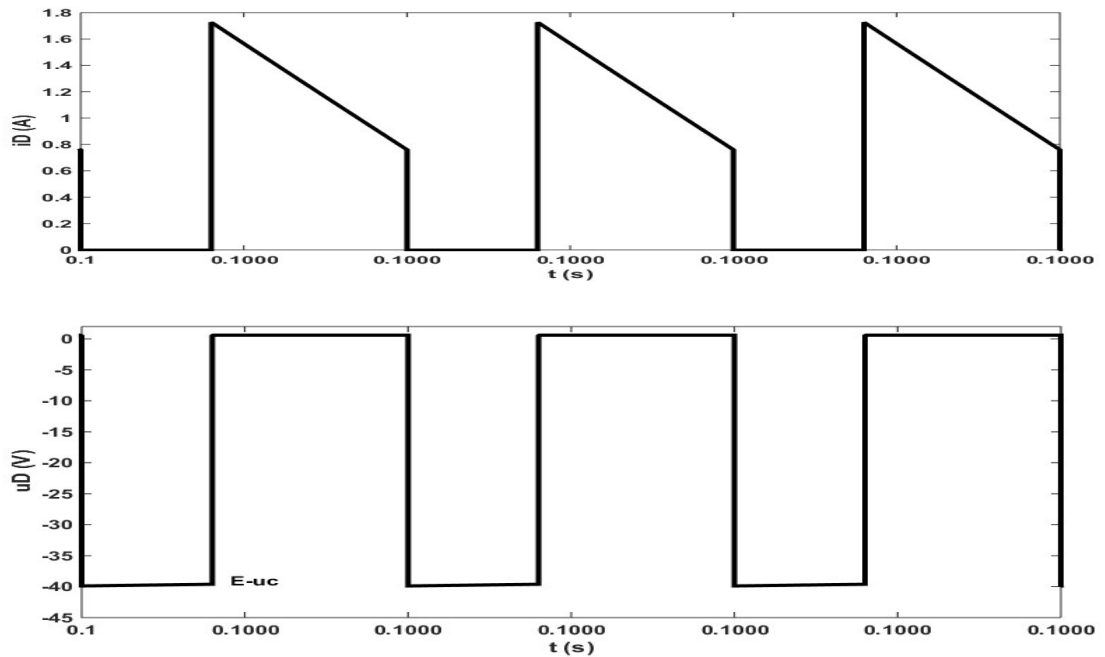


Figure 4.21 Waveforms of the main quantities of an accumulation chopper for 40kHz switching frequency case

#### 4.7. Current-reversible choppers

The schematic diagram is shown in figure 4.22 [6,7,8].

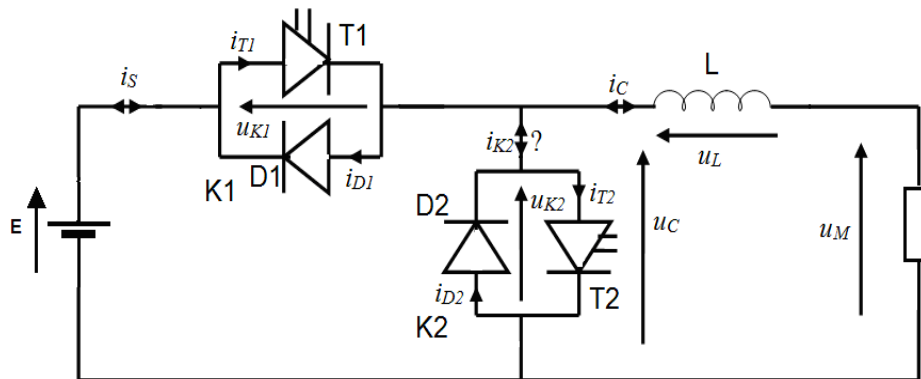


Figure 4.22 Diagram of a reversible current chopper

##### 4.7.1. Study of a Reversible Current Chopper

With this structure, we can consider different types of operation:

**First case:** T1 is controlled and T2 is not controlled (open), D1 is affected and D2 is not. This is called series chopper operation.

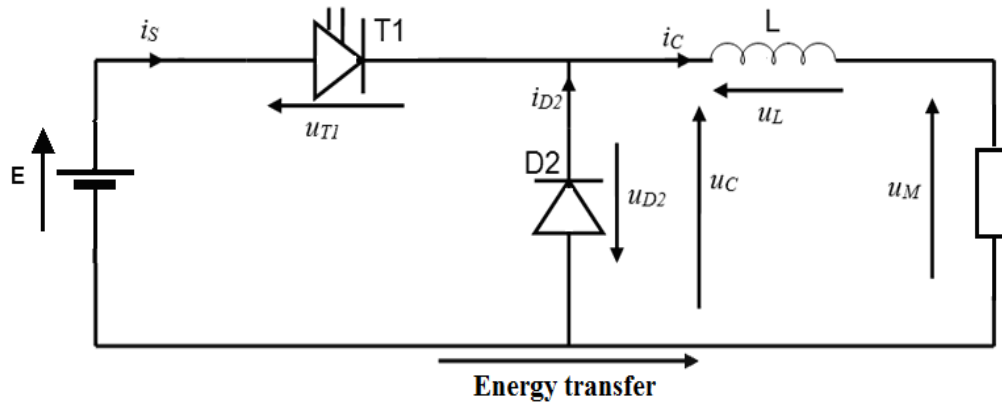


Figure 4.23 Diagram of a Reversible Chopper Operating in Series

**Second case:** T2 is controlled and T1 is not controlled (open), D2 is affected and D1 is not. This is parallel chopper operation.

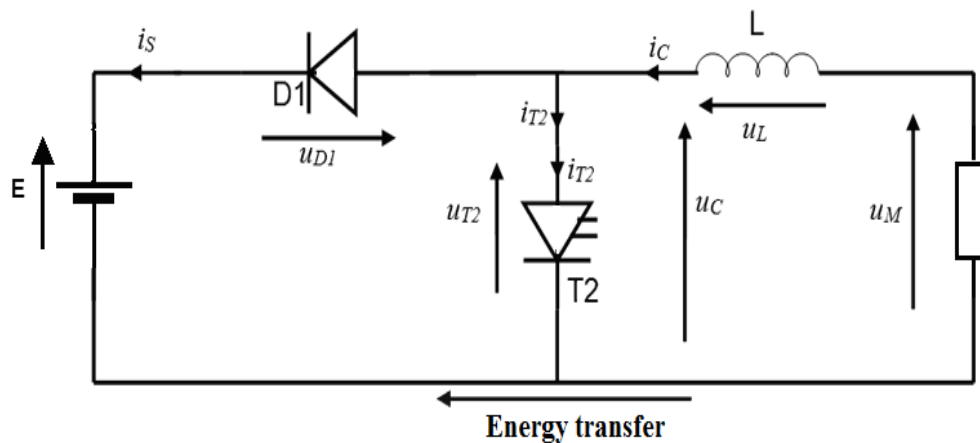


Figure 4.24: Diagram of a reversible chopper operating in parallel

This is true under the following conditions:

- that the source E is current-reversible,
- that the load properly fulfills its source role and is also current-reversible.

This results in a double chopper, or a chopper with two switches, that is current-reversible. Such a structure is well-suited for variable-speed energy recovery in the case of a DC machine.

#### 4.8. Voltage-reversible choppers

The structure is shown in figure 4.25 [6, 7, 8].

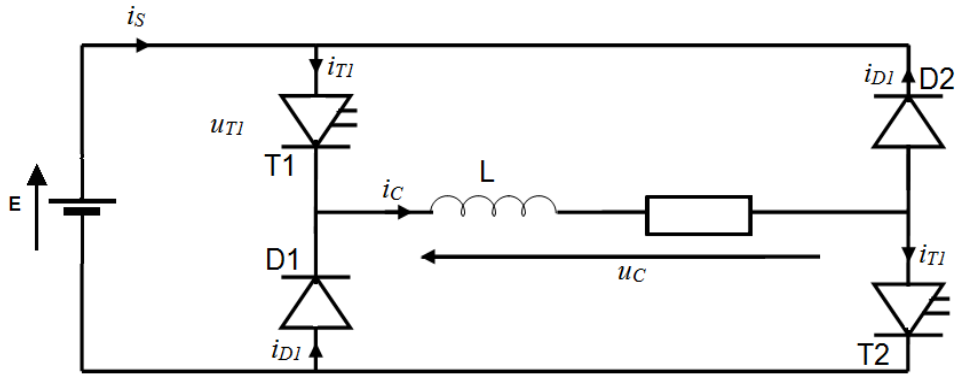
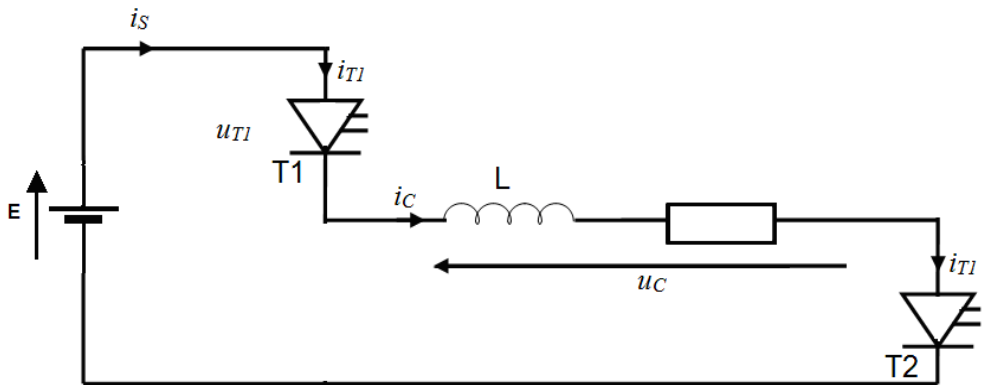


Figure 4.25: Diagram of a voltage reversible chopper

#### 4.8.1. Study of a Reversible Voltage Chopper

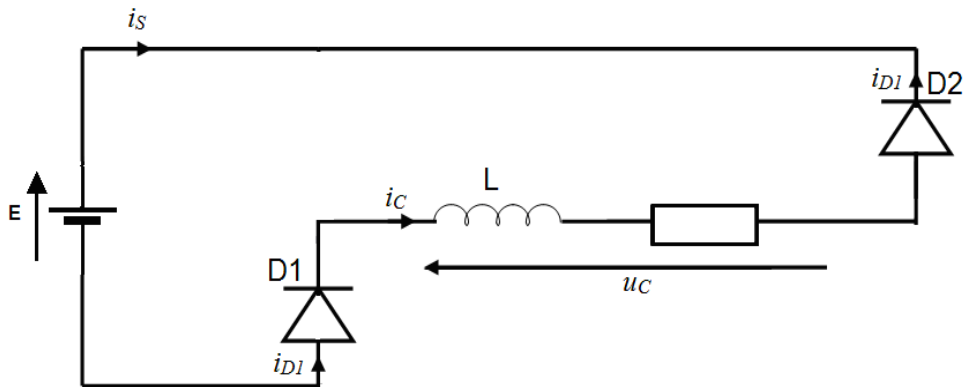
With this structure, we can consider different types of operation :

**First phase  $0 < t < DT$ :** T1 and T2 are controlled (closed), D1 and D2 are open. This is series chopper operation.



$$U_c = E$$

**Second phase  $DT < t < T$ :** T1 and T2 are not controlled (open), D1 and D2 are closed. This is parallel chopper operation.



$$U_c = -E$$

#### 4.8.2. Relationship between input and output voltages

In steady state, the average voltage across the inductor is zero.

$$U_{avg} = \frac{1}{T} \int_0^T U(t).dt = \frac{1}{T} \int_0^{D.T} E.dt + \frac{1}{T} \int_{D.T}^T (-E).dt = D.E - (1-D).E$$

So

$$U_{avg} = (2.D - 1).E$$

### 4.9. H-Shaped Choppers or 4-Switch Choppers

The most comprehensive and versatile structure is the 4-switch chopper, which is arranged as follows:

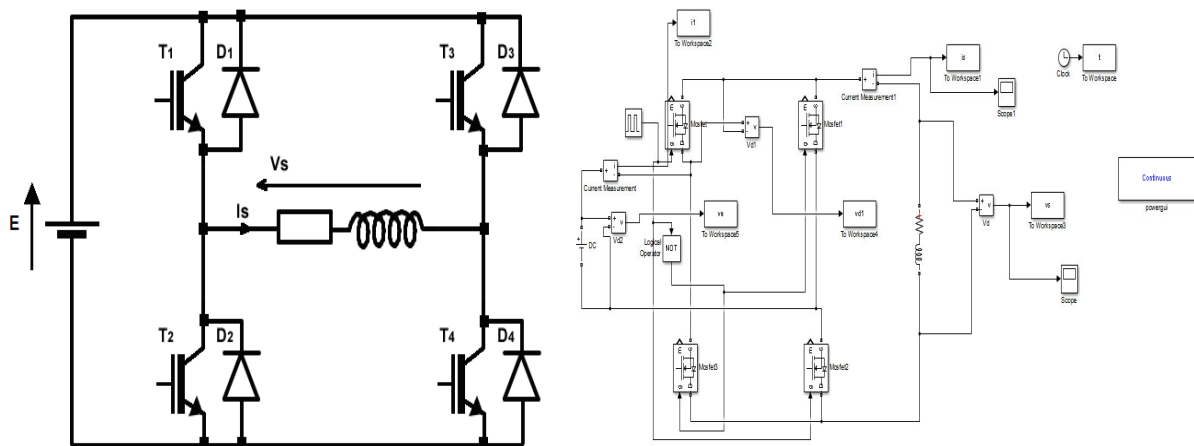


Figure 4.26 Diagram of an H-Chopper

Energy transfer can occur in both directions, with both voltage and current reversibility. Consider, for example, a DC voltage source that is reversible to current, such as a storage battery or a recovery/dissipation system associated with a unidirectional source, and a load consisting, as previously, of a DC machine [6,7,8].

Depending on the control mode adopted, it is then possible to identify the different operating modes of choppers, in terms of voltage and current reversibility.

#### 4.9.1. Operational Analysis

The four operating phases are [6,7,8]:

##### 4.9.1.1.. first Quadrant Operation:

During first quadrant operation, switch T4 remains continuously ON while switch T3 stays OFF. switch T1 is switched ON and OFF to regulate the load voltage. When both T1 and T4 are ON, the load voltage  $V_s$  equals the source voltage  $V_e$ , and a positive load current  $i_s$  begins to flow. Since both  $V_s$  and  $i_s$  are positive, the system operates in the first quadrant. When T1 is turned OFF, the positive current freewheels through T4 and diode D2, maintaining current continuity. In this mode, the type chopper functions as a step-down (buck) chopper.

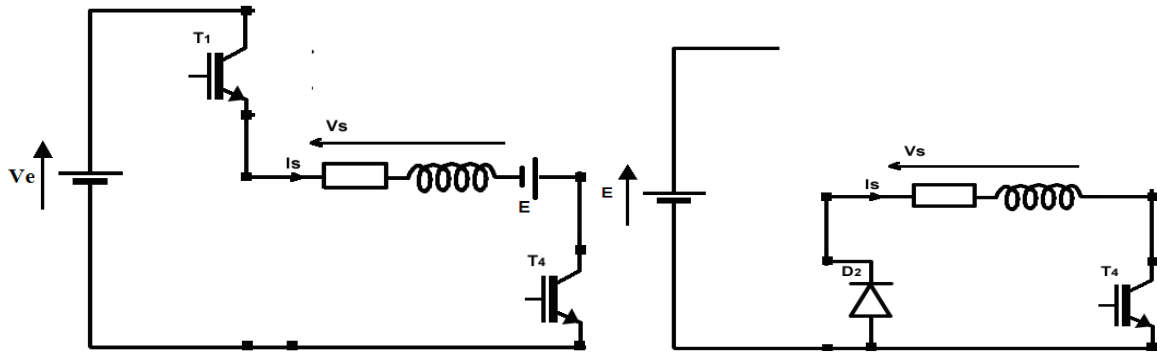


Figure 4.27 Equivalent circuit of a Chopper in first Quadrant

#### 4.9.1.2.. Second Quadrant Operation

In second quadrant operation, switch T2 is turned ON while the other three switches remain OFF. When T2 conducts, a negative current begins to flow through the inductor L, T2, the load, and diode D4. During this period, energy is stored in the inductor. When T2 is turned OFF, the stored energy in the inductor forces current to flow back to the source through diodes D1 and D4. At this point, the voltage across the inductor ( $E + L \frac{di}{dt}$ ) exceeds the source voltage  $V_s$ , enabling reverse power flow. In this mode, the chopper operates as a step-up (boost) converter, transferring energy from the load back to the source [6,7,8].

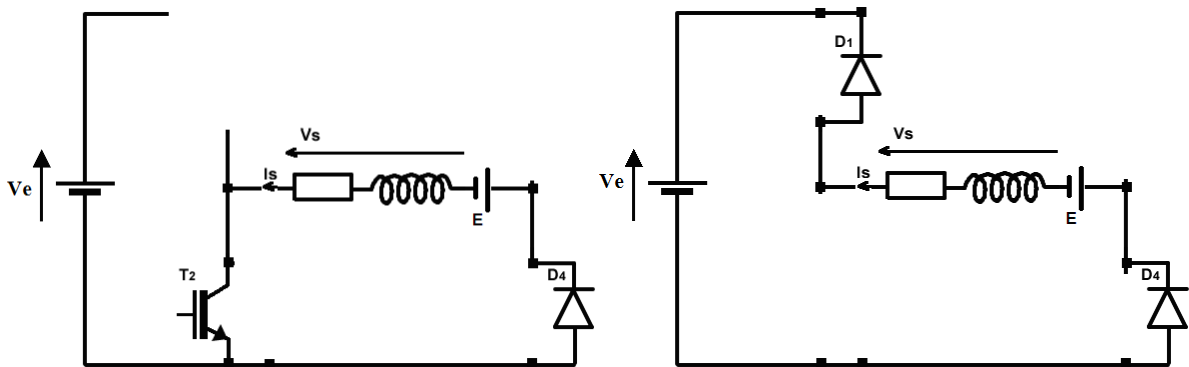


Figure 4.28 Equivalent circuit of a Chopper in second Quadrant

#### 4.9.1.3.. Third Quadrant Operation

In third quadrant operation, switch T1 remains OFF, T2 is kept ON, and switch T3 is actively switched. For this mode to function correctly, the polarity of the load must be reversed. When T3 is turned ON along with T2, the load is connected to the source voltage  $V_s$ , causing a negative load current  $i_e$  to flow. Since both the voltage and current are negative, the system operates in the third quadrant. In this mode, the chopper functions as a step-down (buck) converter, delivering power from the source to the load in the reverse direction.

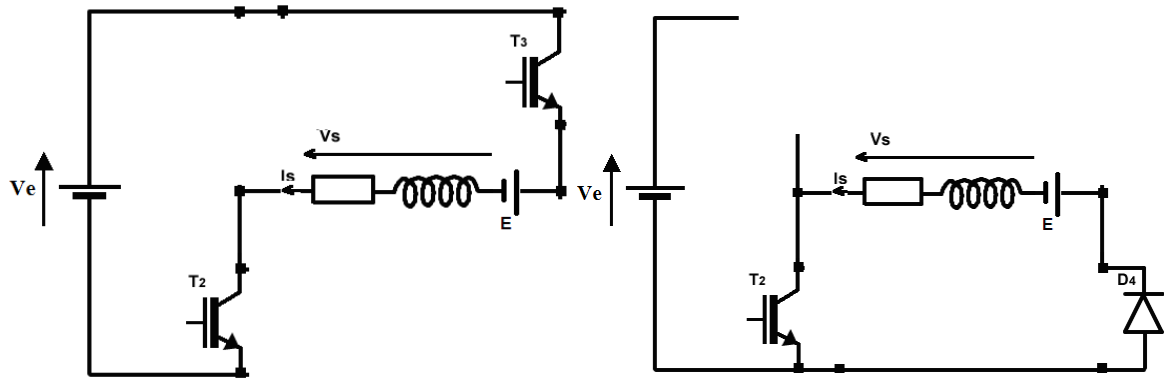


Figure 4.29 Equivalent circuit of a Chopper in third Quadrant

#### 4.9.1.4.. Fourth Quadrant Operation

In fourth quadrant operation, switch T4 is actively switched while T1, T2, and T3 remain OFF. When T4 is turned ON, a positive current begins to flow through T4, diode D2, the load E, and the inductor L, allowing energy to be stored in the inductor. When T4 is turned OFF, the energy stored in the inductor is returned to the source via diodes D2 and D3. During this process, the load voltage becomes negative while the current remains positive, confirming fourth quadrant operation. In this mode, the chopper functions as a step-up (boost) converter, with power flowing from the load back to the source [6, 7, 8].

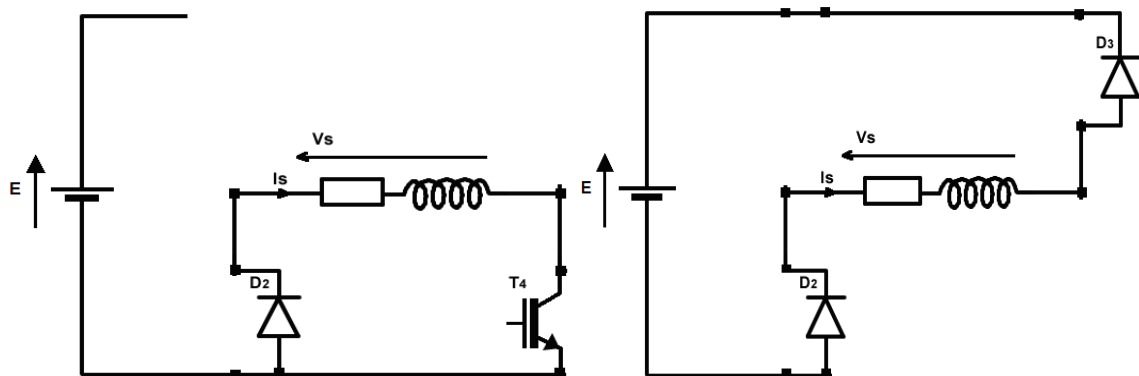


Figure 4.30: Equivalent circuit of a Chopper in fourth Quadrant

The below figure illustrates the characteristics of the chopper in four quadrants.

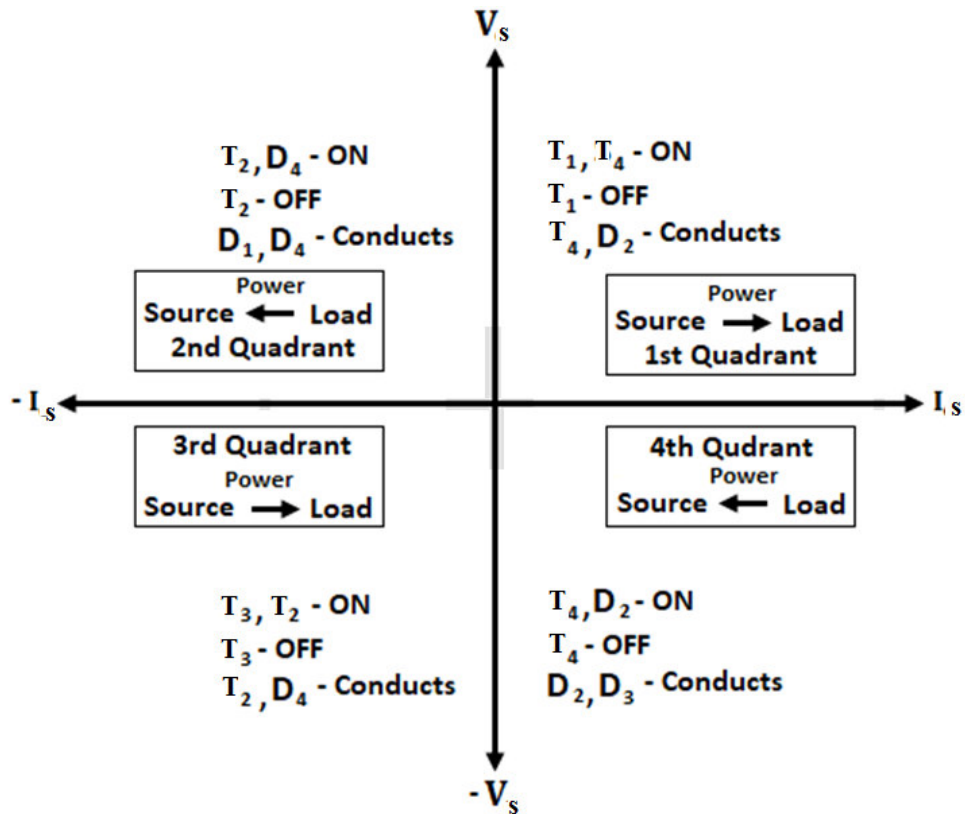


Figure 4.31 Characteristics of the chopper in four quadrants

#### 4.9.2. Relationship between input and output voltages

In steady state, the average voltage across the inductor is zero.

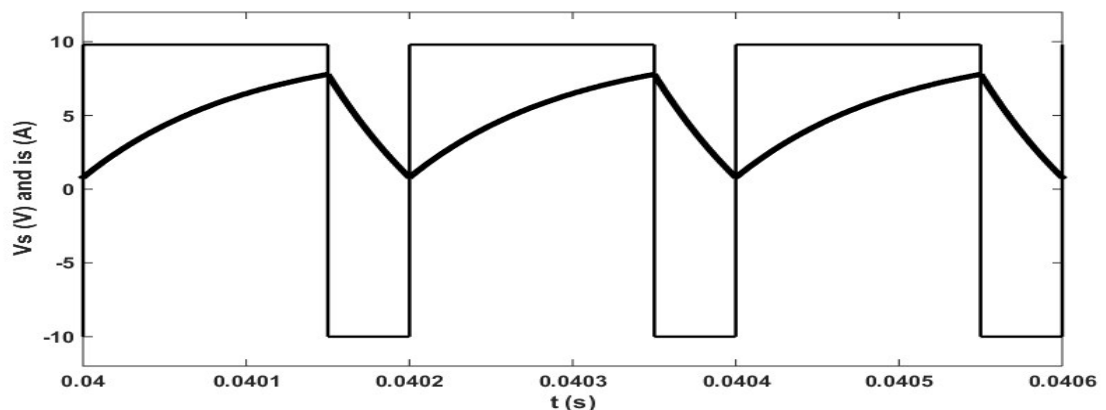
$$U_{cmoy} = \frac{1}{T} \int_0^T U(t).dt = \frac{1}{T} \int_0^{D.T} E.dt + \frac{1}{T} \int_{D.T}^T (-E).dt = D.E - (1-D).E$$

So

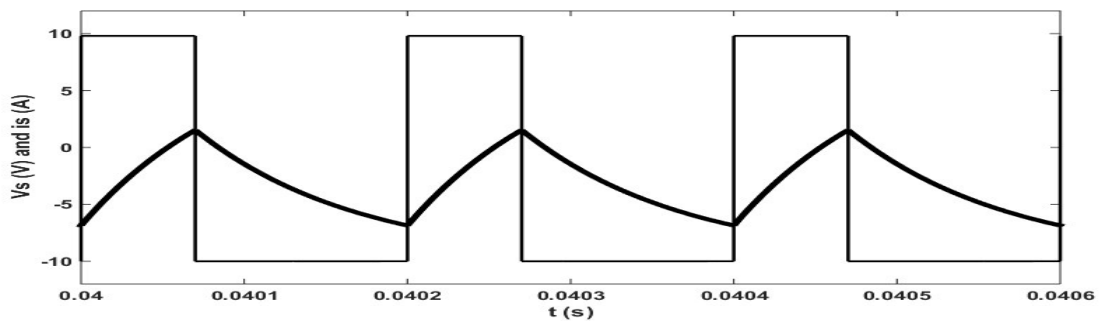
$$U_{cmoy} = (2.D - 1).E$$

The simulation results are illustrated in the following figure for two different duty cycle.

for  $D=0,75$



for  $D=0,35$



**Figure 4.32** Waveforms of the main quantities of an H-chopper for 5kHz switching frequency case

### 4.10. Conclusion:

The choppers we studied are used in multiple applications. They have the advantage of providing a rapidly variable output voltage (if the switching frequency allows). The two- and four-quadrant versions, derived from the H-bridge structure, are 100% compatible with DC motors, from which they instantly recover kinetic energy. They will therefore be very useful in all robotics applications, where very high dynamic range is required. They have even supplanted thyristor rectifiers in this field, whenever the power demand remains modest.

# **Chapter 5: DC-AC Electrical Energy Conversion**

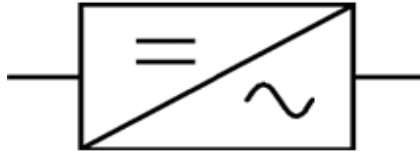
## Chapter 5: DC-AC Electrical Energy Conversion

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### 5.1. Introduction

An inverter is a static DC-AC converter; it produces an AC voltage that can be adjusted in frequency and RMS value from a given DC voltage.

### 5.2. Symbol:



### 5.3. Classification

We classify them according to:

#### 5.3.1. Nature of the power supply:

##### 5.3.1.1. Current inverter:

If the DC stage is considered a current source, the associated DC-AC converters are current inverters.

Current inverters are connected to an AC current source, figure 5.1.

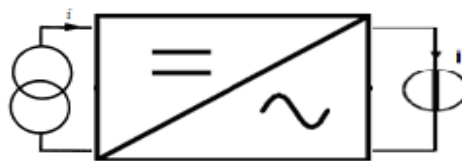


Figure 5.1 Current Inverter

##### 5.3.1.2. Voltage Inverter:

If the DC stage is considered a voltage source, the associated DC-AC converters are voltage inverters.

Voltage inverters are connected to an AC voltage source, figure 5.2.

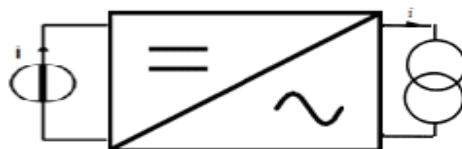


Figure 5.2 Voltage Inverter

#### 5.3.2. The number of load phases: we will distinguish between:

- single-phase inverters

## Chapter 5: DC-AC Electrical Energy Conversion

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- three-phase inverters.

### 5.4. Waveform:

The alternating current waveform of the output voltage is determined by the system. Depending on the shape of this output voltage, inverters are classified into several categories:

**5.4.1. Two-state inverter (voltage in square waves  $+U$ ,  $-U$ ):** The effective value of the output voltage is not adjustable and depends on the DC input voltage [7, 8, 9].

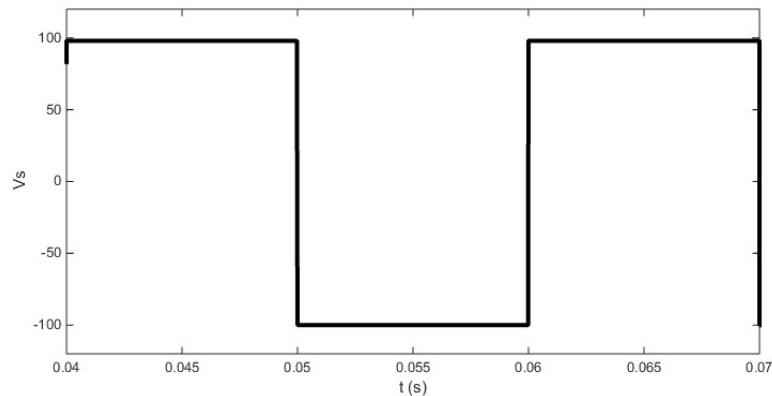


Figure 5.3 Two-state inverter

**5.4.2. Three-state inverters ( $+U$ ,  $0$ ,  $-U$ ):** The effective value of the output voltage can be adjusted by adjusting the pulse duration [7, 8, 9].

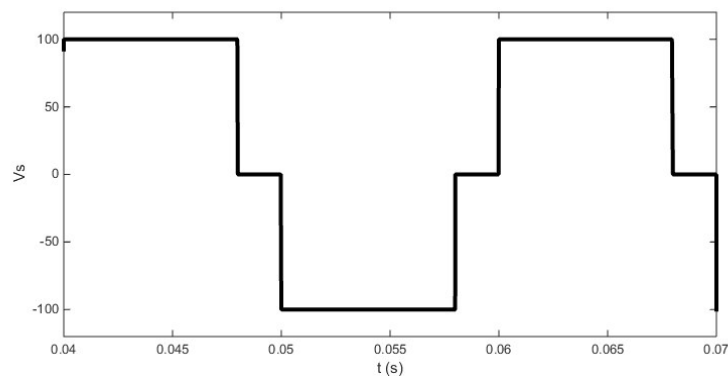


Figure 5.4 Three-state inverter

**5.4.3. Pulse Width Modulation (PWM) inverters:** The output waveform consists of a train of pulses of variable width and spacing. This reduces the harmonic content. It is even possible to obtain an output waveform close to a sine wave [7, 8, 9].

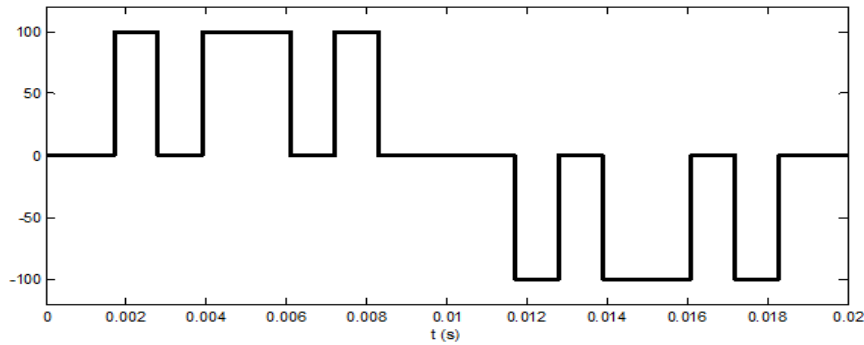


Figure 5.5 PWM Inverter

**5.4.4. Inverters with a step-like output voltage:** The output waveform consists of the sum or difference of variable-width pulses, and its general shape approximates a sinusoid as closely as possible [7, 8, 9].

One of the problems with this system is the large number of elements.

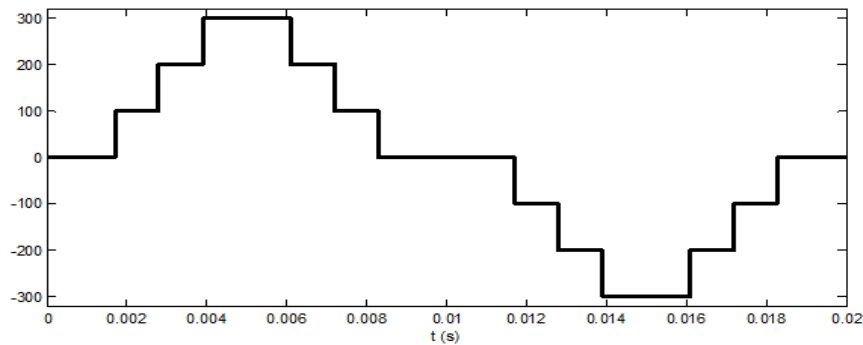


Figure 5.6 Stair-step output voltage

### 5.5. Output signal quality

The spectrum of a rectangular signal includes a fundamental wave (rank  $n = 1$ , angular frequency  $\omega$ ) and harmonic waves (rank  $n > 1$ , angular frequency  $\omega_n = n \cdot \omega$ ) of varying amplitude. In what follows, we compare the performance of each inverter type to the ideal case (pure sine wave with angular frequency  $\omega$ ) by calculating the spectrum of the generated signal.

The quality of the resulting voltage waveform will be assessed by the THD, or harmonic ratio related to the fundamental (ideal THD = 0%) [7, 8, 9].

$$THD = \frac{\sqrt{\sum_{h=0}^{\infty} U_{heff}^2}}{U_{1eff}} = \frac{\sqrt{U_{eff}^2 - U_{1eff}^2}}{U_{1eff}} \text{ in voltage}$$

$$THD = \frac{\sqrt{\sum_{h=0}^{\infty} I_{heff}^2}}{I_{1eff}} = \frac{\sqrt{I_{eff}^2 - I_{1eff}^2}}{I_{1eff}} \text{ in current}$$

### 5.6. Inverters are used in several industrial applications:

#### 5.6.1. Controlling the rotation speed of AC machines:

The speed of a synchronous motor is determined by the pulsation of static currents. To change speed, the frequency of the supply voltages must therefore be changed. This requires rectifying the main voltages and then inverting it to the desired frequency [7, 8, 9].

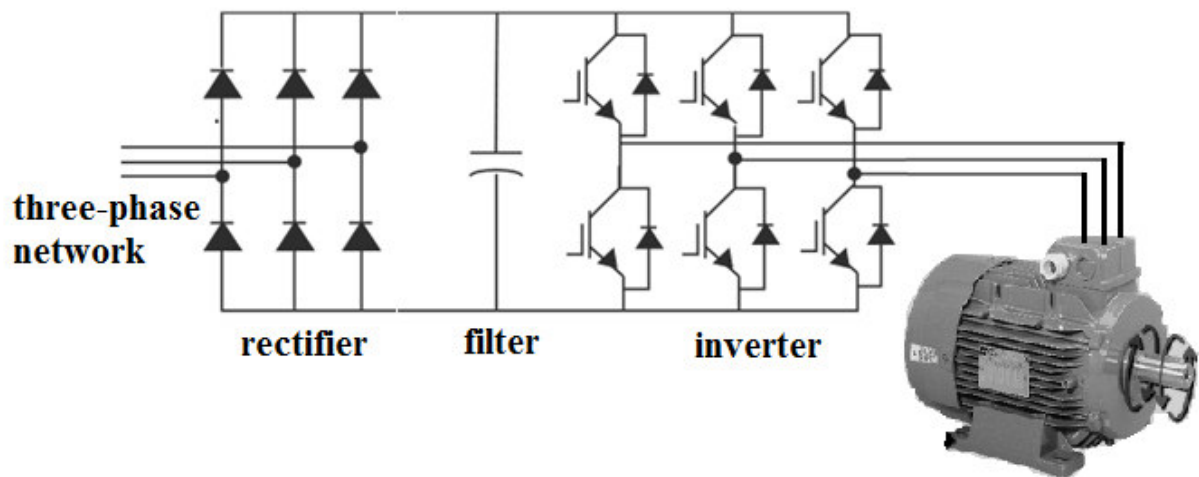


Figure 5.7 Variable supply for an AC machine

#### 5.6.2. Ensuring continuity of emergency power supplies:

In the event of a power outage, the DC voltage stored in the batteries is converted into AC voltage via the inverter, thus maintaining power supply to the equipment. This is particularly important in fields such as IT, where a power outage could lead to data loss and service interruptions [7, 8, 9].

#### 5.6.3. Transportation applications:

Rail transport: The power units currently being developed are powered by induction AC machines. To control their rotation speed, it is necessary to be able to vary their power supply frequency. This is achieved using an inverter, for example, in TGV (high-speed train) systems, trains, and tramways.

Air and maritime transport: Every aircraft produces its own electrical energy needed to operate its on-board equipment.

## Chapter 5: DC-AC Electrical Energy Conversion

### 5.6.4. Uninterruptible Power Supply:

Inverters can provide 24-hour power supply in the event of a power outage.

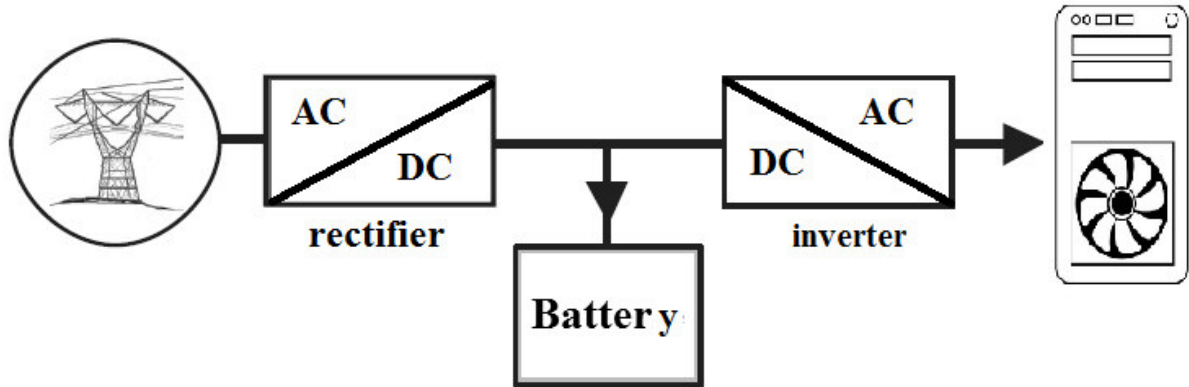


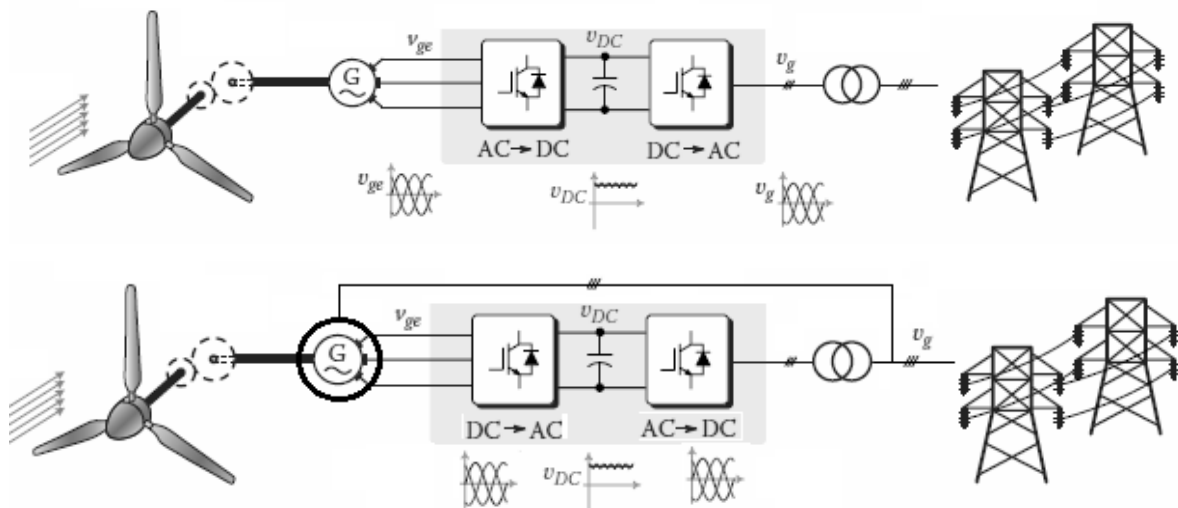
Figure 5.8 Uninterruptible Power Supply

### 5.6.5. Equipment protection:

Inverters protect various equipment in different areas from disturbances.

### 5.6.6. Renewable energy installations:

The connection between renewable energies (wind, photovoltaic, fuel cells, and wave) and the grid is made through inverters, which also allow for the adaptation of the energy delivered by the system and the energy injected into the grid. [7, 8, 9].



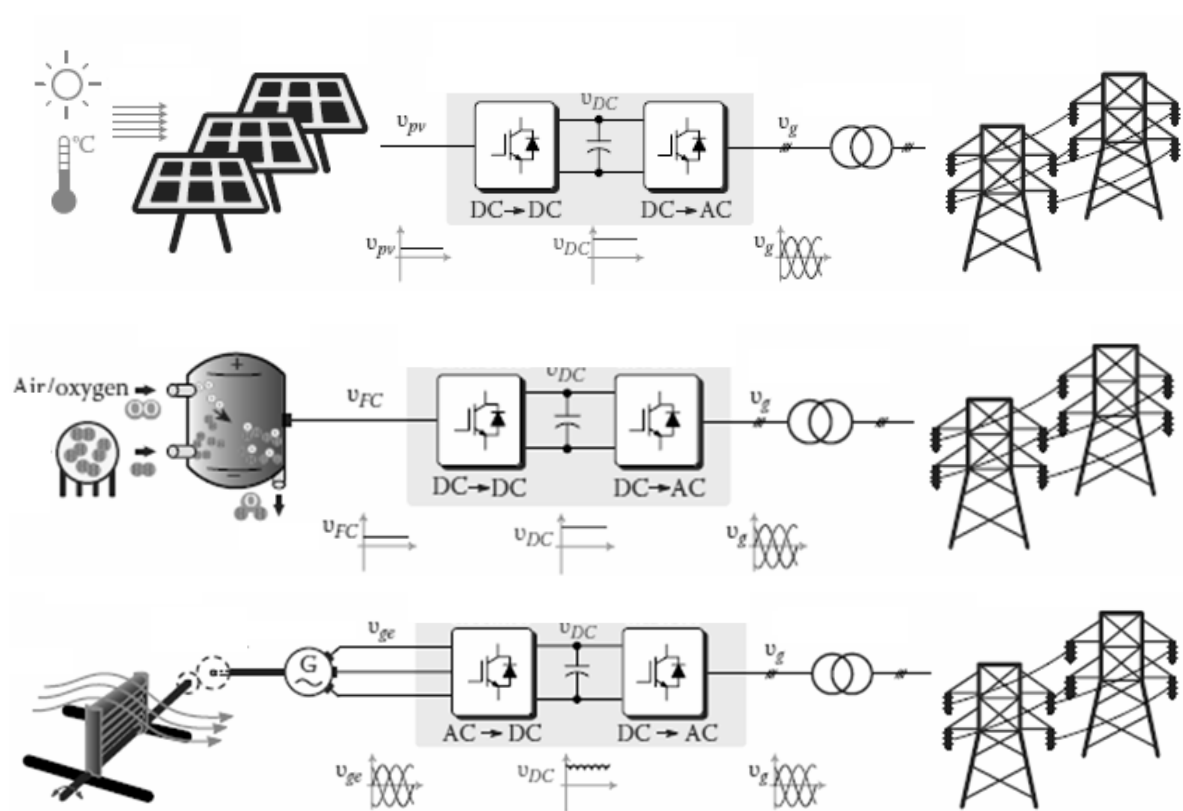


Figure 5.9 Various applications of the inverter in renewable energy systems.

### 5.7. Basic Principle of Single-Phase Power

The basic principle involves alternately connecting a DC source (either voltage or current) to a load, first in one direction and then in the other, to provide it with an AC power supply (either voltage or current). Several structures can be used to achieve this conversion:

- Electronic switch bridge (Fig. 5.10.a): A common configuration for this type of conversion.
- Electronic switch half-bridge (Fig. 5.10.b): This structure requires two power sources to operate.
- Structure using a center-tapped transformer (Fig. 5.10.c): This configuration is equivalent to two loads and can also be used to efficiently achieve the conversion.

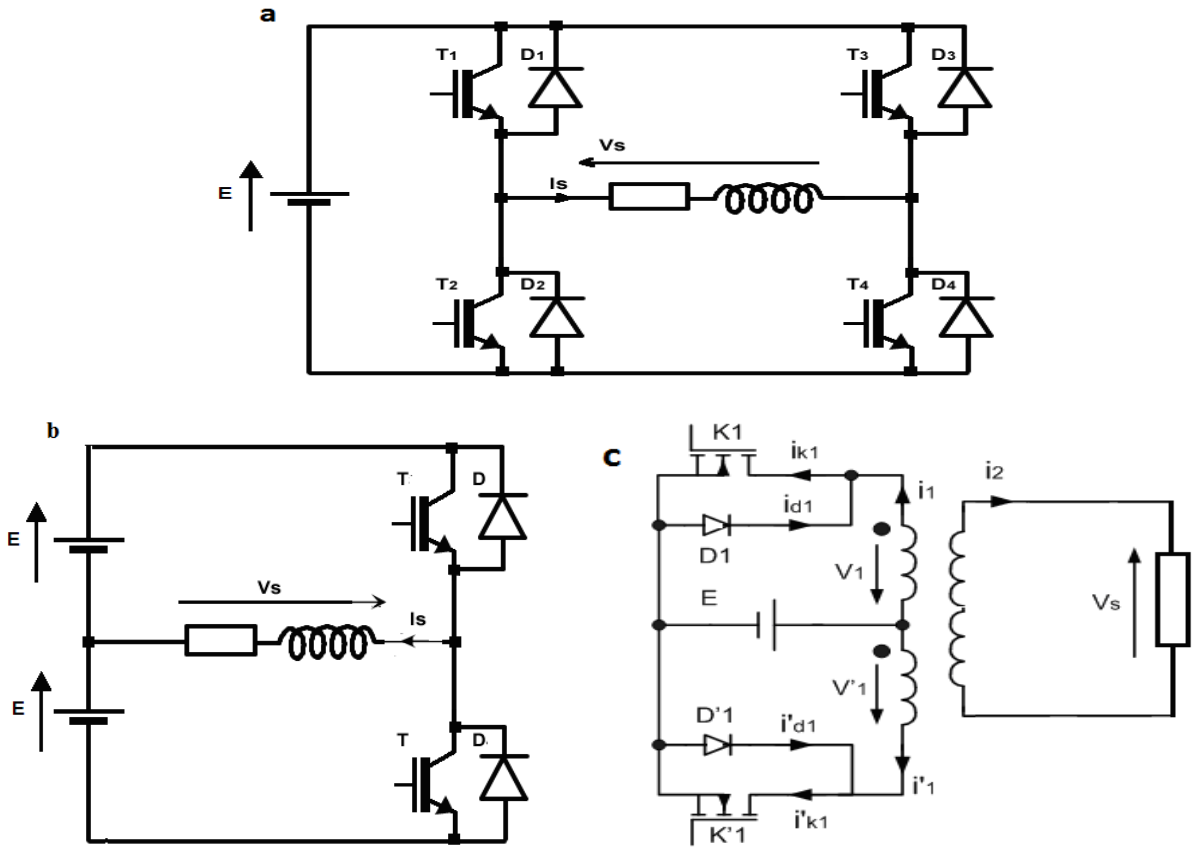


Figure 5.10 Possible structures of a single-phase inverter

### 5.7.1. Operating principle of single-phase voltage inverters

We consider the simplest stand-alone inverter system: a system with two switches with symmetrical control [10, 11, 12].

#### 5.7.1.1. System diagram:

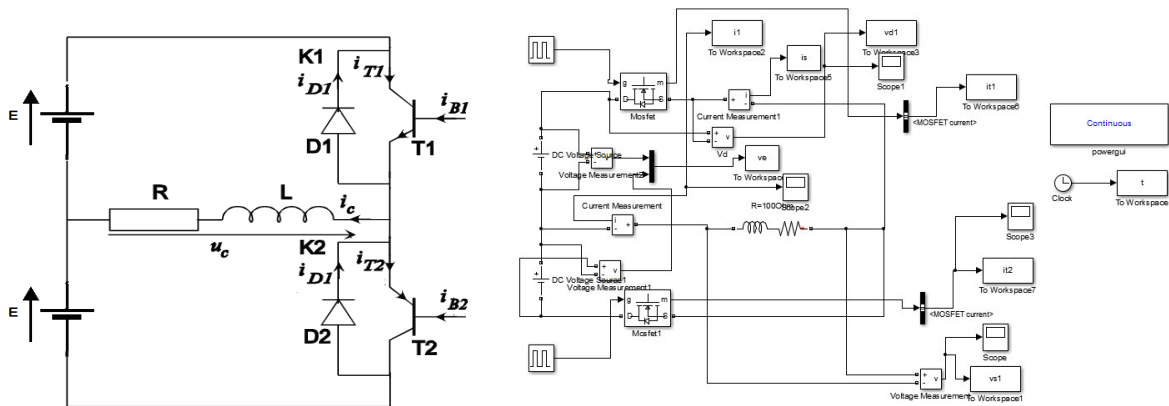


Figure 5.11 Single-phase half-bridge inverter

## Chapter 5: DC-AC Electrical Energy Conversion

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$E$  are two identical ideal DC voltage sources.

$K1$  and  $K2$  are two electronic switches that can be controlled for opening and closing.

$U_c$  is the voltage across the load terminals and  $i_c$  is the current in the load.

The control is symmetrical, meaning that during half of the operating period,  $K1$  is closed and  $K2$  is open, and during the other half,  $K1$  is open and  $K2$  is closed.

### 5.7.1.2. Operational Analysis

1-  $0 < t < T/2$ , switch  $K1$  is closed and  $K2$  is open.

$$U_c = E$$

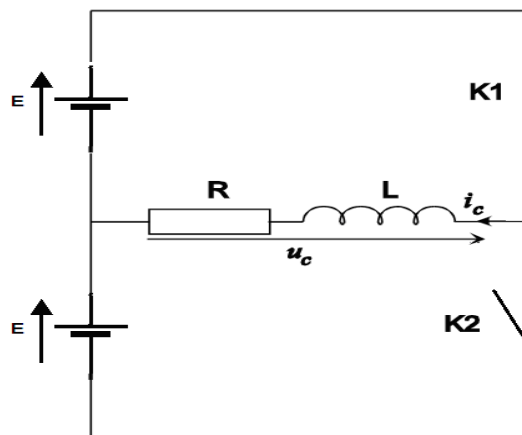


Figure 5.12 Single-phase half-bridge inverter for  $t \in [0, T/2]$

2-  $T/2 < t < T$ , switch  $K2$  is closed and  $K1$  is open.

$$U_c = -E$$

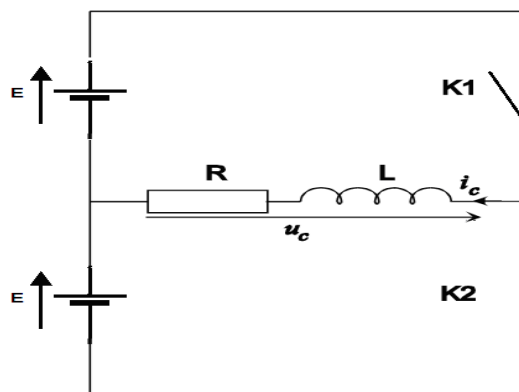
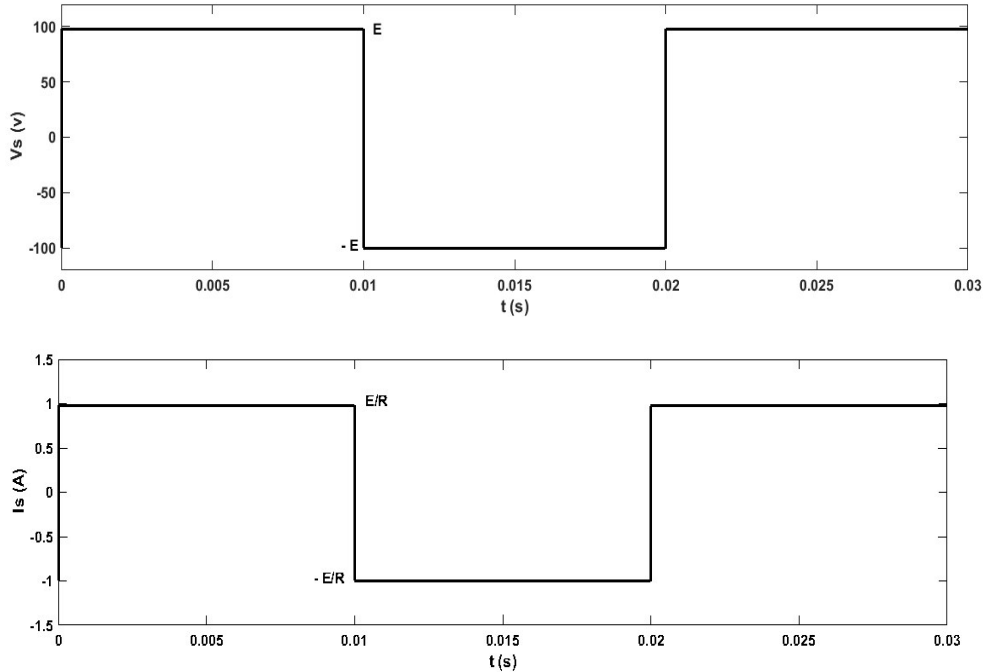


Figure 5.13 Single-phase half-bridge inverter for  $t \in [T/2, T]$

### 5.7.1.3. Waveforms of the main quantities

We use the previous setup. The load is a resistor R.



**Figure 5.14** Voltage and current timing diagrams of a half-bridge inverter on an R load

## 5.7.2. Characteristic quantities of the system

### Period and frequency

The period and frequency of the voltage across the load terminals and the current flowing through the load are determined by the switch control; this is therefore a stand-alone inverter.

### Average value of the voltage across the load terminals

The signal is alternating, the average value of the voltage across the load terminals is zero.

$$U_{cavg} = \frac{1}{T} \int_0^T u_c(t) \cdot dt = \frac{1}{T} \int_0^{\frac{T}{2}} E \cdot dt + \frac{1}{T} \int_{\frac{T}{2}}^T (-E) \cdot dt = 0$$

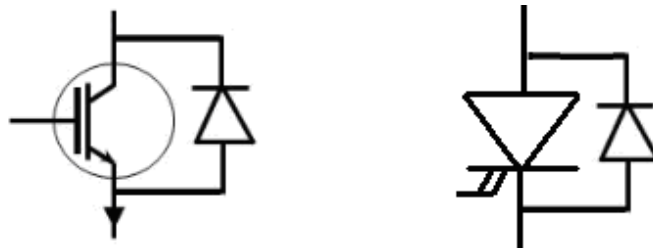
### Effective value of the voltage across the load terminals

It is determined using the area method:

$$U_{ceff} = \sqrt{\frac{1}{T} \int_0^T u_c^2(t) \cdot dt} = \sqrt{\frac{1}{T} \int_0^{\frac{T}{2}} E^2 \cdot dt + \frac{1}{T} \int_{\frac{T}{2}}^T (-E)^2 \cdot dt} = E$$

**5.7.3. Changing the nature of the load:**

An inductive load creates a phase shift between the voltage and current, meaning that  $u_c$  and  $i_c$  do not cancel each other out simultaneously. Consequently, the current through the switches can flow in both directions (positive and negative). It is therefore necessary to adapt the structure of the switches to allow bidirectional current flow. To achieve this, each transistor is connected with a diode connected in antiparallel [7, 8, 9].



The switches consist of an electronic switch that can be controlled to open and close (such as a transistor or GTO thyristor) and a diode in antiparallel.

**5.7.3.1. Operational analysis:**

**1-  $0 \leq t < T/2$ ,** the control forces K1 to be closed and K2 to be open.

$$U_c = E = R.i_c + L \frac{di_c}{dt}$$

This equation has a general solution:

$$i_c(t) = \frac{E}{R} + (i_{\min} - \frac{E}{R}).e^{\frac{-R}{L}t}$$

The voltage across the load terminals is positive. The current flows either through K1 or D1, depending on its sign. The current in the load,  $i_c$ , is zero at time  $t_1$ .

**For  $0 \leq t < t_1$ :** the current in the load is negative,  $i_c < 0$ .

The current flows through diode D1:  $i_{D1} = -i_c$ . Transistor K1 does not conduct. The instantaneous power  $p=U_c.i_c < 0$ : there is energy transfer from the load to the voltage source.

**For  $t_1 \leq t < T/2$ :** the current in the load is positive,  $i_c \geq 0$ .

The current flows through transistor K1:  $i_{K1} = i_c$ . Diode D1 is off. The instantaneous power  $p=U_c.i_c \geq 0$ : there is energy transfer from the source to the load. This is a power supply phase. **2-**

**$T/2 \leq t < T$ :** K2 closed and K1 open therefore

$$U_c = -E = R.i_c + L \frac{di_c}{dt}$$

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This equation has a general solution:

$$i_c(t) = \frac{-E}{R} + \left(i_{\max} + \frac{E}{R}\right) \cdot e^{\frac{-R}{L}\left(t - \frac{T}{2}\right)}$$

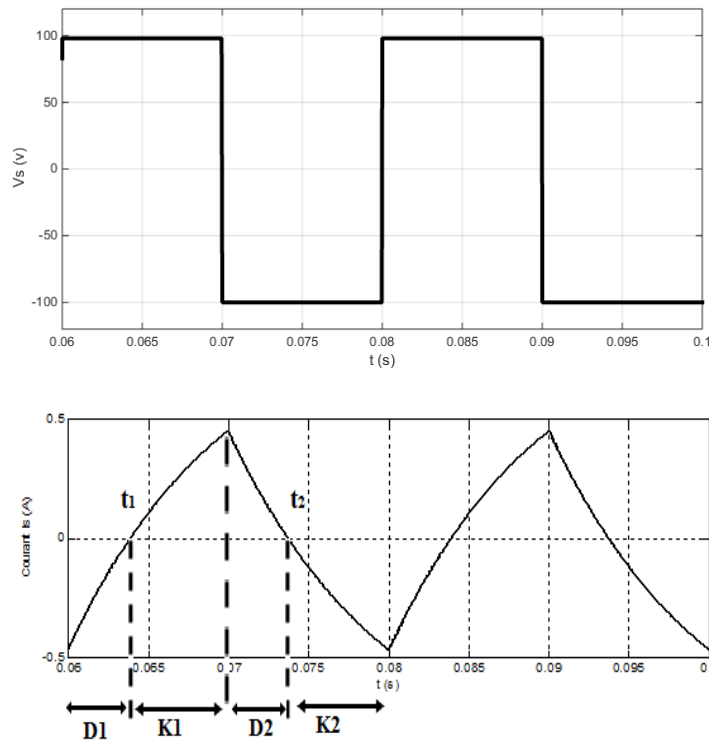
The voltage across the load terminals is negative. The current flows either through K2 or D2, depending on its sign. The current in the load,  $i_c$ , is zero at time  $t_2$ .

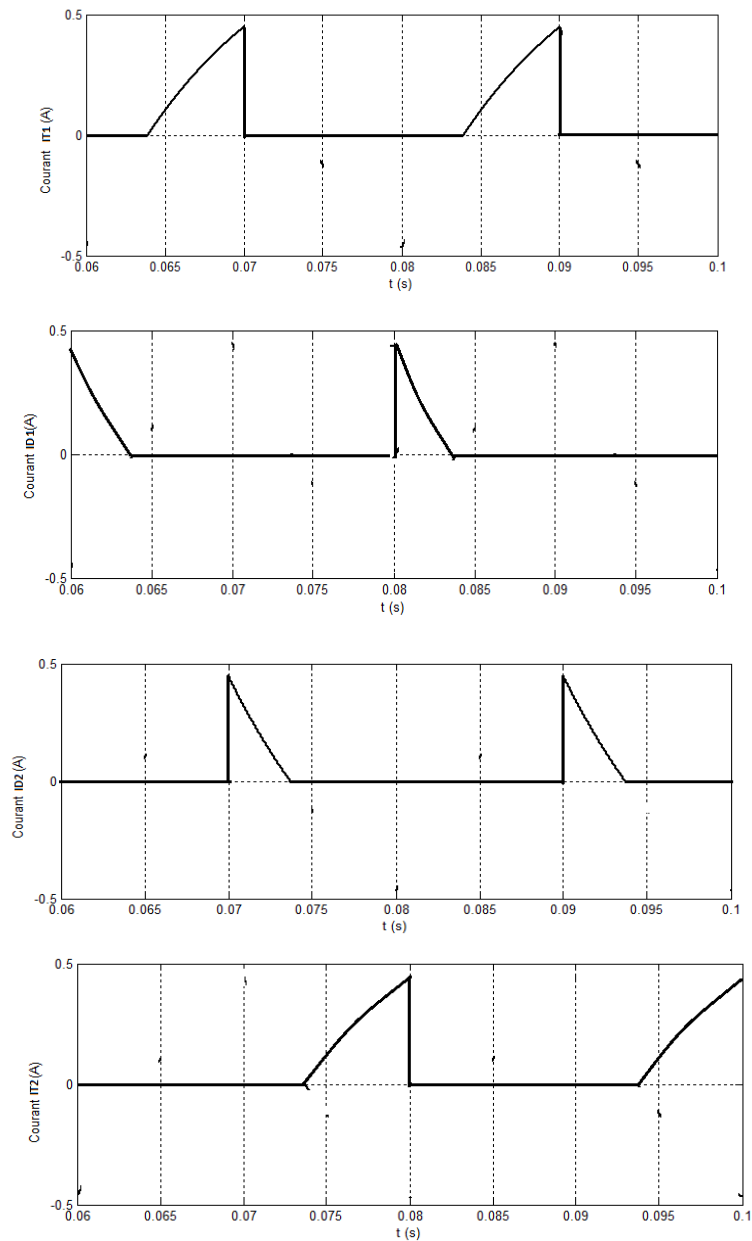
**For  $T/2 \leq t < t_2$ :** the current in the load is positive,  $i_c > 0$ .

The current flows through diode D2, indicating that  $i_{D2} = i_c$ . Transistor K2 remains off. The instantaneous power  $p = U_c \cdot i_c < 0$ : this indicates an energy transfer from the load to the voltage source. This phase corresponds to energy recovery.

**For  $t_2 \leq t < T$ :** the current in the load is negative,  $i_c \leq 0$ .

The current flows through transistor K2, with  $i_{K2} = -i_c$ . Diode D2 is off. The instantaneous power  $p = U_c \cdot i_c$  is positive or zero, indicating an energy transfer from the source to the load. This phase corresponds to a power supply to the load.





**Figure 5.15** Voltage and current timing diagrams of a half-bridge inverter on an RL load

### 5.8. Single-phase bridge inverter (four switches)

A single-phase bridge inverter (Fig. 5.16) requires the use of bidirectional electronic switches. This is due to the phase shift between the current  $i_S$  and the voltage  $u_S$ . To ensure conduction in both directions, each unidirectional switch is associated with a diode connected in antiparallel. The symbol used represents a unidirectional switch that can be controlled for both opening and closing [10, 11, 12].

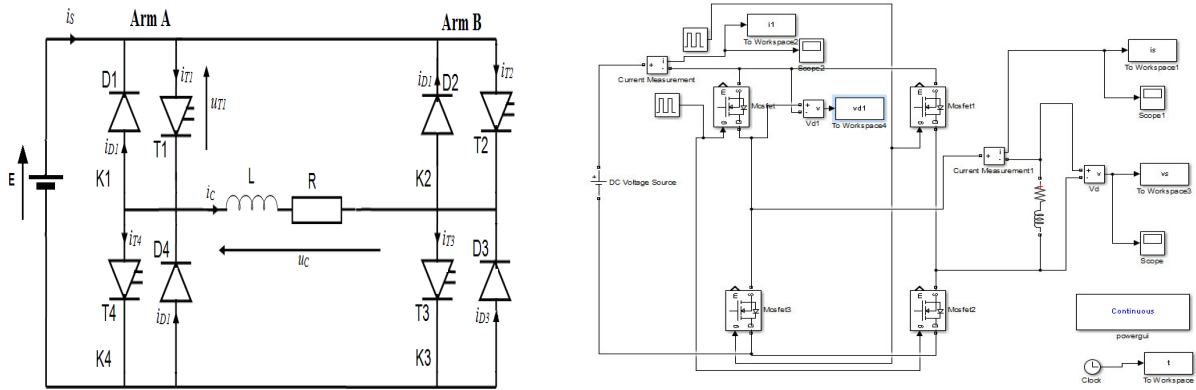


Figure 5.16 Single-phase H-bridge inverter

The circuit consists of two inverter arms: arm A consisting of K1 and K4, and arm B consisting of K2 and K3.

- ◆ The source is a DC voltage generator that is reversible into current.
- ◆ Switches T1, T2, T3, and T4 are open- and closed-controllable switches. D1, D2, D3, and D4 are ideal diodes.

5.8.1. Study of the power section operation

5.8.2. Full-Wave Control:

In this control mode, transistors T1 and T3 are activated simultaneously: they are saturated during the positive half-wave of the output voltage, then blocked during the negative half-wave. Conversely, T2 and T4 remain blocked during the positive half-wave and are saturated during the negative half-wave.

5.8.2.1. Operational Analysis

The switch control requires periodic operation with an adjustable period T.

Each alternation begins with a recovery phase and ends with an accumulation phase.

For  $0 \leq t < T/2$ : T1 and T3 are closed and T2 and T4 are open, therefore

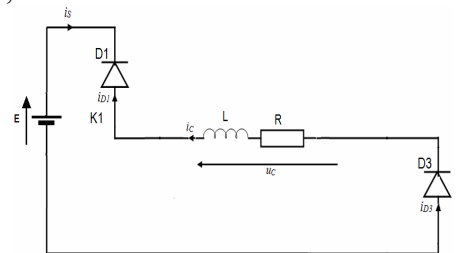
$$Ri + L \cdot \frac{di}{dt} = u_c = +E$$

For  $0 \leq t < t_1$ : the current in the load is negative  $i < 0$ .

The current flows through diodes D1 and D3:  $i_{D1} = i_{D3} = -i$ .

Switches T1 and T3 do not conduct.

The instantaneous power  $p = u \cdot i < 0$ : there is energy transfer from the load to the voltage source..



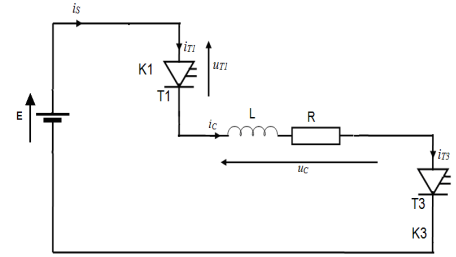
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**For  $t_1 \leq t < T/2$ :** the current in the load is positive  $i \geq 0$ .

The current flows through switches T1 and T3:  $i_{T1} = i_{T3} = i$ .

Diodes D1 and D3 are off.

The instantaneous power  $p = u \cdot i \geq 0$ : reflects an energy transfer from the source to the load. This phase corresponds to a power supply to the load.



**For  $T/2 \leq t < T$ :** T2 and T4 closed and T1 and T3 open therefore

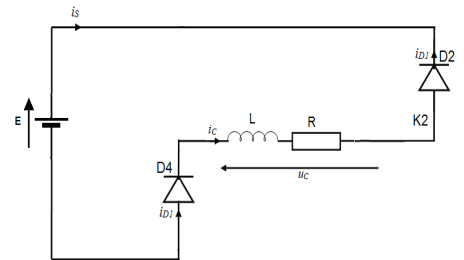
$$R \cdot i + L \cdot \frac{di}{dt} = u_c = -E$$

**For  $T/2 \leq t < t_2$ :** the current in the load is positive  $i > 0$ .

The current flows through diodes D2 and D4:  $i_{D2} = i_{D4} = i$ .

Switches T2 and T4 do not conduct.

The instantaneous power  $p = u \cdot i < 0$ : which indicates an energy transfer from the load to the voltage source.

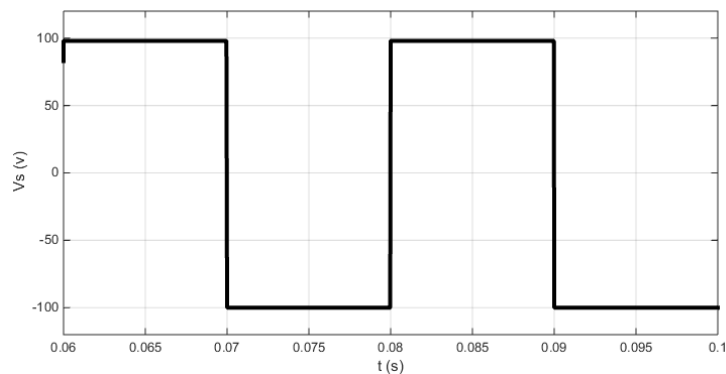
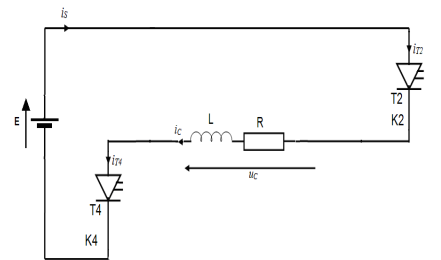


**For  $t_2 \leq t < T$ :** the current in the load is negative  $i \leq 0$ .

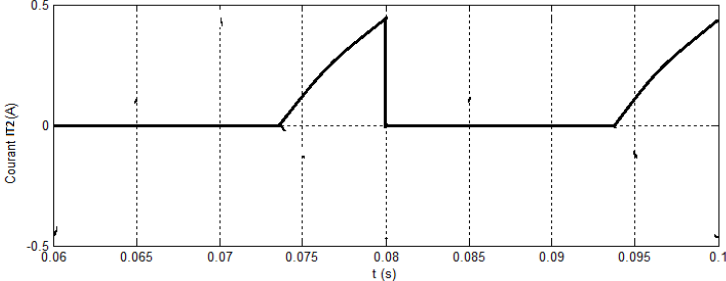
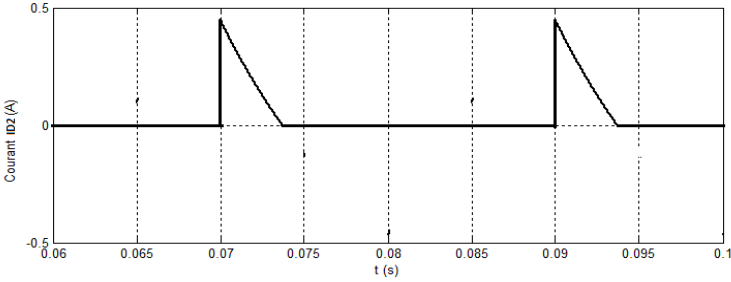
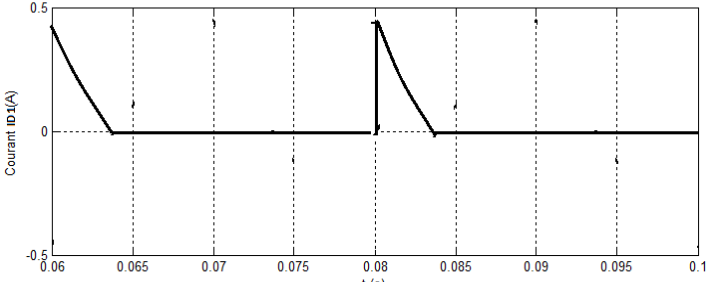
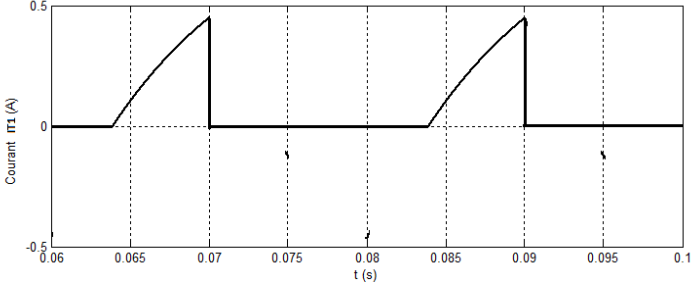
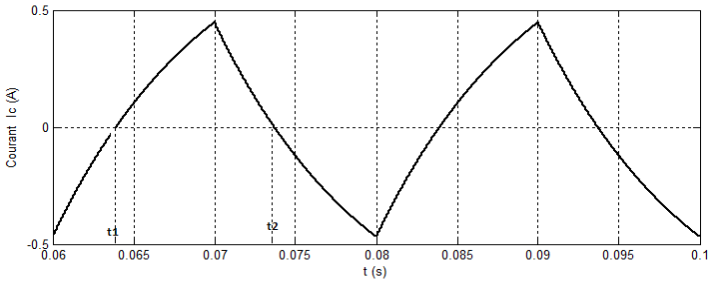
The current flows through diodes T2 and T4:  $i_{T2} = i_{T4} = -i$ .

Switches D2 and D4 are off.

The instantaneous power  $p = u \cdot i \geq 0$ : there is energy transfer from the source to the load..



# Chapter 5: DC-AC Electrical Energy Conversion



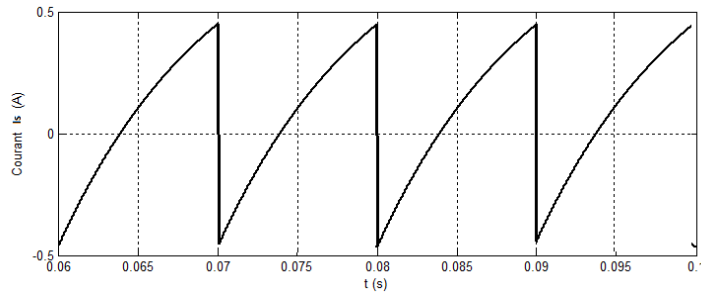


Figure 5.17 Voltage and current timing diagrams of a single-phase inverter on an RL load

**5.8.3. Ripple voltage spectrum:**

The voltage  $u(t)$  is a symmetrical square wave. The Fourier series decomposition yields:

$$u(t) = \frac{4.E}{\pi} \cdot \sum_{k=0}^{\infty} \frac{1}{(2.k + 1)} \cdot \sin((2.k + 1).w.t)$$

The THD is very poor, around 48%

$$THD = \sqrt{\sum_{k=0}^{\infty} \frac{1}{(2.k + 1)^2}} = 48\%$$

**5.8.4. Square Wave Inverter or Offset Control:**

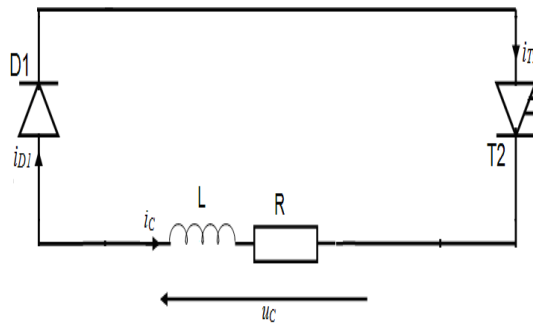
The diagram remains identical to the one presented previously, but this time the switch control is offset. This control offset partially reduces the harmonics present in the output signal, which improves the converter's overall performance [7, 8, 11, 12].

**5.8.5. Analysis of the Operation of an Inverter with Offset Control**

The switch control requires periodic operation with an adjustable period  $T$ . The control of switches T1 and T3 is offset by a duration  $\alpha$  relative to the control of switches T2 and T4:

**For  $0 \leq t < \alpha$ :** D1 and T2 closed and T1, T3 and T4 open, so the load is short-circuited  $u = 0$ .

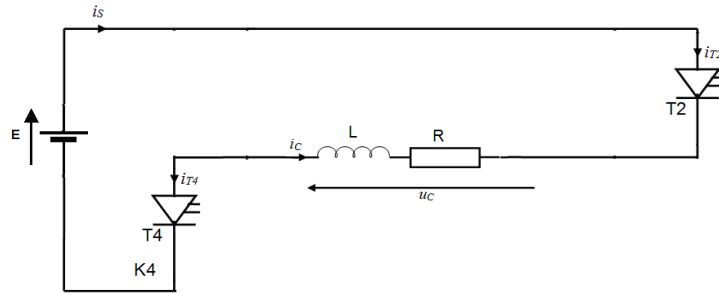
The current intensity in the load is negative



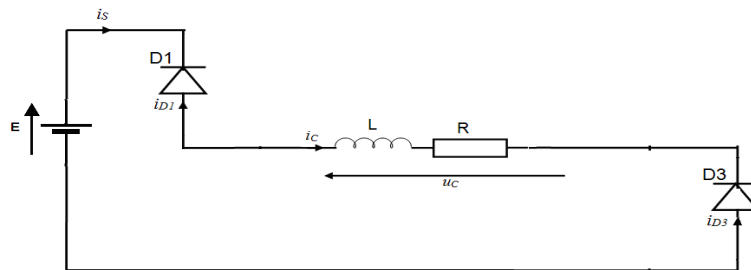
$u_c=0.$

## Chapter 5: DC-AC Electrical Energy Conversion

For  $\alpha \leq t < T/2$ : T1 and T3 are closed and T2 and T4 are open, so  $u_c = E$ .

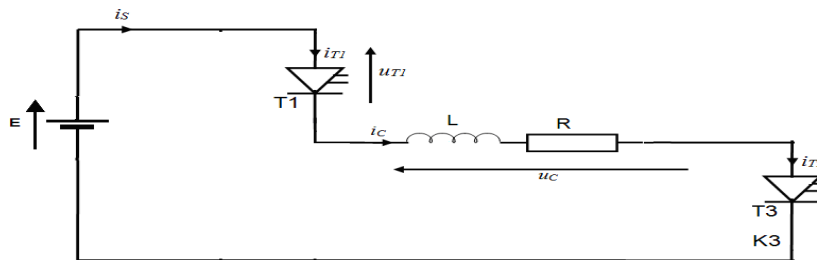


For  $\alpha \leq t < t_1$ : the current in the load is negative  $i < 0$ .  
The current flows through diodes D1 and D3.



$u_c = E$ .

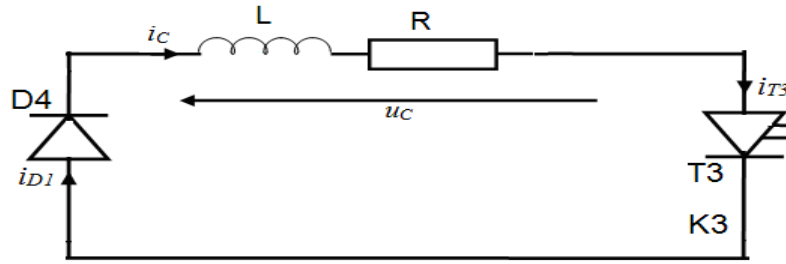
For  $t_1 \leq t < T/2$ : the current in the load is positive  $i \geq 0$ .  
The current flows through transistors T1 and T3.



For  $T/2 \leq t < T/2 + \alpha$ : T3 and D4 are closed and T1, T3 and T2 are open, so the load is short-circuited. The current in the load is positive.

$u_c = 0$ .

## Chapter 5: DC-AC Electrical Energy Conversion

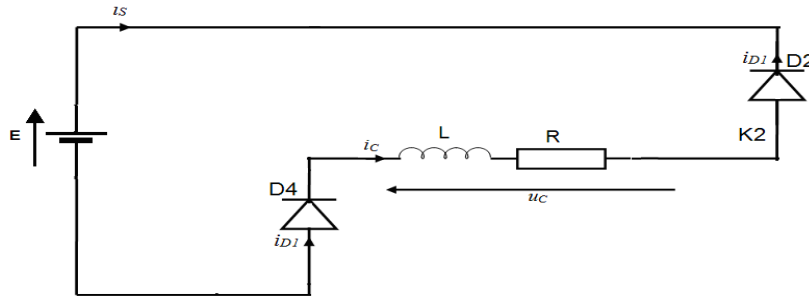


**For  $T/2 + \alpha \leq t < T$ :** T2 and T4 are closed and T1 and T3 are open, so

$$u_c = -E.$$

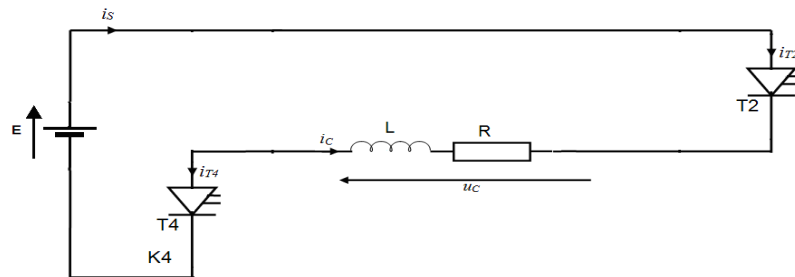
**For  $T/2 \leq t < t_2$ :** the current in the load is positive  $i > 0$ .

The current flows through diodes D2 and D4: this is a recovery phase.



**For  $t_2 \leq t < T$ :** the current in the load is negative  $i \leq 0$ .

The current flows through transistors T2 and T4: this is a supply phase.



$$u_c = -E.$$

### 5.8.5.1. Characteristic quantities of the assembly [7, 8, 11, 12]

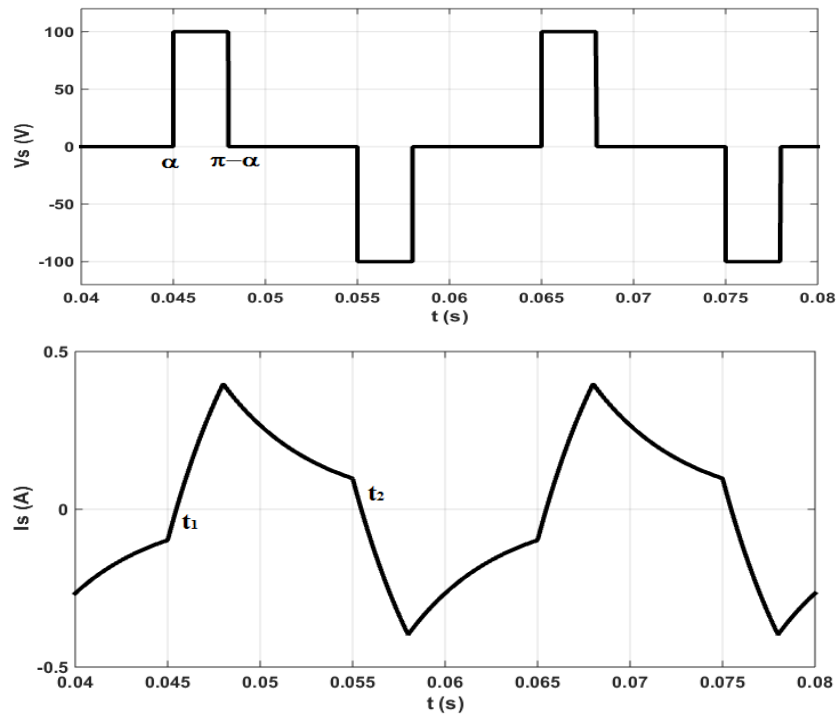
Period and frequency: imposed by the control and adjustable independently of the load.

**Average value of the voltage and current for the load:** zero, the signals are alternating.

**RMS value of the voltage across the load terminals:**

$$U_{ceff} = \sqrt{\frac{1}{T} \int_0^T u_c^2(t) dt} = \sqrt{\frac{2}{T} \int_{\alpha}^{\frac{T}{2}-\alpha} E^2 dt} = E \cdot \sqrt{1 - \frac{2\alpha}{\pi}}$$

## Chapter 5: DC-AC Electrical Energy Conversion



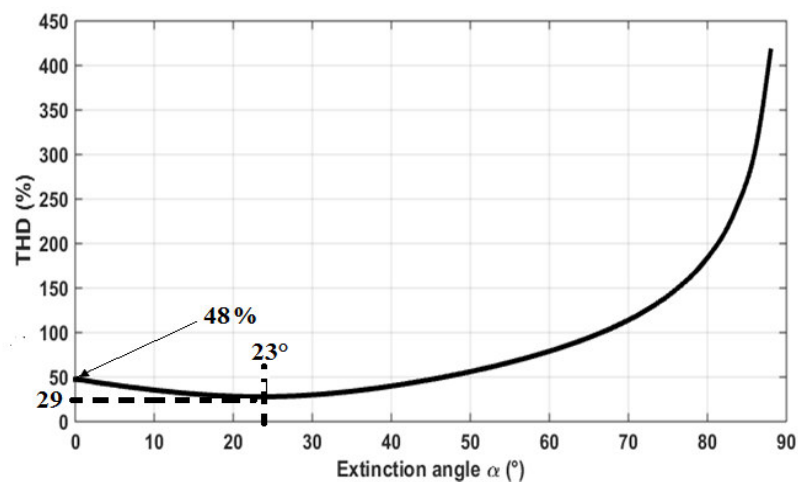
**Figure 5.18** Voltage and current timing diagrams of a single-phase inverter on an RL load

### Waveform Voltage Spectrum (Offset Control): [7, 8, 11, 12]

$$u(t) = \frac{4E}{\pi} \cdot \sum_{k=0}^{\infty} \frac{1}{(2k+1)} \cdot \cos((2k+1)\alpha) \cdot \sin((2k+1)\omega t)$$

THD is

$$THD = \frac{\sqrt{\pi^2 - 2\pi\alpha - 8\cos^2(\alpha)}}{2\sqrt{2}\cos(\alpha)}$$



**Figure 5.19** THD figure

**5.9. PWM Control:**

Pulse Width Modulation (PWM) is a widely used technique for generating pseudo-analog signals from circuits operating in an on-off mode, or more generally with discrete states.

PWM allows a pseudo-analog signal to be produced from a digital or analog environment so that it can be processed by switching components, operating as open or closed switches. By considering only the low-order harmonics, the total harmonic distortion (THD) becomes very low, close to zero. [7,8,11,12]

**5.9.1. Two Level PWM**

The electronic switches are all controlled simultaneously; either T1-T3 are closed and T2-T4 are open, or T1-T3 are open and T2-T4 are closed. The voltage u is odd and symmetrical about the vertical line passing through  $\pi/2$ . In the general case, we have a number m of angles  $\alpha_i$ ,

with  $0 \leq \alpha_i \leq \frac{\pi}{2}$ .

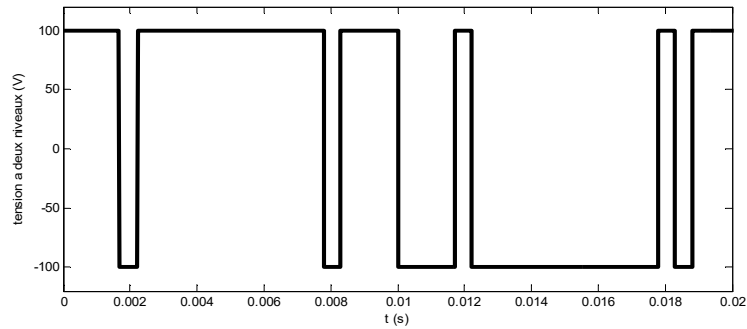


Figure 5.20 PWM-2-Level Wave

**5.9.1.1. Spectrum** [7, 8, 11, 12].

The voltage u is odd, the Fourier series expansion of u is:

$$u(t) = \frac{4.E}{\pi} \cdot \sum_{k=0}^{\infty} [1 + 2 \sum_{i=1}^m (-1)^i \cdot \cos((2.k + 1).\alpha_i)] \cdot \frac{\sin((2.k + 1).w.t)}{2.k + 1}$$

**5.9.2. Three Level PWM**

During the first half-cycle, T3 is closed and T4 is open, while either T1 is closed and T2 is open, or T1 is open and T2 is closed.

During the second half-cycle, T2 is closed and T1 is open, while either T4 is closed and T3 is open, or T4 is open and T3 is closed. The voltage u is odd and symmetrical about the vertical line passing through  $\pi/2$ . In the general case, we have an odd number m with angles  $\alpha_i$ ,

$$0 \leq \alpha_i \leq \frac{\pi}{2}$$

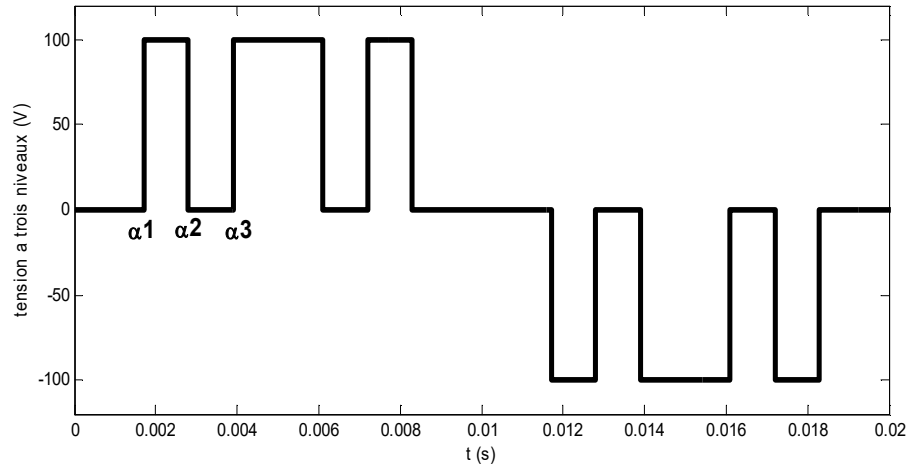


Figure 5.21 PWM-3-Level Wave

### 5.9.2.1. Spectrum. [7, 8, 11, 12]

The Fourier series expansion of u:

$$u(t) = \frac{4.E}{\pi} \cdot \sum_{k=0}^{\infty} \left[ \sum_{i=1}^m (-1)^{i+1} \cdot \cos((2.k + 1).\alpha_i) \right] \cdot \frac{\sin((2.k + 1).w.t)}{2.k + 1}$$

### 5.9.3. Step-wise PWM:

A step-wise voltage of m steps is obtained by summing (usually with transformers) m offset control voltages of height  $E_1/m$ . The offsets  $2\alpha_i$  are between 0 and  $\pi$ .

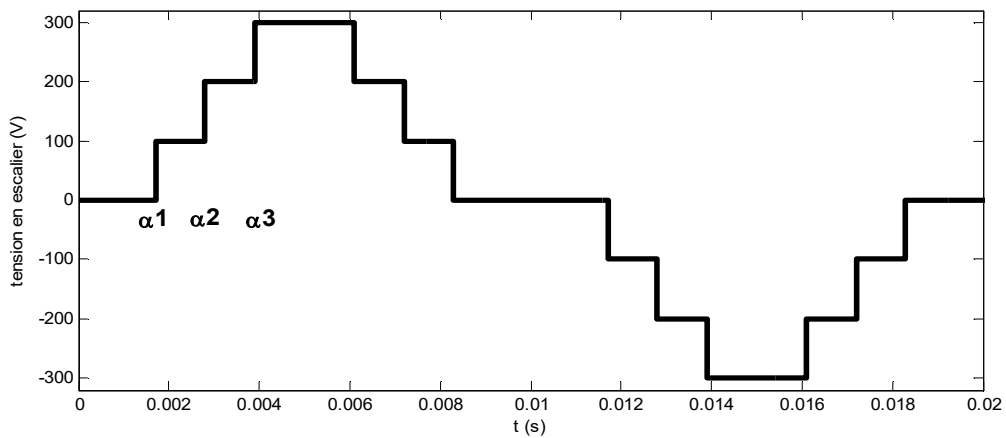


Figure 5.22 Three-step staircase voltage

5.10. Three phase inverters

5.10.1. Principle of the Three-Phase Voltage Inverter

The three-phase bridge inverter consists of a DC voltage source and six switches arranged in a bridge. This DC voltage is generally supplied by a three-phase diode rectifier, followed by a filter. This type of inverter is widely used in pulse-width modulation (PWM) to power balanced three-phase loads with variable voltage and frequency. It can be viewed as the superposition of three single-phase half-bridge inverters (figure 5.23). Each of the three output voltages is a bistable waveform alternating between the values  $-U$  and  $+U$ , with an offset of  $2\pi/3$  between them. Fundamental elements of the inverter, power switches are made up, depending on the power level to be switched, of GTO (Gate Turn-Off), power MOS transistors or IGBT (Insulated Gate Bipolar Transistor), associated in parallel with a diode (figure 5.23). This diode guarantees the continuity of the current when its direction of circulation is reversed [7, 8, 11, 12].

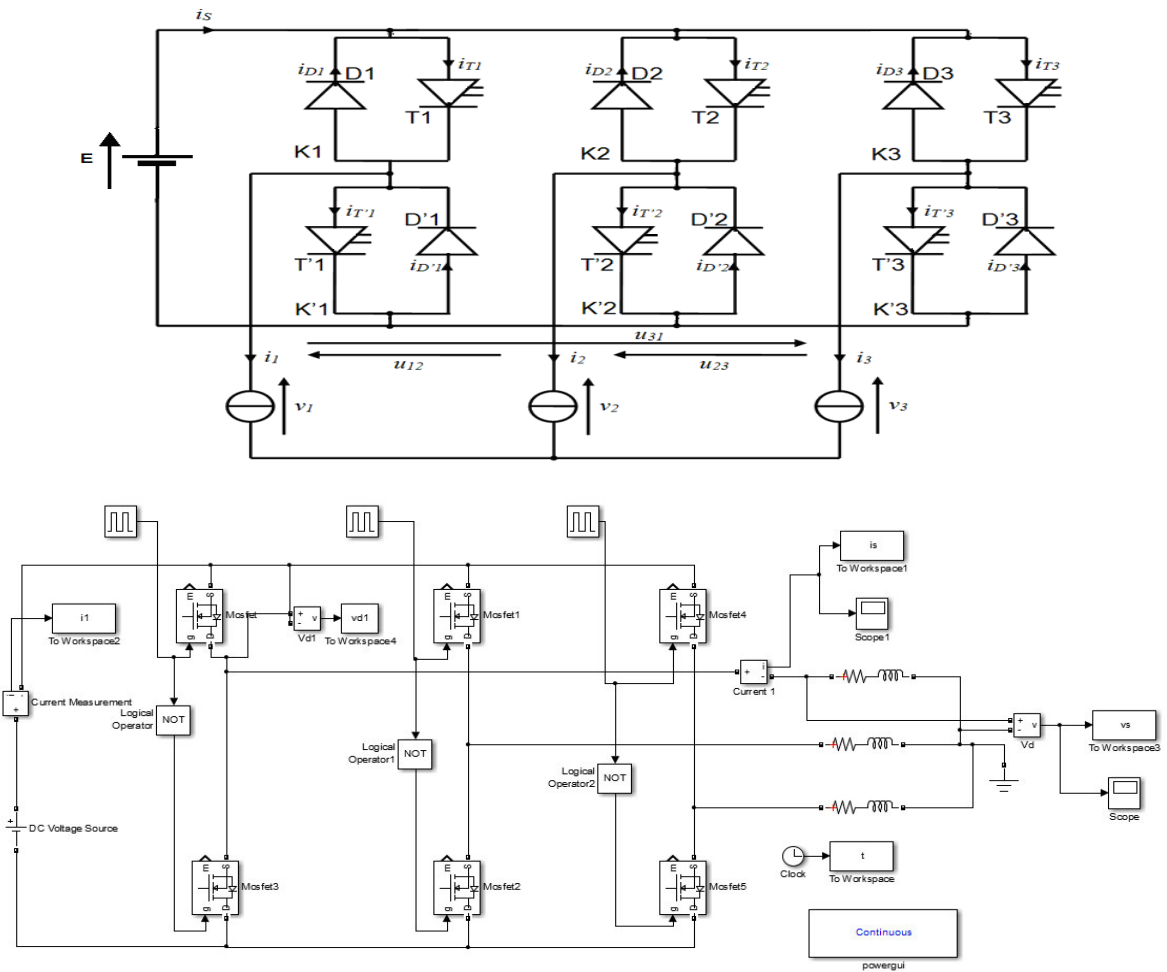


Figure. 5.23 Structure of a three-phase inverter and Simulink model

## Chapter 5: DC-AC Electrical Energy Conversion

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We consider a balanced three-phase load, and to simplify the study, we will assume a star connection. In this structure, several types of control can be applied, the most commonly used being:

- 120° conduction
- 150° conduction
- 180° conduction
- Pulse-width modulation (PWM) control

### 5.10.2. 120° conduction

The switches are controlled for a duration equivalent to one-third of a cycle, with a 120° shift between the sequences of each arm [7, 8, 11, 12]. Thus:

- At any given time, two switches are conducting, while the other four remain off.
- The two switches on the same arm must be controlled in a complementary manner to avoid short-circuiting the voltage source. This control logic generates six conduction sequences per period, as illustrated in figure 5.24.

It is also relevant to model this setup as a combination of three single-phase inverters in a half-bridge configuration, representing the DC source as two equivalent voltage sources  $E/2$ , with a midpoint denoted O.

#### 5.10.2.1. Voltage Study

The voltages  $v_{ao}$ ,  $v_{bo}$ ,  $v_{co}$  measured between points A, B, C and the midpoint are then the voltages delivered by the single-phase inverters. The shape of the phase-to-phase voltages can then be determined by taking into account the following relationships [7, 8, 11, 12]:

$$v_{ab} = v_{ao} - v_{bo}$$

$$v_{bc} = v_{bo} - v_{co}$$

$$v_{ca} = v_{co} - v_{ao}$$

The analysis of the voltage timing diagrams highlights the generation of a three-phase voltage system, of period T, whose phases are offset by 120°. The amplitude of the voltages depends on the type of control used. From these timing diagrams, it is possible to deduce the expressions for the simple voltages applied to the load.

$$u_{ab} = v_a - v_b$$

$$u_{bc} = v_b - v_c$$

$$u_{ca} = v_c - v_a$$

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And by making the member-to-member difference between the first and third relations:

$$u_{ab} - u_{ca} = v_a - v_b - (v_c - v_a)$$

Or again:

$$u_{ab} - u_{ca} = 2.v_a - (v_b - v_c)$$

And so

$$3.v_a = u_{ab} - u_{ca}$$

Hence the expression of the first simple voltage:

$$v_a = \frac{1}{3} \cdot (u_{ab} - u_{ca})$$

By performing a circular permutation of the indices A, B, C, we establish the expressions of the two other simple voltage:

$$v_b = \frac{1}{3} \cdot (u_{bc} - u_{ab})$$

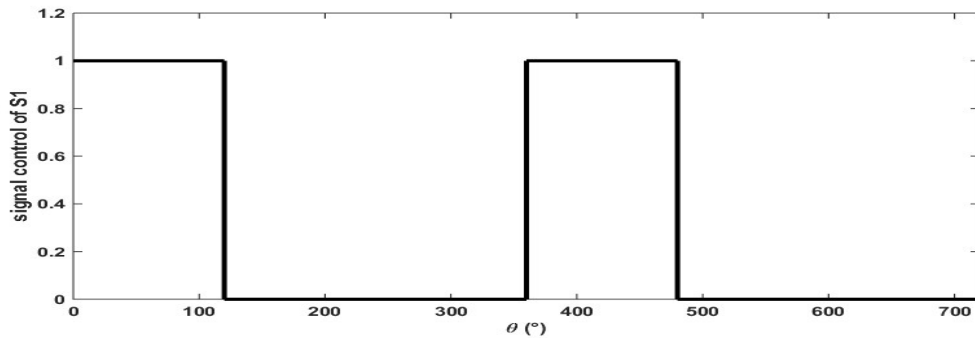
$$v_c = \frac{1}{3} \cdot (u_{ca} - u_{bc})$$

The matrix expression of the simple voltages of the inverter by means of the logical connection functions is obtained from the equations:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} s_a \\ s_b \\ s_c \end{bmatrix} \cdot E$$

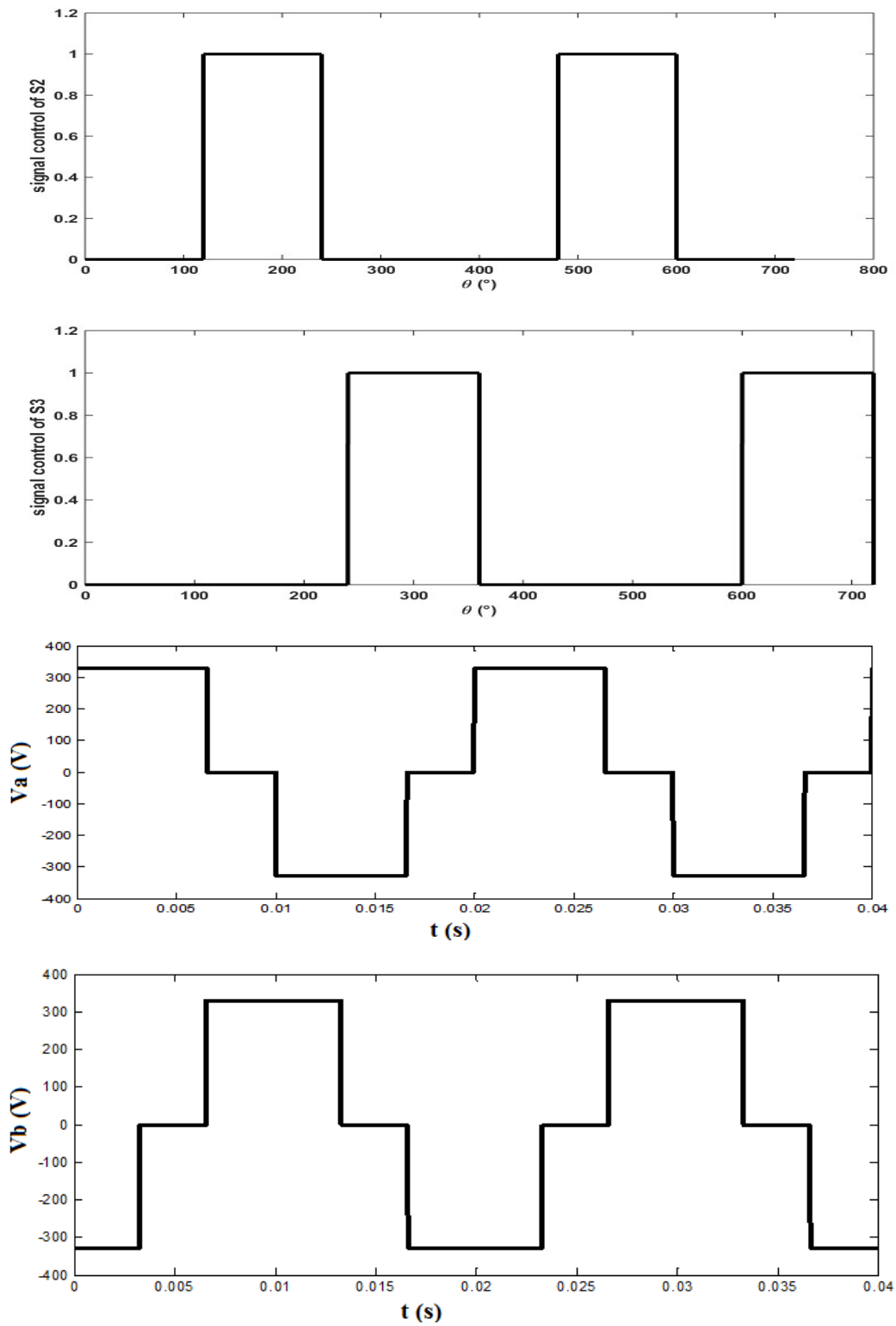
where

$s_i \in [1, 0]$  the connection function of a  $K_{is}$  switch. Or 0 indicates the switch is open and 1 indicates the switch is closed.



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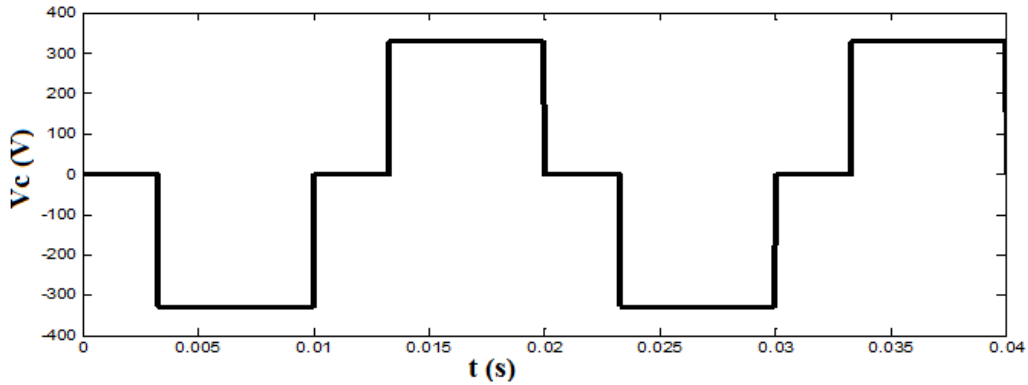


Figure 5.24 Timing diagrams of control signals and output voltages

**5.10.2.2. Characteristic quantities of the circuit [7, 8, 11, 12]**

**Period and frequency:** imposed by the control and adjustable independently of the load.

**Average voltage and current values for the load:** zero, the signals are alternating.

**RMS value of the voltage across the load terminals:**

$$U_{ceff} = \sqrt{\frac{1}{T} \int_0^T u_c^2(t) . dt} = \sqrt{\frac{2}{T} \int_0^{\frac{T}{2}} (\frac{1}{2} E)^2 . dt} = \frac{1}{\sqrt{6}} . E$$

**5.10.2.3. Harmonics of phase-to-neutral voltages**

The phase-to-neutral voltage  $v_a$  is in alternating positive and negative square waves with a period T. The Fourier series decomposition can be expressed as follows [7,8,11,12]:

$$u_a(t) = \sum_{k=1}^{\infty} \frac{2.E}{\pi.(2k+1)} . \cos(\frac{(2k+1).\pi}{6}) \sin(. (2k+1).w.t)$$

$$u_a(t) = \frac{\sqrt{3}.E}{\pi} (\sin(w.t) - \frac{1}{5} . \sin(5.w.t) - \frac{1}{7} . \sin(7.w.t) + \frac{1}{11} . \sin(11.w.t) \dots \dots \dots)$$

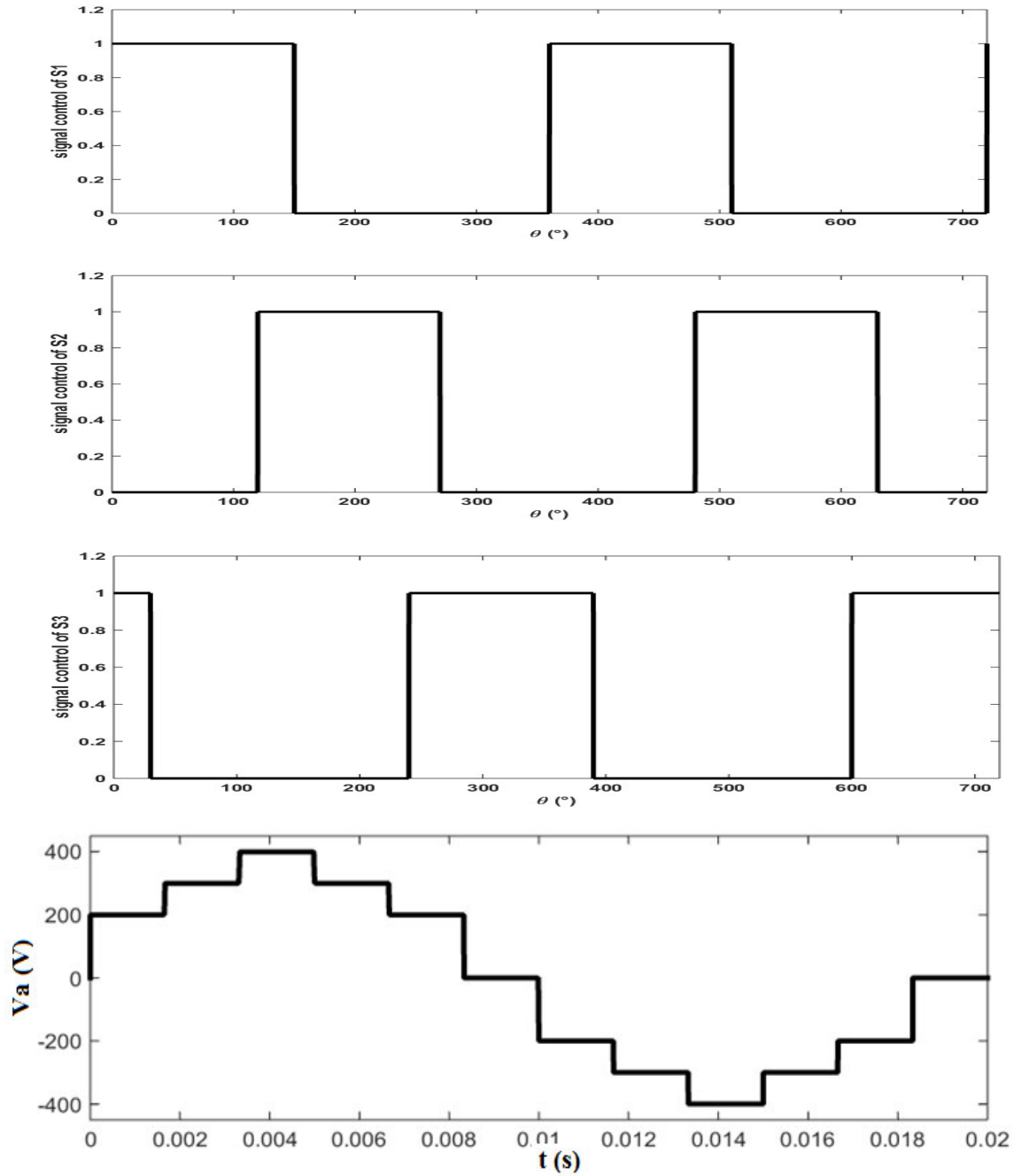
We note that even harmonics and those multiples of three (3, 9, 12, etc.) disappear, while those of ranks 5, 7, 11, 13, etc., remain present in the spectrum.

**5.10.3. 150° conduction**

150° control consists of activating each switch for a duration corresponding to 150 electrical degrees, or 5T/12, where T is the signal period. This method improves the output voltage profile compared to 120° control, particularly by reducing the harmonics present in the spectrum. At any given moment, two switches are conducting, ensuring continuous power to the load. The control signals are offset by 120° from one arm to the other, resulting in a balanced three-phase system.

## Chapter 5: DC-AC Electrical Energy Conversion

150° control is often preferred in applications requiring better voltage quality and increased efficiency.



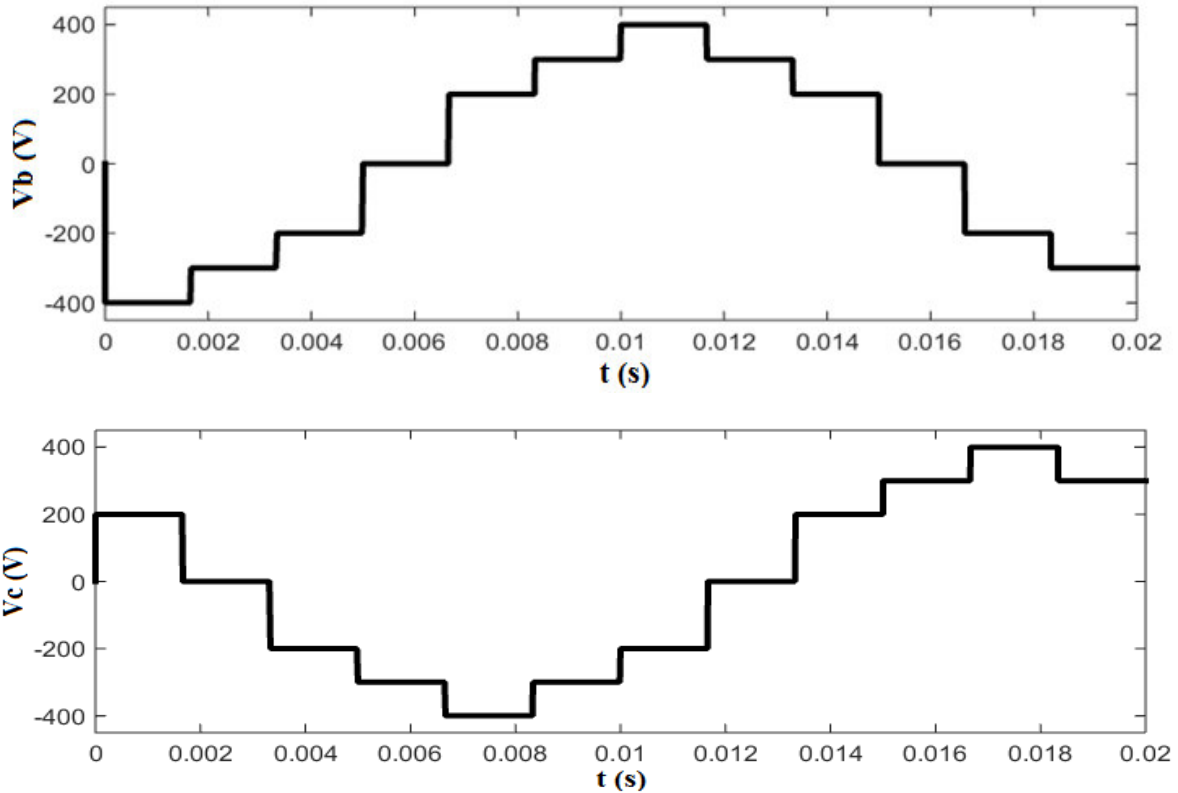


Figure 5.25 Timing diagrams of control signals and output voltages

**5.10.3.1. Characteristic quantities of the circuit [7, 8, 11, 12]**

**Period and frequency:** imposed by the control and adjustable independently of the load.

**Average value of the voltage and current for the load:** zero, the signals are alternating.

**RMS value of the voltage across the load terminals:**

$$U_{ceff} = \sqrt{\frac{1}{T} \int_0^T u_c^2(t) dt} = \frac{\sqrt{7}}{6} \cdot E$$

**5.10.3.2. Harmonics of simple Voltages**

The phase-to-neutral voltage  $u_a$  is formed by alternating positive and negative pulses with a period of T. The Fourier series decomposition can be expressed as follows [7,8,11,12]:

$$u_a(t) = \sum_{k=0}^{\infty} \frac{2E}{3 \cdot (2k+1)\pi} \cdot [2 \cdot \cos\left(\frac{(2k+1)\pi}{12}\right) + \cos\left(\frac{(2k+1)\pi}{4}\right) + \cos\left(\frac{5 \cdot (2k+1)\pi}{12}\right)] \cdot \sin\left((2k+1)\omega t + \frac{(2k+1)\pi}{12}\right)$$

$$u_a(t) = \frac{2 \cdot E}{3 \cdot \pi} (2,89 \cdot \sin(\omega t) + \frac{1}{5} \cdot 0,77 \cdot \sin(5 \cdot \omega t) - \frac{1}{7} \cdot 0,77 \cdot \sin(7 \cdot \omega t) - \frac{1}{11} \cdot 2,89 \cdot \sin(11 \cdot \omega t) \dots \dots \dots)$$

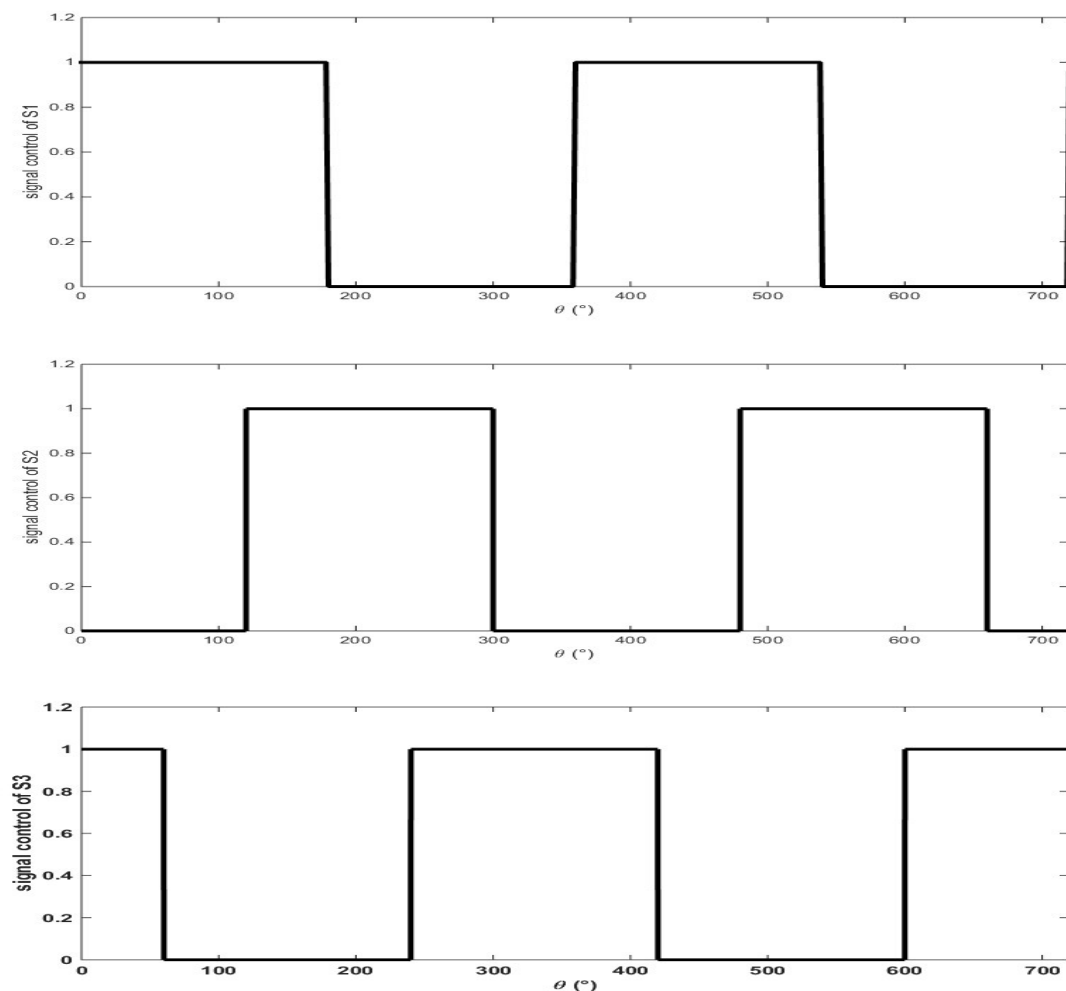
## Chapter 5: DC-AC Electrical Energy Conversion

Harmonics  $n=12k\pm 1$  ( $k=1, 2, 3$ , etc.), such as 11, 13, etc., remain unchanged in 180, 120, and 150 control, and harmonics 5, 7, 17, 19, etc., are extremely reduced in the 150 firing mode compared to other modes.

### 5.10.4. 180° conduction

180° control consists of activating each switch for a duration equivalent to a half-cycle (180° electrical), with a firing offset between the inverter arms [7, 8, 11, 12].

Figure 5.25 illustrates the six conduction sequences that repeat over a period. These sequences, as in the case of 120° control, allow the construction of the phase-to-phase and phase-to-neutral voltage waveforms. The associated timing diagrams reveal that the phase-to-neutral voltages exhibit a staircase-like appearance and form a balanced three-phase system with a period  $T$ , identical to that of the phase-to-phase voltages. The amplitude of these voltages depends on the control configuration used.



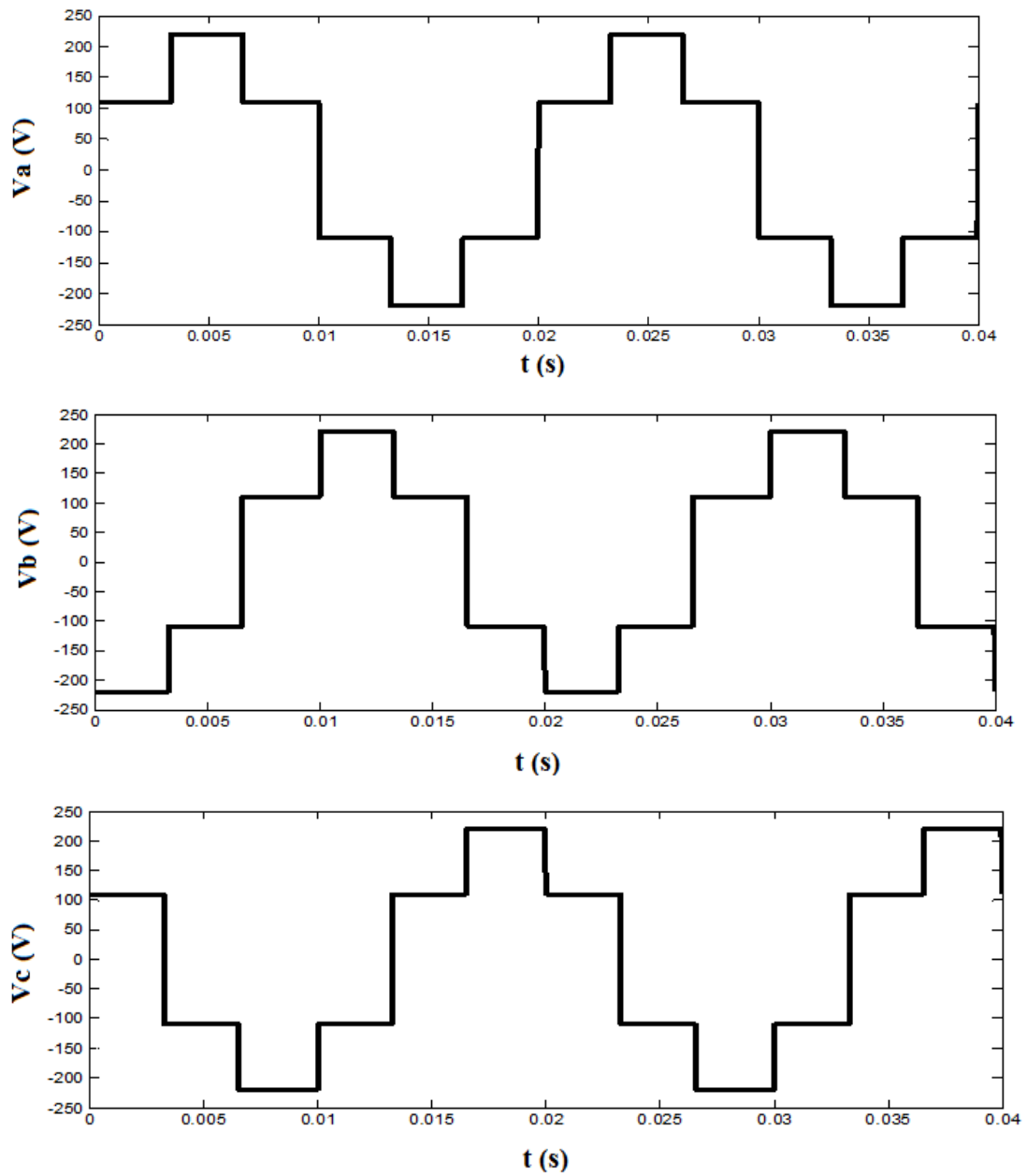


Figure 5.26 Timing diagrams of control signals and output voltages

**5.10.4.1. Characteristic quantities of the assembly [7, 8, 11, 12]**

**Period and frequency:** imposed by the control and adjustable independently of the load.

**Average value of the voltage and current for the load:** zero, the signals are alternating.

**RMS value of the voltage across the load terminals:**

$$U_{ceff} = \sqrt{\frac{1}{T} \int_0^T u_c^2(t) .dt} = \sqrt{\frac{2}{T} \int_0^{\frac{T}{6}} (\frac{1}{3}E)^2 .dt + \frac{2}{T} \int_{\frac{T}{6}}^{\frac{T}{3}} (\frac{2}{3}E)^2 .dt + \frac{2}{T} \int_{\frac{T}{3}}^{\frac{T}{2}} (\frac{1}{3}E)^2 .dt} = \frac{\sqrt{2}}{3} .E$$

**5.10.4.2. Harmonics of simple voltages**

The Fourier series decomposition is expressed by highlighting the fundamental term [7,8,11,12]:

$$u_{an}(t) = \sum_{k=0}^{\infty} \frac{4.E}{3.(2k+1).\pi} .(1 + \cos(\frac{(2k+1).\pi}{3})).\sin((2k+1).w.t)]$$

$$u_a(t) = \frac{2.E}{\pi} (\sin(w.t) + \frac{1}{5} .\sin(5.w.t) + \frac{1}{7} .\sin(7.w.t) + \frac{1}{11} .\sin(11.w.t).....)$$

As with the 120° control, we note that even harmonics are absent, and those that are multiples of three (3, 9, 12, etc.) have disappeared because they are zero sequence components. Only the 5th, 7th, 11th, etc. harmonics remain in the spectrum.

**5.10.5. Pulse Width Modulation (PWM)**

The purpose of analog or digital controls is to generate the desired voltages or currents at the machine terminals. Pulse Width Modulation (PWM) is a widely used technique for synthesizing these quantities from a fixed source, generally a constant-frequency DC voltage, using a static converter. This converter provides the electrical connection between the source and the load.

The control principle is based on controlling the opening and closing times of the switches, as well as the chosen switching sequence. In most applications, the ideal waveform is sinusoidal. PWM allows you to get closer to this effectively while modulating the value of the fundamental component of the output voltage. It also has the advantage of shifting unwanted harmonics to higher frequencies, making them easier to eliminate by filtering [7, 8, 11, 12].

**5.10.5.1. Voltage Inverter Modulation Techniques**

**5.10.5.1.a Sine-Delta Modulation**

► **Principle**

Sine-delta PWM is achieved by comparing a low-frequency modulating wave (reference voltage) to a high-frequency triangular carrier wave. The switching times are determined by the intersection points between the carrier and modulating waves. The switching frequency of the switches is set by the carrier. In three-phase systems, the three sinusoidal reference waves are phase-shifted by at the same frequency f [7, 8, 11, 12].

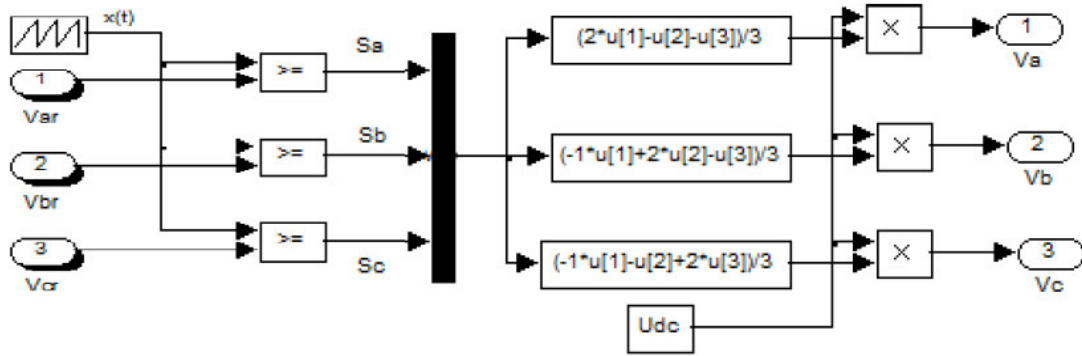


Figure 5.27 Simulink model of PWM control and three-phase inverter

► Modulation characterization

When the reference is sinusoidal, two main parameters characterize PWM control. The first is the frequency modulation index ( $m$ ), defined as the ratio between the modulation frequency (or carrier frequency) and the reference frequency (or modulating frequency). This index directly influences the quality of the output signal, particularly with regard to the harmonic spectrum and filtering ease.

$$m = \frac{f_p}{f_m} \text{ with } m > 1$$

A modulation index  $m$  greater than unity is generally chosen because this results in a shift of harmonics toward higher frequencies. However, in simulations, it has been observed that very high values of  $m$  cause an increase in voltage losses, also known as "voltage dropout." This makes it necessary to optimize the value of  $m$  to achieve an optimal compromise between signal quality and system performance.

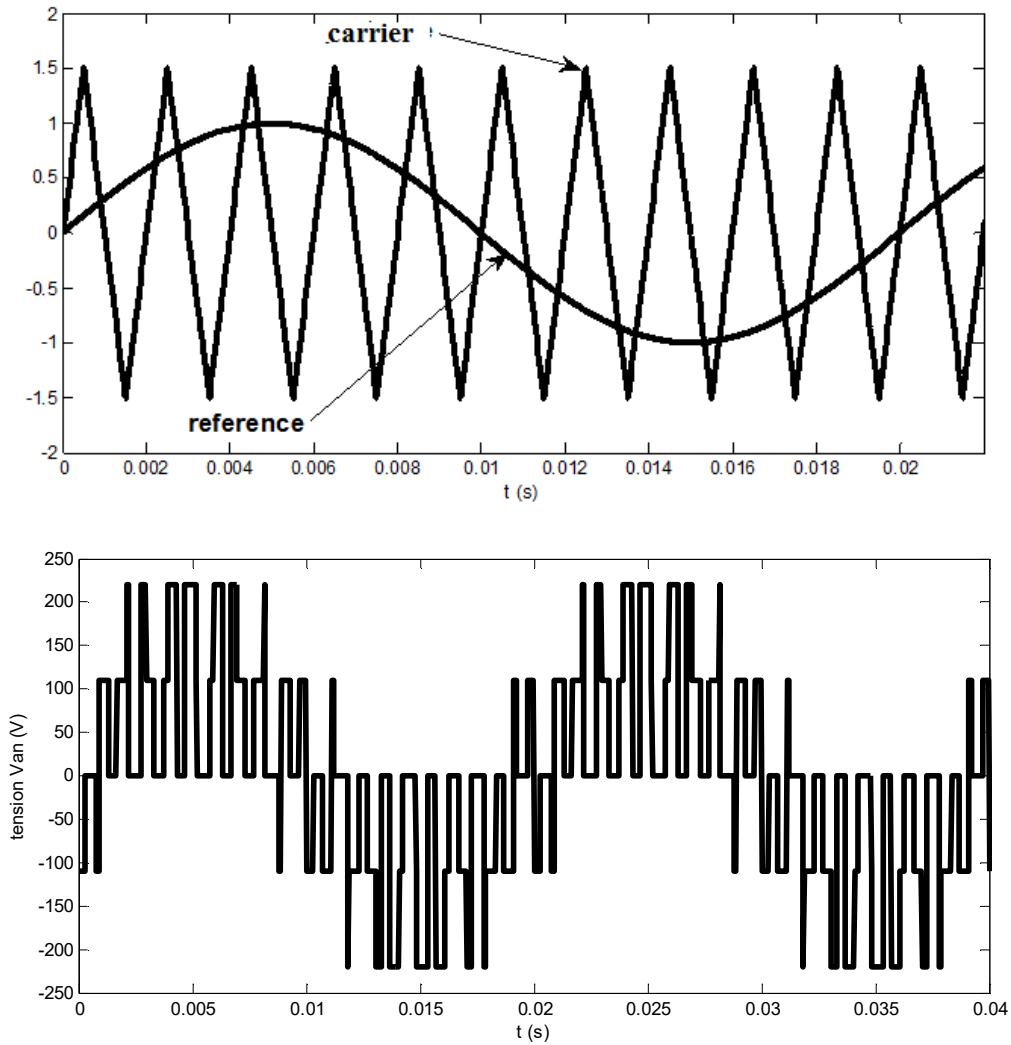
- The voltage adjustment coefficient  $r$ , equal to the ratio of the reference voltage amplitude to the carrier voltage amplitude.

$$r = \frac{U_m}{U_p}$$

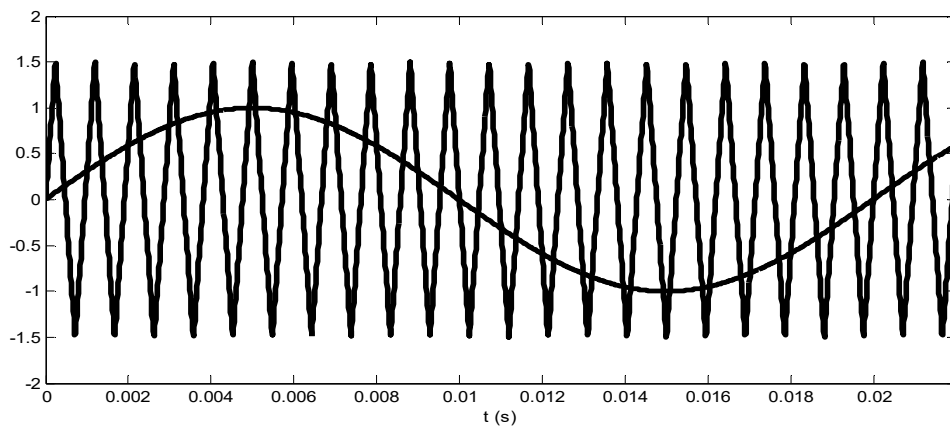
Figures 5.28 and 5.29 illustrate the simulation of the output voltage  $v_{an}$  for  $r=0.67$  and  $m=10$  and 21, with sinusoidal three-phase input voltages at a frequency of 50 Hz. The simulation reveals that increasing the modulation index allows shifting the harmonics of the inverter output voltage to higher frequencies. However, this increase also leads to an increase in the number of

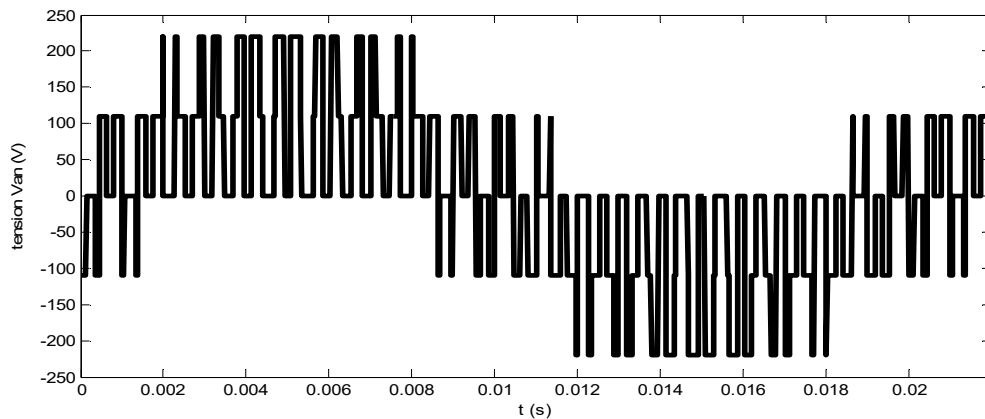
## Chapter 5: DC-AC Electrical Energy Conversion

switching operations per period, which is given by  $2m$ , which increases the switching losses per period. In addition, it reduces the minimum opening-closing cycle of the switches.



**Figure 5.28** The sine-triangle PWM signal for  $r=0.67$  and  $m=10$





**Figure 5.29** The sine-triangle PWM signal for  $r=0,67$  and  $m=21$

### 5.11. Conclusion

DC-AC converters, or inverters, transform a DC voltage source into an AC voltage source. This transformation is based on fast and robust semiconductor control devices.

The structure of an inverter depends primarily on the type of power sources between which it is connected. Since the alternation of the power sources at its access points must be respected, a distinction is made between:

- Voltage inverters connecting a DC voltage source to an AC current source,
- Current inverters placed between a DC current source and an AC voltage source.

The type of power sources is defined from a switching perspective.

Fixed-frequency voltage inverters are primarily used:

- To create safety power supplies delivering a constant-frequency sinusoidal voltage,
- To connect DC generators (e.g., photovoltaic panels) to the industrial grid or to provide AC-to-DC conversion from the grid (reverse operation of an inverter). Variable frequency voltage inverters are used to create variable speed drives with AC motors.

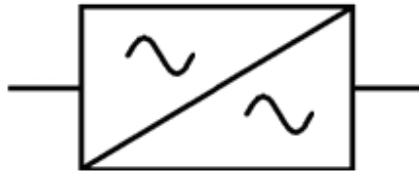
# **Chapter 6: AC-to-AC Electrical Energy Conversion**

## Chapter 6: AC-to-AC Electrical Energy Conversion

### 6.1. Definition:

The conversion of electrical energy supplied in AC form to supply an alternating current load can be performed either without changing the frequency or with it. In the first case, when there is no change in frequency, it is called a dimmer. In the second case, when the output frequency is different from the input frequency, it is called a cycloconverter.

### 6.2. Symbol:



### 6.3. Single-Phase Dimmer

#### 6.3.1. Construction of a dimmer:

It consists of two parts integrated into a single unit: the power section and the control section.

- The power section includes either two thyristors mounted in a head-to-tail configuration for high-power applications (over 10 kW), or a triac for lower power applications [7, 8, 11, 12].
- The control section, for its part, groups together various electronic circuits responsible for generating the signals required to control the thyristors, based on an external control order.

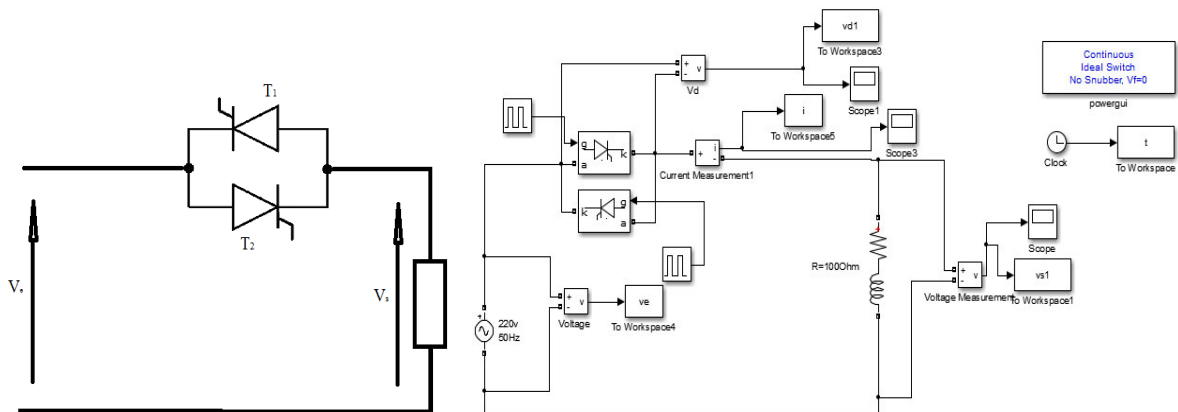


Figure 6.1 Single-phase dimmer

#### 6.3.2. Phase angle dimmer

This is a device that, while powered by a sinusoidal voltage of constant RMS value, delivers a non-sinusoidal alternating current to the load, having the same frequency as the supply voltage, but with an adjustable RMS value.

## Chapter 6: AC-to-AC Electrical Energy Conversion

### 6.3.2.1. Operating principle of a phase-angle dimmer

#### 6.3.2.1.a.. Discharge on resistive load

In this type of dimmer, the signal applied to the control input is analog. Operation is based on the controlled phase shift of thyristor firing: thyristor Th1 is triggered during the positive half-wave of the mains voltage, with a delay angle  $\alpha$  relative to the zero crossing. Symmetrically, thyristor Th2 is triggered during the negative half-wave, also with the same delay angle  $\alpha$  [7, 8, 11, 12].

-  $0 < \theta < \alpha$ , Th1 and Th2 are blocked.

$$V_s = 0$$

-  $\alpha < \theta < \pi$  Th1 conducts and Th2 is blocked.

$$V_s = V_e$$

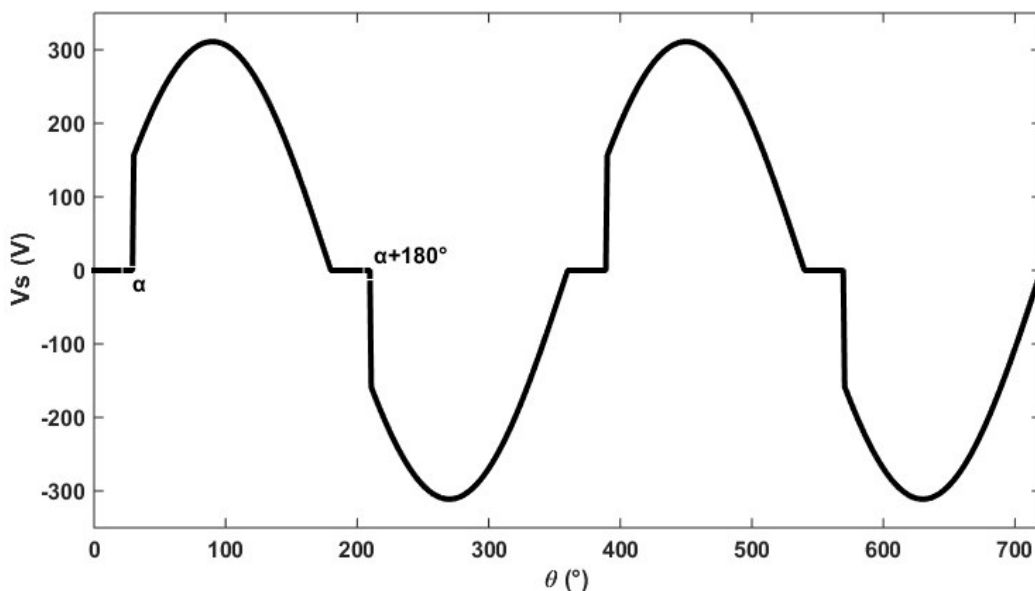
-  $\pi < \theta < \alpha + \pi$  Th1 and Th2 are blocked.

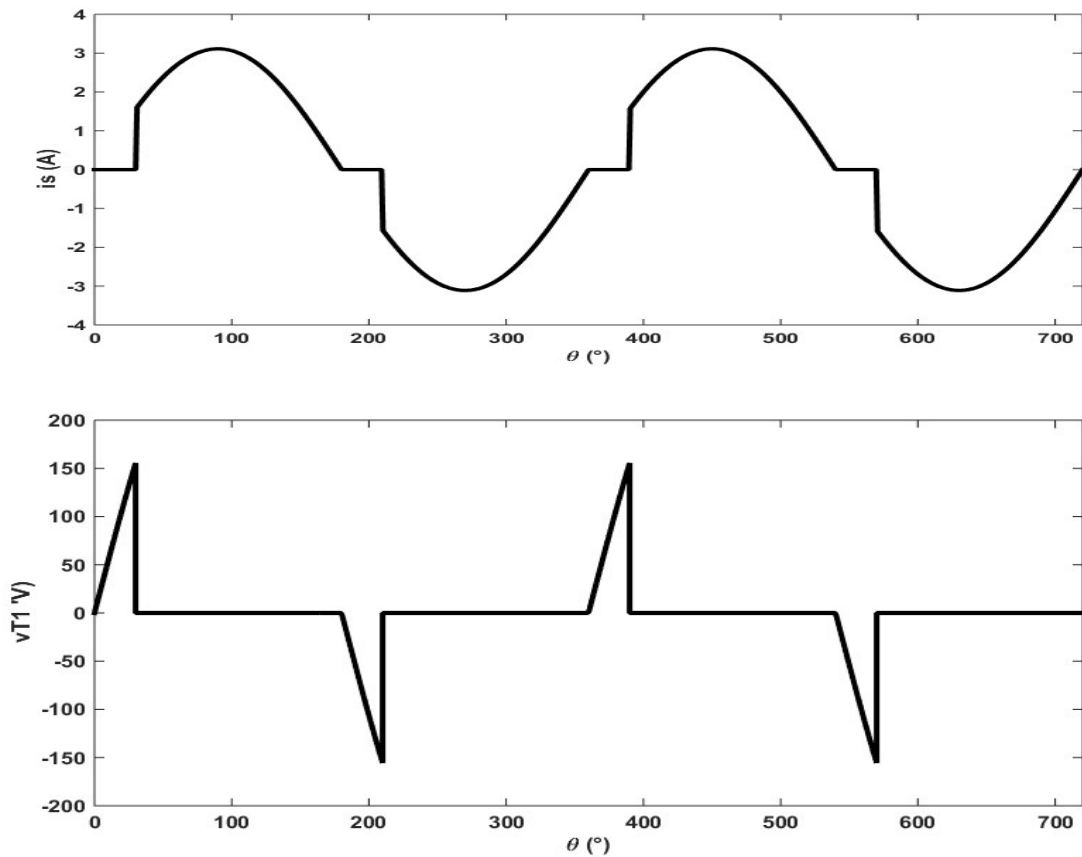
$$V_s = 0$$

-  $\alpha + \pi < \theta < 2\pi$  Th2 conducts and Th1 is blocked.

$$V_s = V_e$$

We then obtain the following voltage at the terminals of the load:





**Figure 6.2** Voltage and current timing diagrams of a single-phase dimmer on a load R for  $\alpha=30^\circ$

### 6.3.2.1.b.. Main relationships [7, 8, 11, 12]

- Average voltage value across the load terminals:

$$u_{cmoy} = \frac{1}{T} \int_0^T v(t) dt = 0$$

- Value of the effective voltage at the terminals of the load:

$$u_{seff} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt} = \sqrt{v_{effe}^2 \left(1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}\right)}$$

Average power dissipated in the load:

$$p = \frac{1}{T} \int_0^T v(t) \cdot i(t) dt = \frac{1}{T} \int_0^T v(t) \cdot \frac{v(t)}{R} dt$$

$$p = \frac{v_{effe}^2}{R} \left(1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}\right)$$

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### 6.3.2.2. Areas of use for this type of dimmer:

- Heating: regulation of the thermal power delivered to heating resistors.
- Lighting: variation of light intensity, particularly in incandescent lighting systems.
- Speed variation of low-power inductions motors, such as those used in drills, vacuum cleaners, or other household appliances (a few hundred watts) [7, 8, 11, 12].

### 6.3.2.3. Disadvantages:

- The voltage applied to the load is a non-sinusoidal alternating waveform, resulting in a non-sinusoidal current draw. This distortion generates a significant presence of harmonics in the current injected into the network, which can disrupt other electrical equipment.
- The relationship between the average power dissipated in the load and the control signal is not linear, which complicates precise power control and may require additional correction or regulation devices [7,8,11,12].

### 6.3.2.4. Flow on an inductive load

Due to the inductive effect, conduction continues after the end of the alternation, up to  $\theta_1$ , the instant of cancellation of the current  $i(\theta)$ , when the angle becomes less than  $\varphi$ , the argument of the receiver. Operation depends on the nature of the signals applied to the gates:

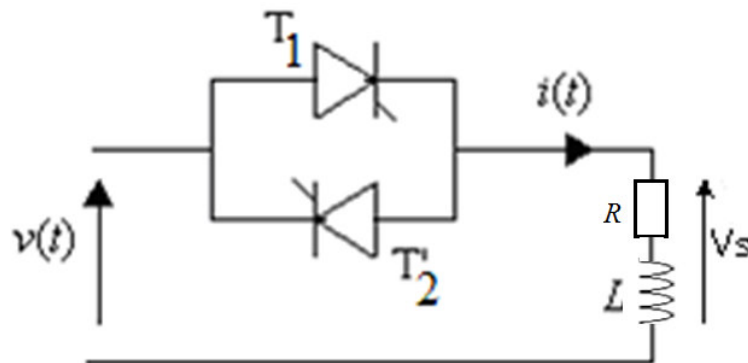


Figure 6.3 Single-phase dimmer on an RL load

There are two cases:

#### 6.3.2.5.a. case: 1 for $\varphi < \alpha < \pi$

The current in the load  $i_{sg}$  the sum of a free component  $i_{sh}$  characterizing the transient regime and a forced component  $i_{sp}$  characterizing the permanent regime

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$$R.i + L.\frac{di}{dt} = u_c = V_{\max} \cdot \sin(\omega.t)$$

The  $i_{sg}$  component is the solution to the equation

$$i_{sg} = i_{sh} + i_{sp}$$

The  $i_{sh}$  component is a solution to the equation without a right-hand side

$$R.i + L.\frac{di}{dt} = 0$$

$$i_{sh} = K.e^{\frac{-R}{L}.t}$$

The particular solution is

$$R.i + L.\frac{di}{dt} = u_c = V_{\max} \cdot \sin(\omega.t)$$

$$i_{sp} = i_{s\max} \cdot \sin(\theta - \varphi)$$

$$i_{s\max} = \frac{V_{\max}}{Z}, \quad \text{tg}(\varphi) = \frac{L.\omega}{R} = Q, \quad \theta = \omega.t \quad \text{and} \quad Z = \sqrt{(L.\omega)^2 + R^2}$$

The general solution is:

$$i_{sg} = \frac{V_{\max}}{Z} \cdot (\sin(\theta - \varphi) + \sin(\varphi - \alpha) \cdot e^{\frac{-(\theta - \alpha)}{Q}})$$

### ► Operational Analysis

#### For $\alpha < \theta < \theta_1$

TH1 remains conductive, it turns off before TH2 fires at  $\pi + \alpha$

#### For $\theta_1 < \theta < \pi + \alpha$

No thyristor is fired,  $i(\theta) = 0$ , and  $v_{TH1}(\theta) = -v_{TH2}(\theta) = v(\theta)$ .

#### For $\pi + \alpha < \theta < 2\pi$

TH2 becomes conductive, it turns off at  $\theta = \theta_1 + \pi$  when the current flowing through it is zero.

#### For $\theta = 2\pi + \alpha$

If TH1 is fired again, the phenomenon becomes periodic; it is therefore possible to adjust the current acting on  $\alpha$  for  $\varphi < \alpha < \pi$ .

6.3.2.4. Waveform of the different quantities

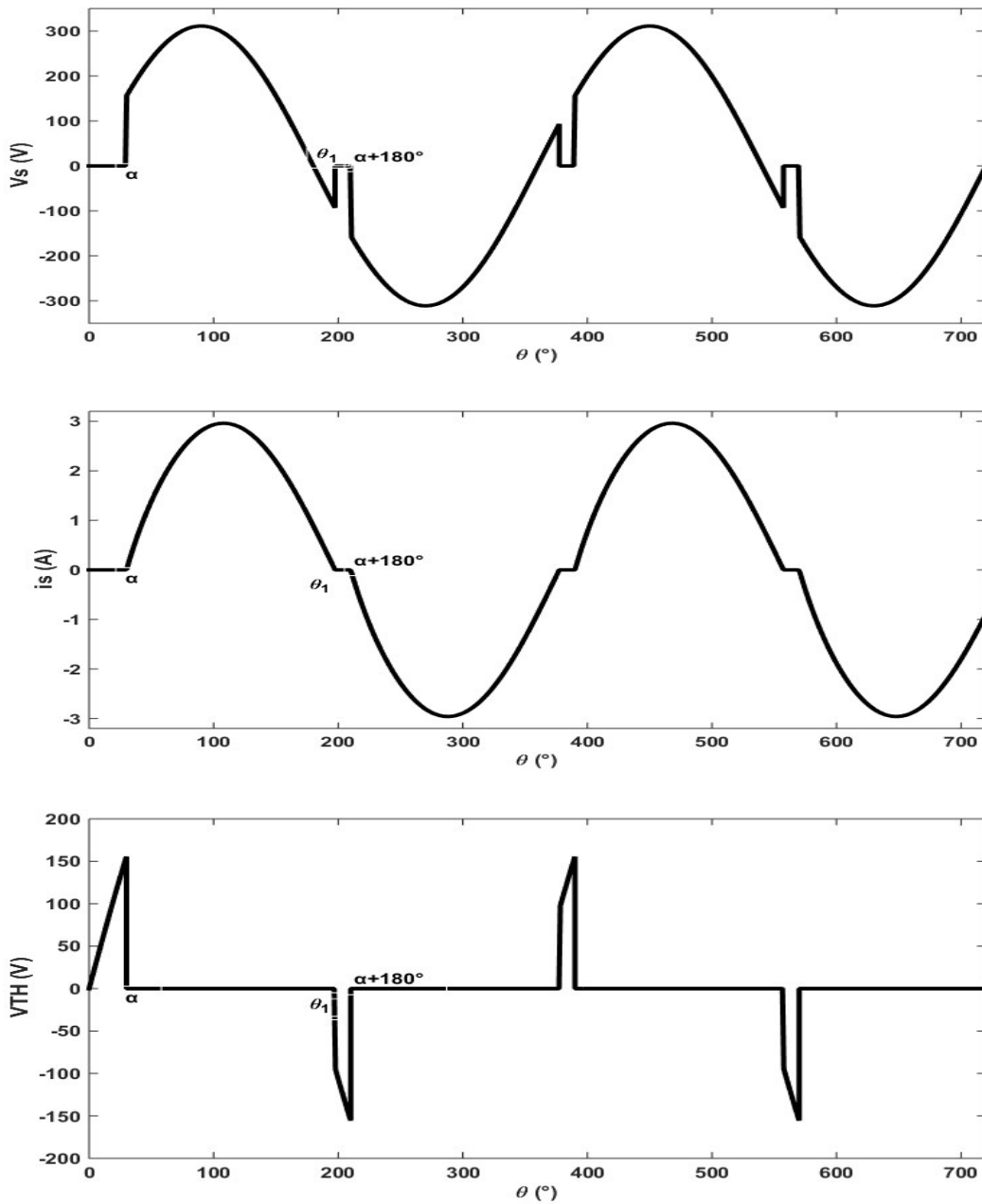


Figure 6.4 Voltage and current timing diagrams of a single-phase dimmer on an inductive load

6.3.2.5.a. case 2: for  $\alpha < \varphi$

When the angle becomes less than  $\varphi$ , the operation depends on the nature of the signals applied to the triggers:

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### ► Case of short-duration trigger pulse [7, 8, 11, 12]

Th1 begins to conduct. The current  $i(\theta)$  is, again

$$i_c = \frac{V_{\max}}{Z} \cdot (\sin(\theta - \varphi) + \sin(\varphi - \alpha)) \cdot e^{\frac{-(\theta - \alpha)}{Q}}$$

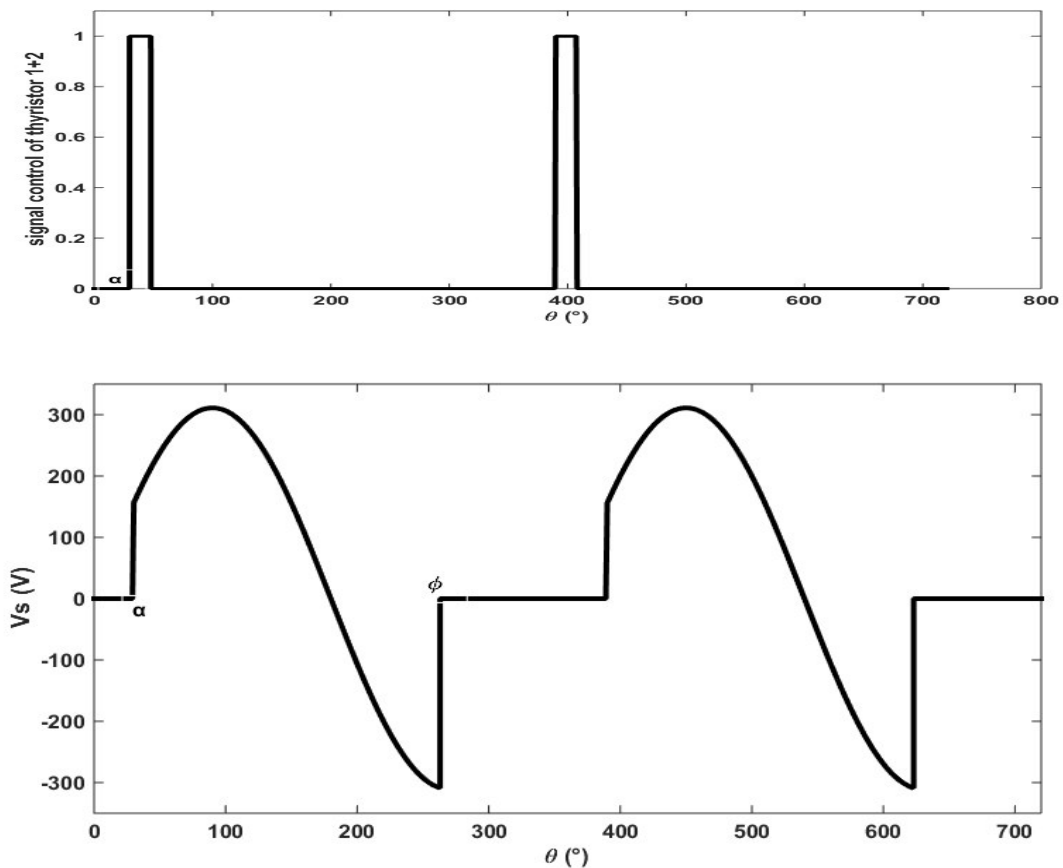
The current is zero for  $\theta_1$  greater than  $+\pi$ .

The pulse sent to the gate of thyristor TH2 for  $\theta = +\pi$  finds this rectifier with a negative anode voltage, and is therefore ineffective. When  $V_{\text{TH2}}(\theta)$  becomes positive for  $\theta = \theta_1$ , there is no longer any current flowing to the gate of TH2.

The circuit operates as a half-wave rectifier with a unidirectional output current.

$$i_c = \frac{V_{\max}}{Z} \cdot (\sin(\theta - \varphi) + \sin(\varphi - \alpha)) \cdot e^{\frac{-(\theta - \alpha)}{Q}}$$

### ► Waveform of the different quantities



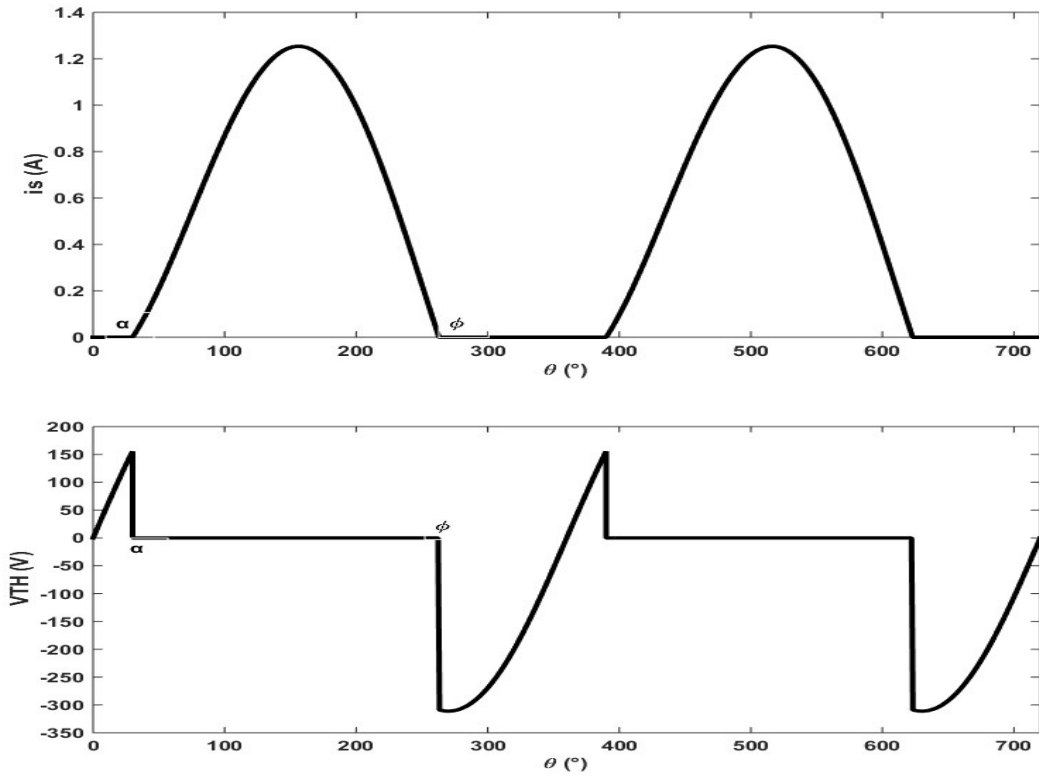


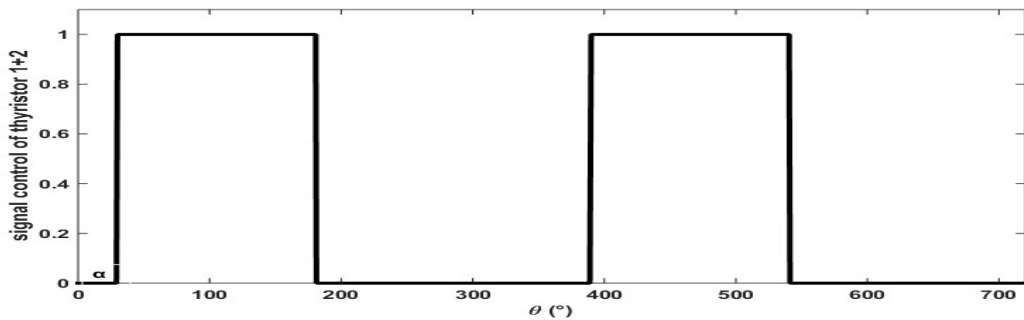
Figure 6.5 Voltage and current timing diagrams of a single-phase dimmer on an inductive load.

► Sufficient width trigger pulse case [7, 8, 11, 12]

Thyristor TH1 is triggered, and it remains conductive until the instant  $\theta = \theta_1$  as before. For  $\theta = \theta_1$ , the voltage across thyristor TH2 becomes positive, and its gate, powered since  $\theta = +\pi$ , still receives a turn-on current; this rectifier therefore begins to conduct. After a few periods, the current  $i(\theta)$  becomes sinusoidal.

The transition of the angle to a value less than  $\phi$  is now without any drawback.

► Waveform of the different quantities



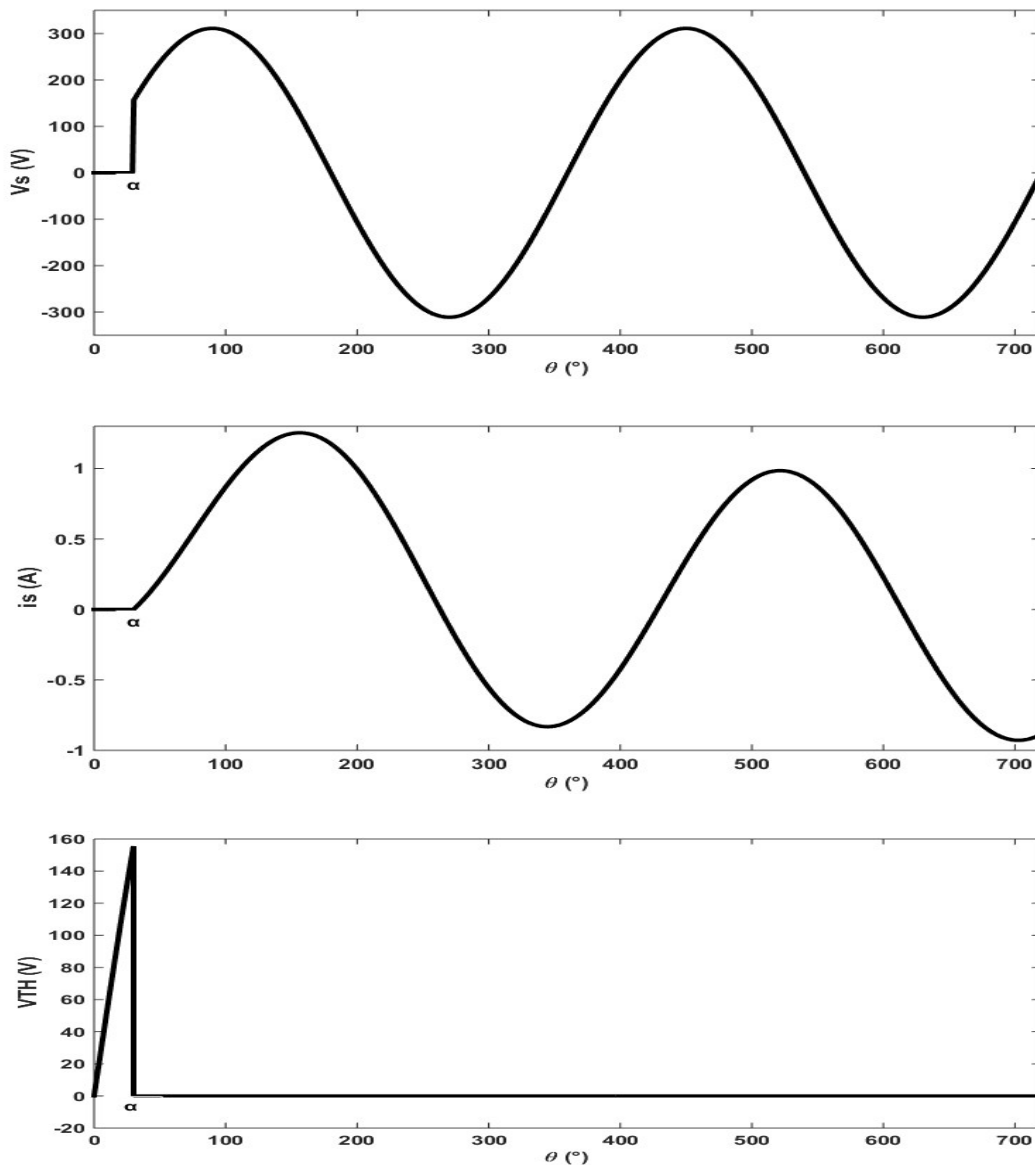


Figure 6.6 Voltage and current timing diagrams of a single-phase dimmer on an inductive load.

### 6.3.3. Burst Firing Dimmer

Operating principle of a single-phase burst dimmer operating on a resistive load. In this type of dimmer, the signal sent to the dimmer's control input is a discrete type.

Thyristor Th1 and thyristor Th2 are continuously triggered during the conduction period  $T_{on}$  and are then blocked until the end of the modulation period. The following voltage is then obtained across the load terminals [7, 8, 11, 12].



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They can be configured in different ways, depending on the coupling between the switches and the load [7, 8, 11, 12].

### 6.4.1. Coupling and Configuration of Three-Phase Dimmers:

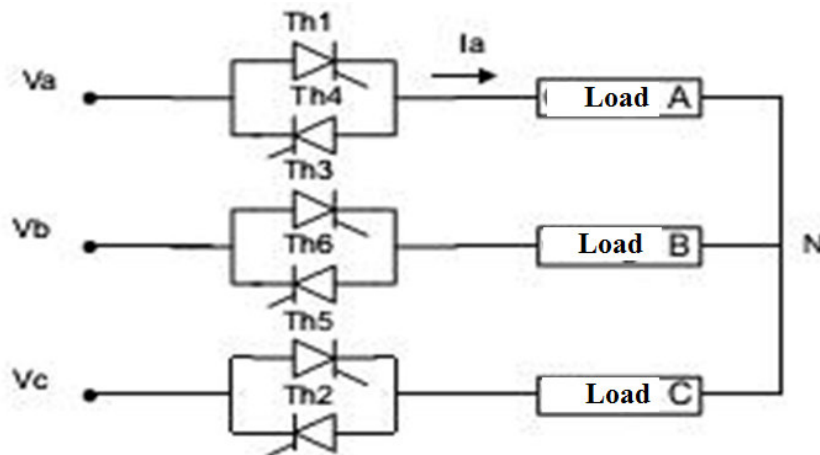
#### 6.4.1.1. Delta or Star Connection:

A three-phase dimmer typically uses three sets of two thyristors, connected between the three-phase supply and the load, and connected either in a delta or a star connection.

- Delta: This configuration is often used for better voltage stability and power control.
- Star: The supply and load are connected in a star connection, with accessible common points. This allows these points to be connected to form three equivalent single-phase dimmers, thus offering the advantages of a three-phase dimmer while maintaining the simplicity of single-phase coupling [7,8,11,12].

#### 6.4.1.2. Advantages of a Three-Phase Dimmer with Star Connection:

When the power supply and load are already configured in a star configuration, and the common points are accessible, these points can be easily connected to create a three-phase dimmer configuration. This system allows the load to be distributed evenly between the phases and optimizes the dimmer's performance while offering the advantages of a single-phase dimmer.



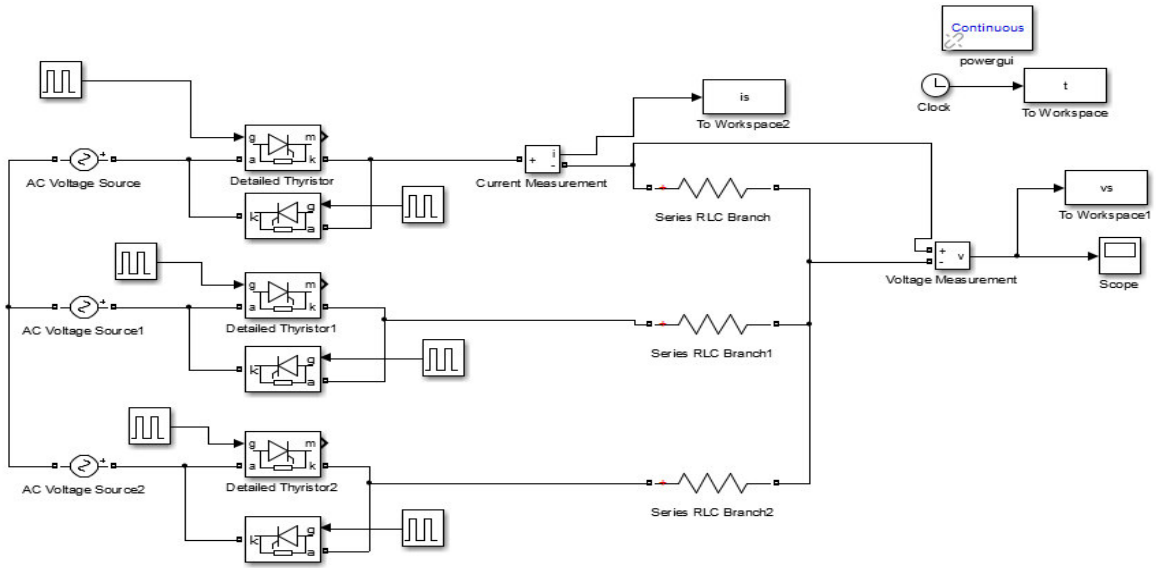


Figure 6.8 : Three-phase dimmer circuit with star-connected load

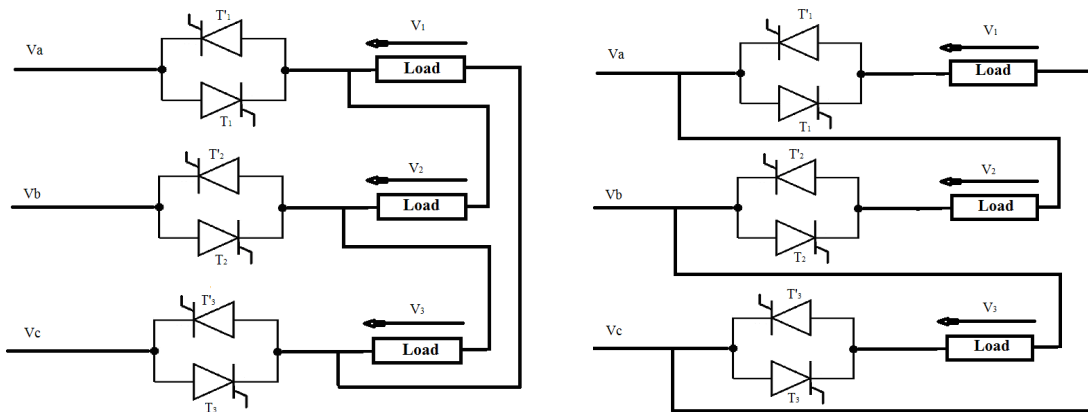


Figure 6.9 Three-phase dimmer circuit with delta-connected load

**6.4.2. Load flow:**

Given the circuit structure, isolated conduction of a switch is impossible. Therefore, there can only be three possibilities:

a) One switch is controlled: No current conduction is possible (no current loop)

b) Two switches are controlled:

Since the impedances in each conducting phase are identical, the corresponding phase-to-phase voltage is distributed equally between the two elements concerned. Thus, for example, if [TH1;TH4] and [TH2;TH5] are conductive, we have

$$V_1(\theta) = \frac{1}{2}(V_A(\theta) - V_B(\theta)), \quad V_1(\theta) = -V_2(\theta)$$

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[TH3;TH6] blocked. The law of meshes, applied to phases 2 and 3 gives.

$$V_c(\theta) - V_{T3}(\theta) + V_2(\theta) - V_B(\theta) = 0$$

As  $V_3(\theta) = 0$  and  $V_2(\theta) = \frac{1}{2}(V_B(\theta) - V_A(\theta))$ , we deduce, all calculations made, that

$$V_{T3}(\theta) = \frac{2}{3}V_c(\theta)$$

c) Three switches are controlled:

The installation then corresponds to a balanced three-phase installation.

We will limit ourselves to plotting the output voltages for example  $V_1$ , the others being identical at offsets of  $2\pi/3$ .

The complete study shows that, depending on the value of the firing delay angle of each thyristor, three operating modes are possible.

**1<sup>st</sup> Case :**  $0 < \alpha < \frac{\pi}{3}$

there are 3 or 2 conductive thyristors:

$\alpha < \theta < \frac{\pi}{3}$  TH1, TH5 and TH3 drivers from where  $V_1(\theta) = V_A(\theta)$ ,  $V_2(\theta) = V_B(\theta)$  et  $V_3(\theta) = V_c(\theta)$

$\frac{\pi}{3} < \theta < \alpha + \frac{\pi}{3}$  TH1 and TH5 drivers from where  $V_1(\theta) = \frac{1}{2}(V_A(\theta) - V_B(\theta))$ ,  $V_1(\theta) = -V_2(\theta)$  et  $V_3(\theta) = 0$

The output voltage has the effective value

$$u_{seff} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt} = \sqrt{\frac{2}{T} \int_{\alpha}^{\frac{\pi}{3}} v_A^2(t) dt + \frac{2}{T} \int_{\frac{\pi}{3}}^{\frac{\pi}{3} + \alpha} \left(\frac{1}{2}(v_A(t) - v_B(t))\right)^2 dt + \frac{2}{T} \int_{\frac{\pi}{3} + \alpha}^{\frac{2\pi}{3}} v_A^2(t) dt + \frac{2}{T} \int_{\frac{2\pi}{3}}^{\frac{2\pi}{3} + \alpha} \left(\frac{1}{2}(v_A(t) - v_C(t))\right)^2 dt + \frac{2}{T} \int_{\frac{2\pi}{3} + \alpha}^{\pi} v_A^2(t) dt}$$

$$u_{seff} = \sqrt{v_{effe}^2 \left(1 - \frac{3\alpha}{2\pi} + \frac{3}{4\pi} \cdot \sin(2\alpha)\right)}$$

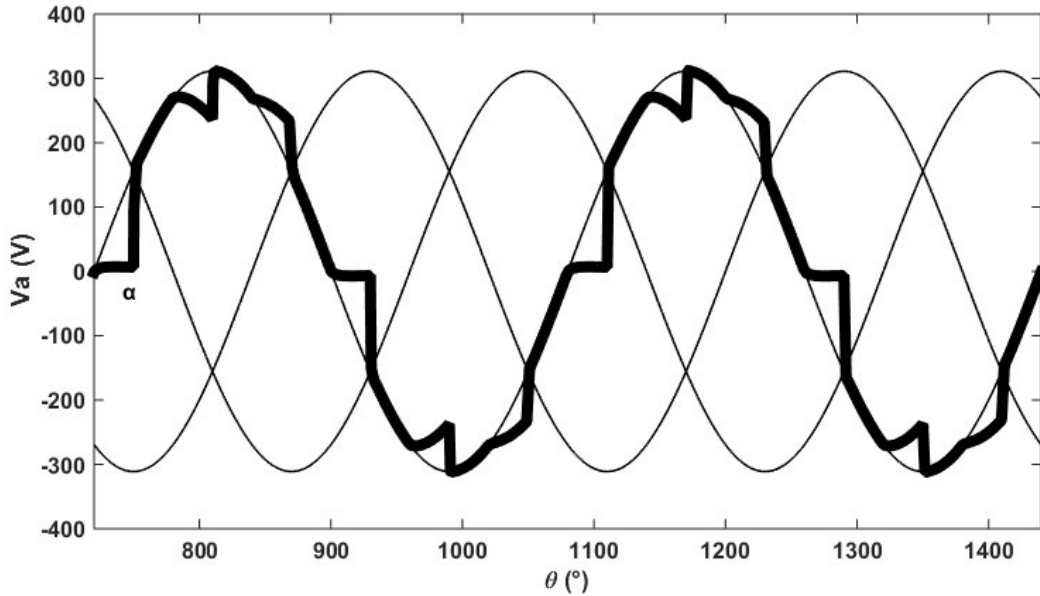


Figure 6.10 waveform of  $V_1(\theta)$  for  $\alpha = \pi/6$

2<sup>nd</sup> Case :  $\frac{\pi}{3} < \alpha < \frac{\pi}{2} + \alpha$

there are 2 conductive thyristors

$\alpha < \theta < \frac{\pi}{3} + \alpha$  TH1 and TH5 conduct where  $V_1(\theta) = \frac{1}{2}(V_A(\theta) - V_B(\theta))$ ,  $V_1(\theta) = -V_2(\theta)$  et  $V_3(\theta) = 0$

$\alpha + \frac{\pi}{3} < \theta < \frac{2\pi}{3}$  TH1 and TH5 conduct where  $V_1(\theta) = \frac{1}{2}(V_A(\theta) - V_B(\theta))$ ,  $V_1(\theta) = -V_3(\theta)$  et  $V_2(\theta) = 0$

The output voltage has the effective value

$$\begin{aligned}
 u_{seff} &= \sqrt{\frac{1}{T} \int_0^T v^2(t) dt} = \sqrt{\frac{2}{T} \int_{\alpha}^{\frac{\pi}{3} + \alpha} \left(\frac{1}{2}(v_A(t) - v_B(t))\right)^2 dt + \frac{2}{T} \int_{\frac{\pi}{3} + \alpha}^{\frac{2\pi}{3} + \alpha} \left(\frac{1}{2}(v_A(t) - v_C(t))\right)^2 dt} \\
 &= \sqrt{\frac{2}{T} \int_{\alpha}^{\frac{5\pi}{6}} \left(\frac{1}{2}(V_{max} \sin(\omega t)) - V_{max} \sin(\omega t - \frac{2\pi}{3})\right)^2 dt + \frac{2}{T} \int_{\frac{\pi}{3} + \alpha}^{\frac{7\pi}{6}} \left(\frac{1}{2}(V_{max} \sin(\omega t)) - V_{max} \sin(\omega t - \frac{4\pi}{3})\right)^2 dt} \\
 u_{seff} &= \sqrt{v_{eff}^2 \left(\frac{1}{2} + \frac{3\sqrt{3}}{4\pi} \cdot \sin\left(2\alpha + \frac{\pi}{6}\right)\right)}
 \end{aligned}$$

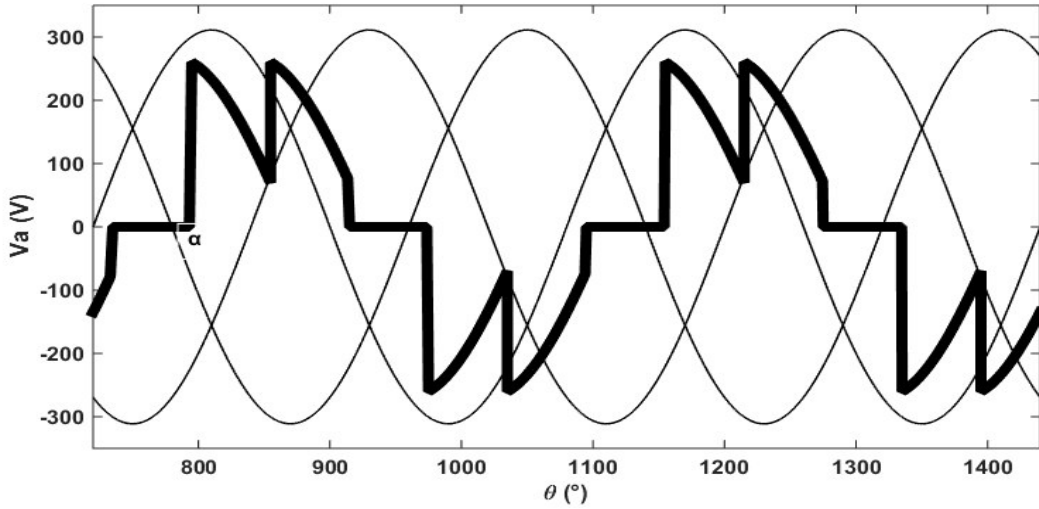


Figure 6.11 waveform of  $V_1(\theta)$  for  $\alpha=5\pi/12$

**3<sup>rd</sup> Case :**  $\frac{\pi}{2} < \alpha < \frac{5\pi}{6}$

there are always 2 or 0 thyristors passing; it is necessary to send "conformation pulses": when a rectifier is unlocked, it is necessary at the same time to send a pulse to the trigger of the one which had entered into conduction one sixth of a period previously

$\alpha < \theta < \frac{5\pi}{6}$  TH1 and TH5 conduct where  $V_1(\theta) = V_2(\theta) = V_3(\theta) = 0$

$\alpha + \frac{\pi}{3} < \theta < \frac{5\pi}{6} + \frac{\pi}{3}$  TH1 and TH6 conduct where  $V_1(\theta) = \frac{1}{2}(V_A(\theta) - V_B(\theta))$

$V_1(\theta) = V_3(\theta) - V_2(\theta) = 0$

The output voltage has the effective value

$$u_{seff} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt} = \sqrt{\frac{2}{T} \int_{\alpha}^{\frac{5\pi}{6}} \left(\frac{1}{2}(v_A(t) - v_B(t))\right)^2 dt + \frac{2}{T} \int_{\frac{\pi}{3} + \alpha}^{\frac{7\pi}{6}} \left(\frac{1}{2}(v_A(t) - v_C(t))\right)^2 dt}$$

$$= \sqrt{\frac{2}{T} \int_{\alpha}^{\frac{5\pi}{6}} \left(\frac{1}{2}(V_{max} \sin(\omega t)) - V_{max} \sin(\omega t - \frac{2\pi}{3})\right)^2 dt + \frac{2}{T} \int_{\frac{\pi}{3} + \alpha}^{\frac{7\pi}{6}} \left(\frac{1}{2}(V_{max} \sin(\omega t)) - V_{max} \sin(\omega t - \frac{4\pi}{3})\right)^2 dt}$$

$$u_{seff} = \sqrt{v_{efes}^2 \left(\frac{5}{4} - \frac{3\alpha}{2\pi} + \frac{3}{4\pi} \cdot \sin(2\alpha + \frac{\pi}{3})\right)}$$

with

$$\frac{1}{2}(V_{\max} \sin(\omega t)) - V_{\max} \sin(\omega t - \frac{2\pi}{3}) = V_{\max} \frac{\sqrt{3}}{2} \cos(\omega t - \frac{\pi}{3}) = V_{\max} \frac{\sqrt{3}}{2} \sin(\omega t + \frac{\pi}{6})$$

and

$$\frac{1}{2}(V_{\max} \sin(\omega t)) - V_{\max} \sin(\omega t - \frac{4\pi}{3}) = V_{\max} \frac{\sqrt{3}}{2} \cos(\omega t - \frac{2\pi}{3}) = V_{\max} \frac{\sqrt{3}}{2} \cos(\omega t + \frac{\pi}{3}) = V_{\max} \frac{\sqrt{3}}{2} \sin(\omega t - \frac{\pi}{6})$$

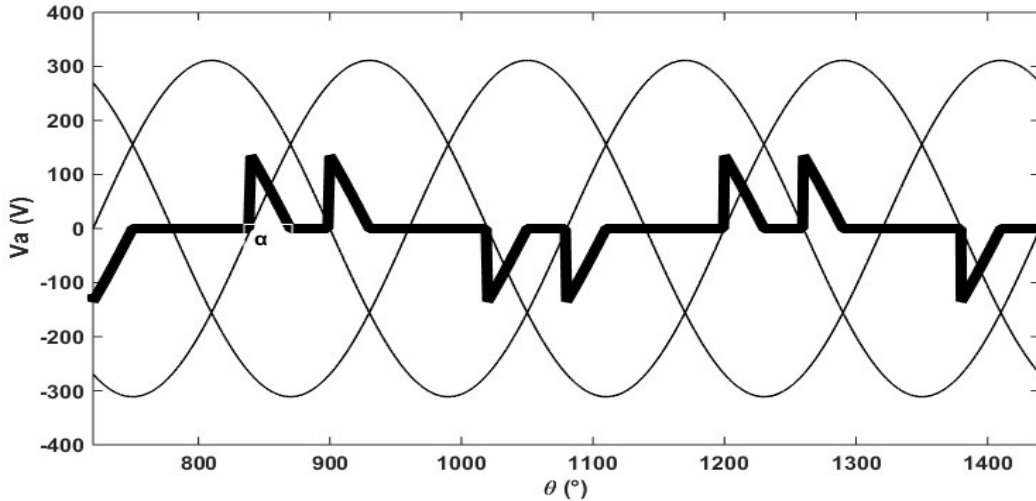


Figure 6.12 Waveform of of  $V_1(\theta)$  for  $\alpha=2\pi/3$

### 6.5. Cycloconverters

Cycloconverters are naturally commutated devices that allow one or more voltages at a lower frequency, generally much lower than that of the power supply, to be obtained from a given frequency network. Because they operate only as a step-down converter, cycloconverters are only one component of the direct frequency converter family, which also includes frequency multipliers [7, 8, 11, 12].

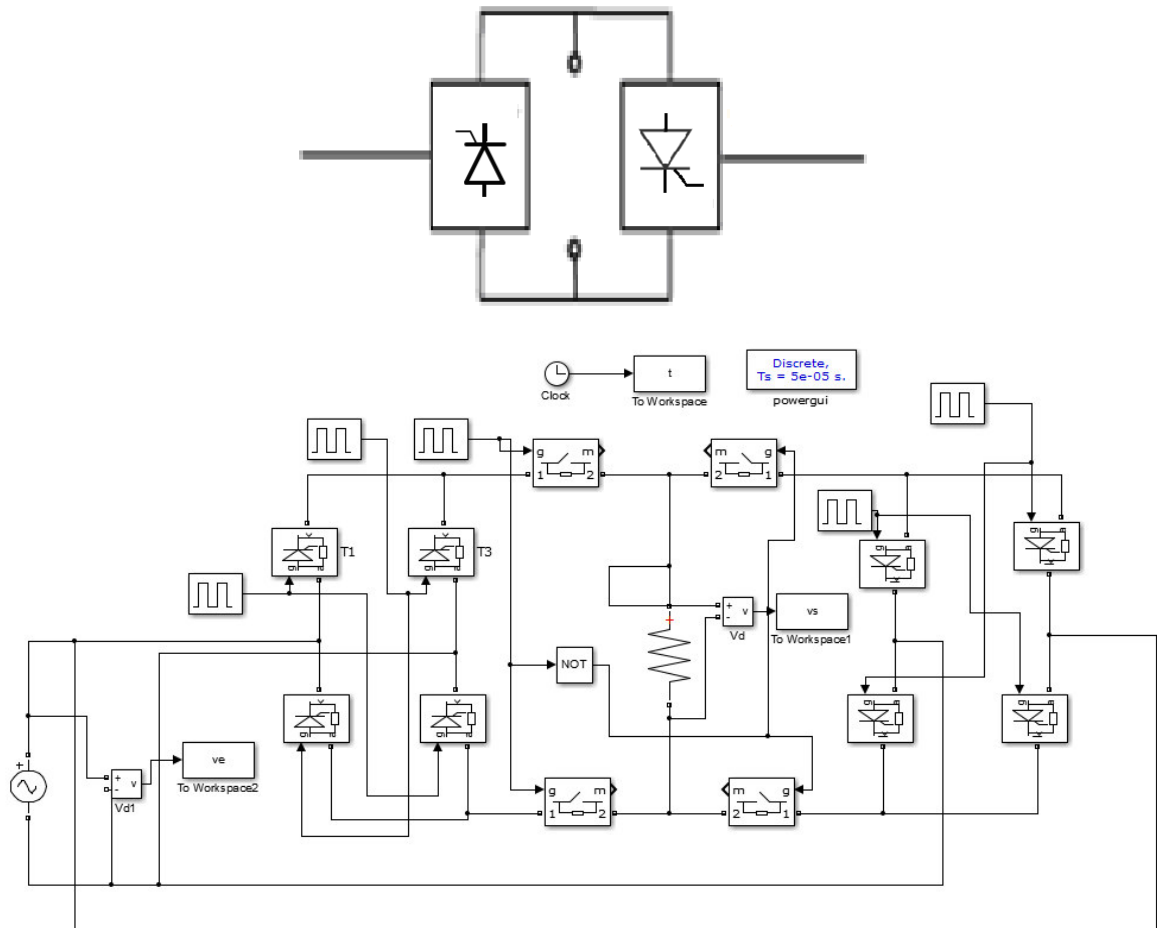
#### 6.5.1. Operating Principle

Output voltages are generated using portions of sinusoids from the power supply network. Precisely slicing these sinusoids allows for signals with a specific frequency and amplitude, while maintaining a reasonable harmonic content.

The structure resembles the basic schematics of rectifier circuits. Indeed, it is sufficient to modulate the triggering delay angle according to the rhythm of the low frequency to obtain, at the circuit output, a voltage shape similar to that shown above. However, since these devices can only supply unidirectional currents, each phase of the cycloconverter is composed of two

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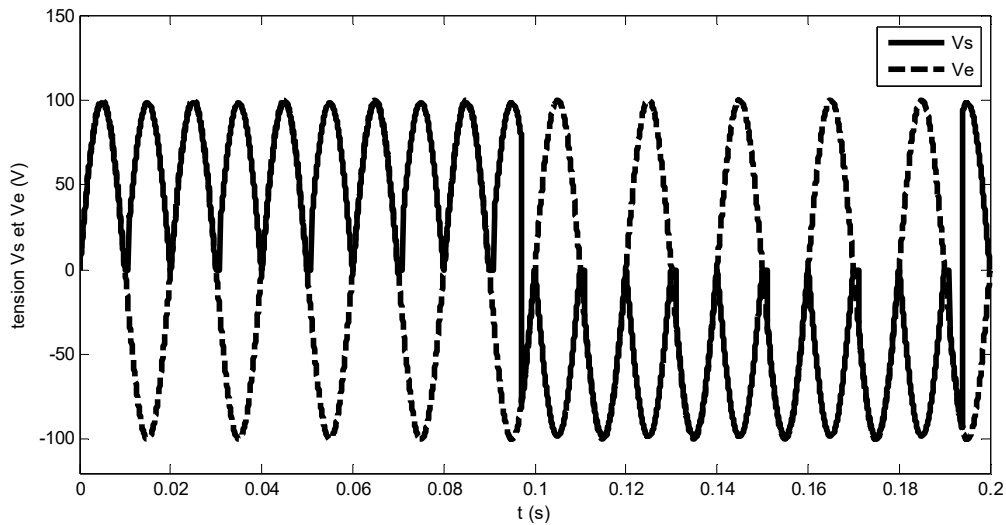
rectifiers connected in inverse parallel (as shown in figure 6.13), each delivering an alternation of the output current. This immediately implies that the cycloconverter is inherently reversible, since its components are also reversible [7, 8, 11, 12].



**Figure 6.13** Single-phase cycloconverter

It should also be noted that, as with controlled rectification, the various back-to-back groups can operate with or without circulating current.

For example, from a three-phase network with a frequency of  $f_0$ , it is possible to create a signal with a frequency of  $f_0/10$ , as illustrated below.



**Figure 6.14** Timing diagram of a cycloconverter output voltage for a 50Hz frequency source

### 6.5.2. Example of assemblies used

Cycloconverters are classified according to their pulsation index, which, like rectifier assemblies, corresponds to the number of commutations occurring during a power supply cycle. It is easy to understand that generating signals with a low harmonic content is easier as this index increases. However, since the number of thyristors also increases proportionally, cycloconverter configurations are mainly divided into one of the following two categories [7, 8, 11, 12].

### 6.5.3. Cycloconverters with a pulsation index equal to 3

As shown below, these consist of combinations of P3-type rectifiers. The inductors, whose role is to limit the amplitude of the circulating currents, can obviously be removed if this operating mode is not used [7, 8, 11, 12].

As shown below, cycloconverters consist of combinations of P3-type rectifiers. The inductors, whose role is to limit the amplitude of the circulating currents, can obviously be removed if this operating mode is not used.

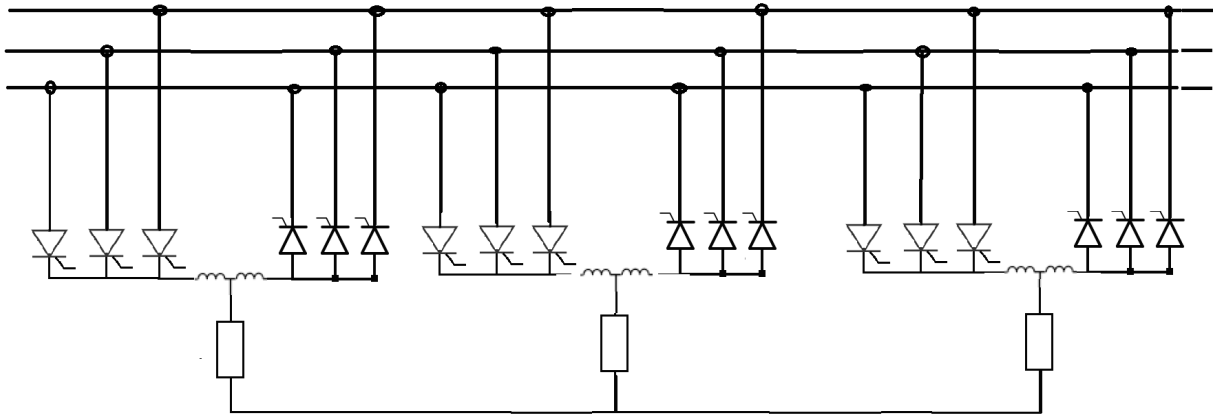
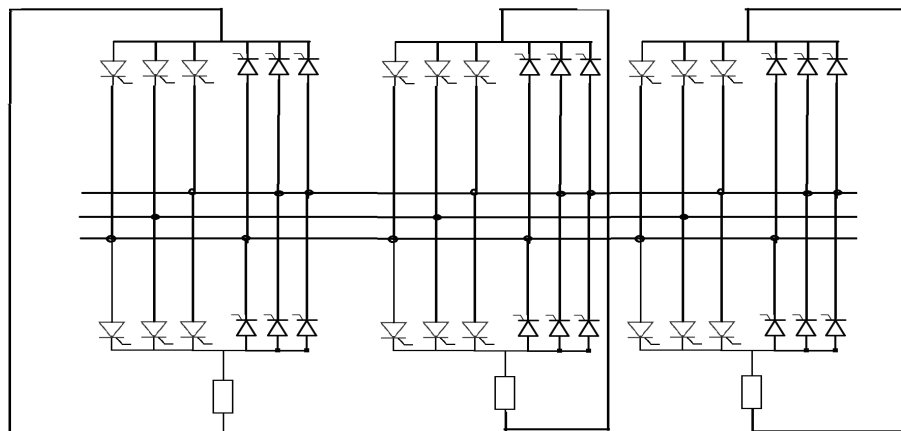


Figure 6.15 Three-phase cycloconverter

#### 6.5.4. Cycloconverters with a pulse index of 6

These are the most commonly used devices. Several configurations are possible, but we will only mention the basic arrangement here, consisting of PD3-type rectifiers (see figure 6.16). It should be noted that this arrangement can only be used when the load actually consists of three independent elements [7, 8, 11, 12].



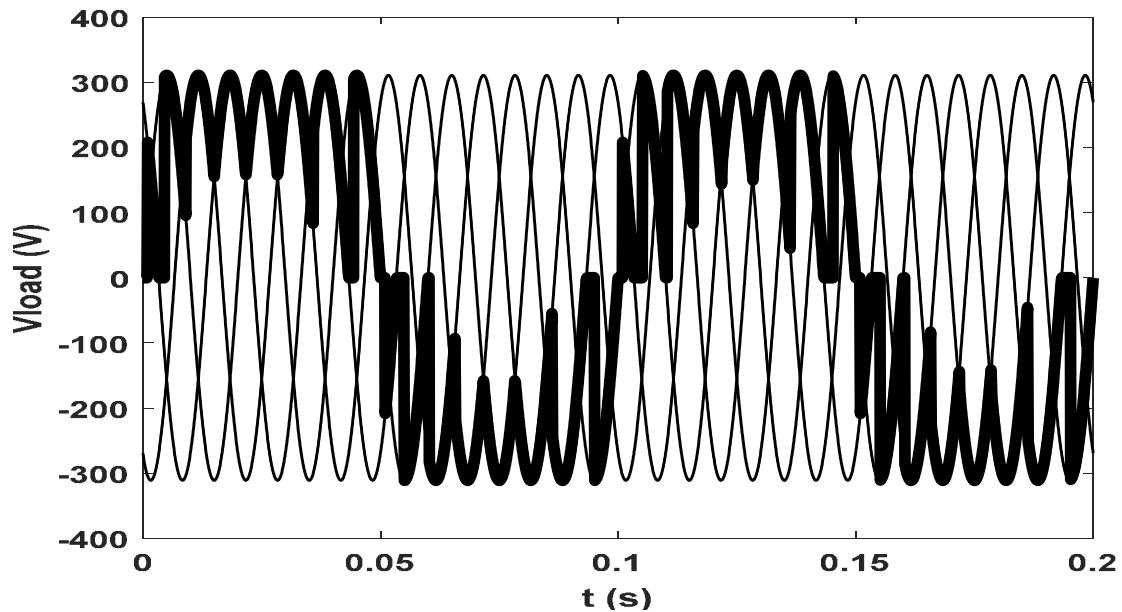


Figure 6.16 Three-phase bridge cycloconverter

### 6.5.5. Cycloconverter Applications

Due to the complexity of these devices, cycloconverter applications are primarily limited to high-power applications. In addition to their use in variable speed drives for AC machines, cycloconverters can be used in the following areas:

#### 6.5.5.1. Source Frequency Conversion

The optimal operating frequency for power generators is generally higher than the industrial frequency. For autonomously powered equipment, it is possible to operate the source at the frequency that optimizes its performance. Then, using one or more cycloconverters, this frequency can be converted to match the frequency required to operate the various load components. This optimizes generator efficiency while ensuring compatibility with consumption systems at standard industrial frequencies [7, 8, 11, 12].

#### 6.5.5.2. Reactive Power Generation

One possible technique is to use a "high-frequency base". The operating principle is illustrated opposite. The cycloconverter, powered by high-frequency signals (generated here using simple oscillating circuits, since the assembly does not need to supply active power), delivers energy to the network via inductances  $L$ . It is controlled so that its output voltages have the same angular frequency  $\omega_0$  as that of the network and are in phase with it. Since the output currents are in quadrature with the voltages, the reactive power supplied by the cycloconverter can be expressed

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in the following form  $\frac{(U_1 - U) \cdot U}{L\omega_0}$ , with  $U_1$ , the effective value of the fundamental of the phase-to-phase output voltage of the cycloconverter, and  $U$ , the corresponding effective value for the network. By adjusting the amplitude of  $U_1$ , it is therefore possible to vary the reactive power supplied or absorbed by the device [7, 8, 11, 12].

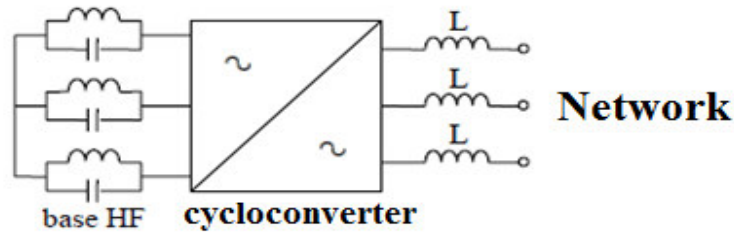


Figure 6.17 Application of a cycloconverter

### 6.5.5.3. Connecting Two Networks of Different Frequency

The basic single-line diagram is shown below. Depending on the direction of power transfer, the cycloconverters can either absorb or supply active power at the output. As in the previous application, the high-frequency (HF) base generates the high-frequency signals required for the input of each cycloconverter. The filters, for their part, reduce the harmonics present in the output currents, thus ensuring a better signal quality [7, 8, 11, 12].

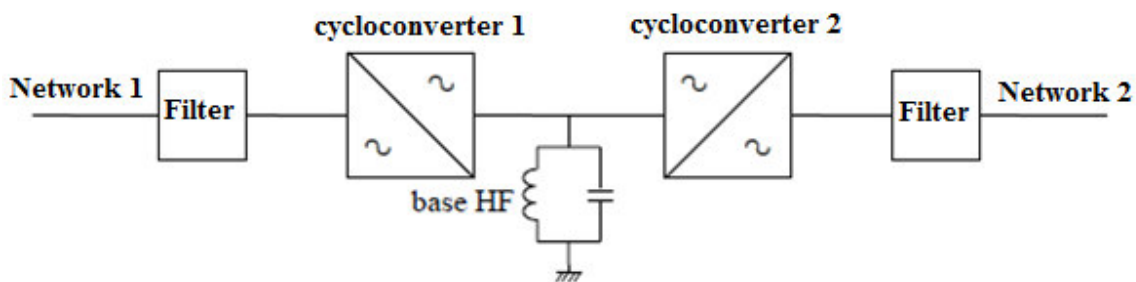


Figure 6.18 Application of a cycloconverter

## 6.6. Conclusion

AC-AC converters are used to control the flow of an AC source to an AC receiver, with or without frequency change. They ensure a constant connection between the source and the load, and can also interrupt this connection, which corresponds to switching operation. In addition, these converters make this connection intermittent, which allows the regulation of the current intensity that the source supplies to the receiver, a function performed in dimmer mode.

# **Chapter 7: Synthesis of Static Converters**

### 7.1. Introduction

The search for static converter structures adapted to the specific problems to be solved has given rise to numerous studies, both in high-tech sectors and in industry. Therefore, converter design is increasingly becoming the result of thoughtful reasoning. Current approaches to static converter synthesis are generally based on the nature of the energy conversion, such as DC-DC [3, 4, 5], AC-DC [7], etc. Other approaches rely on the analysis of all possible operating sequences of a converter [2], taking into account the voltage and/or current reversibilities of the input and output sources. Electrical energy conversion consists of transforming the form and characteristics of the energy supplied by the source to adapt it to the application. Controlled power levels range from a few watts to several megawatts.

### 7.2. Switching power supply

Switching power supply designs as we know them today are directly derived from stabilized power supplies, but seek to minimize the two main drawbacks of linear power supplies:

- The size and weight of the transformer.
- Low efficiency due to power dissipation in the regulator.

From a functional standpoint, the regulator acts as a voltage-controlled DC-DC converter. The power dissipated by the regulator results from the linear nature of its operation, meaning that it simultaneously presents a non-zero voltage across its terminals and a non-zero current flowing through it. The same function can be achieved by using a chopper (for example, in series) associated with a filter, the whole being voltage-controlled.

In this case, the power transistor, which modifies the output voltage, operates in switching mode, and its operating losses are considerably reduced. In fact, the component is either blocked (the current flowing through it is zero), or in conduction (the voltage across its terminals is close to zero), which makes it possible to optimize efficiency [7, 8, 11, 12].

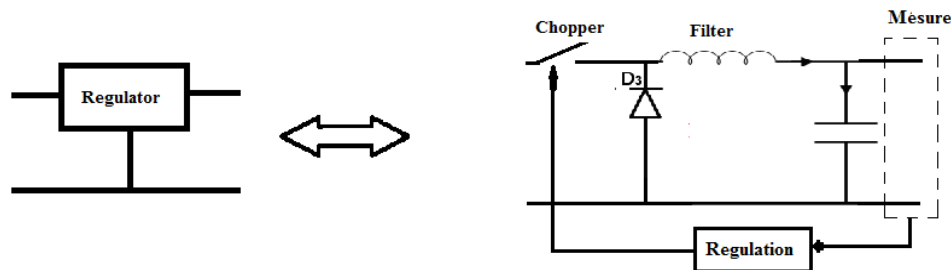


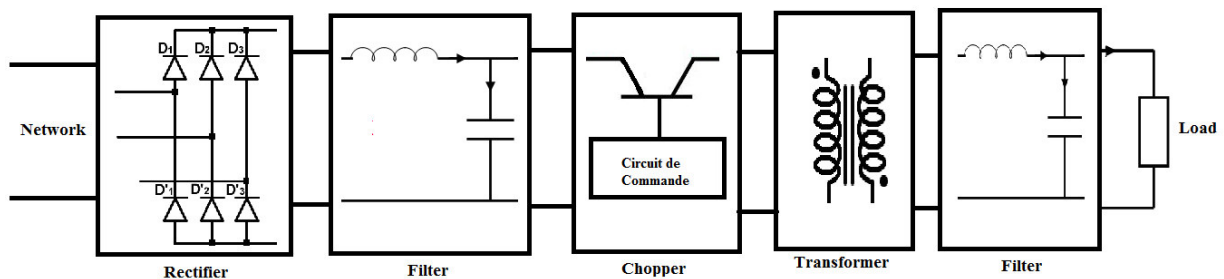
Figure 7.1 Chopper adjustment system

## Chapter 7: Synthesis of Static Converters

Using a chopper, filter, and regulator combination not only significantly improves power supply efficiency, but also indirectly reduces the size and volume of the transformer. Indeed, to reduce the size of a magnetic circuit, particularly for a transformer, the most effective method is to increase its operating frequency.

In a switching power supply, the transformer can be placed between the chopper and the filter. Thus, it is powered by an AC voltage source (the chopper's output voltage) while operating at the chopper's frequency, which is generally between a few tens and a few hundred kilohertz.

The structure of a switching power supply using a series chopper can then be represented as follows.



**Figure 7.2** Structure of a switching power supply

We will discuss and examine three main families of switching power supplies, each based on a specific type of chopper:

- Flyback power supplies: Based on the inductive storage chopper, these are commonly used for low- to medium-power applications, with galvanic isolation between the input and output.
- Forward power supplies: Based on the one-quadrant series chopper, these offer greater efficiency and are used for higher-power applications while ensuring more stable output voltage regulation.
- Push-pull power supplies: Based on the bridge chopper principle, these power supplies are often used for medium- to high-power applications, offering good performance in terms of efficiency and stability.

Each of these designs has advantages and disadvantages depending on the specifics of the application.

### 7.3. Inductive Storage Switching Power Supply (FLYBACK) [7, 8, 11, 12]

The design is based on the inductive storage chopper, whose inductance is doubled in a coupled magnetic structure that provides galvanic isolation but whose dimensioning is that of an inductor.

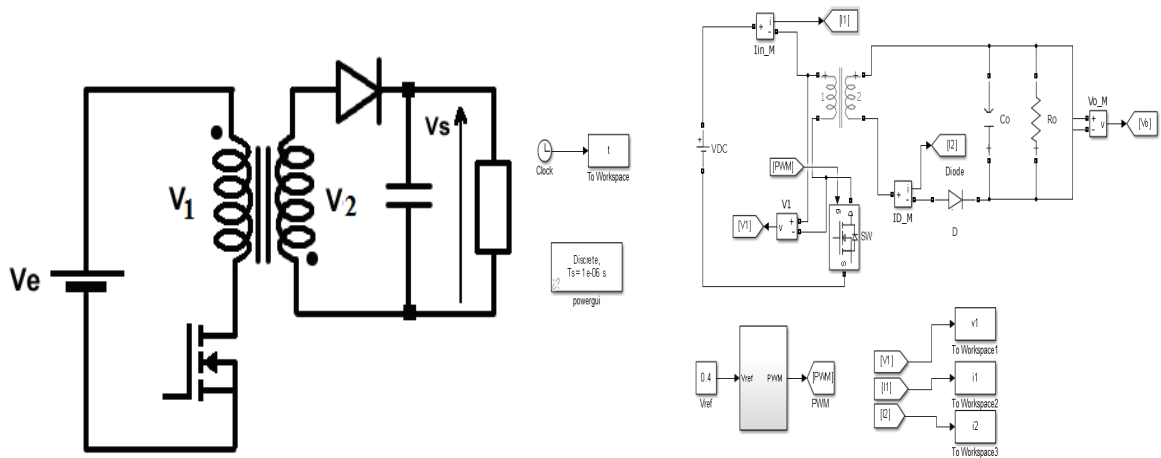


Figure 7.3 FLYBACK switching power supply

### 7.3.1. Operating principle in continuous mode

For  $0 < t < DT$  switch closed and diode blocked

$$V_2 = \frac{-n_2}{n_1} \cdot V_e$$

At  $t=0$   $i_1(0) = i_{1\min}$

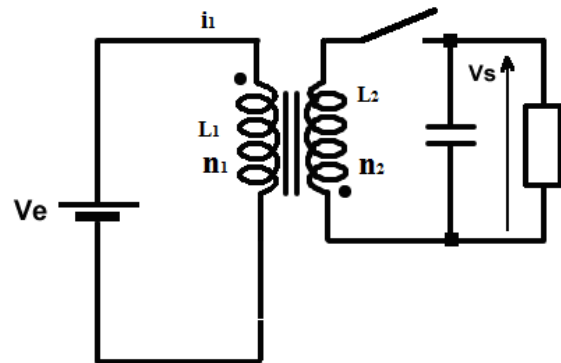
$$V_e(t) = V_1(t) = L_1 \cdot \frac{di_1}{dt}(t)$$

so

$$i_1(t) = \frac{V_e}{L_1} \cdot t + i_{1\min}$$

At  $t=DT$

$$i_1(D.T) = i_{1\max} = \frac{V_e}{L_1} \cdot D.T + i_{1\min}$$



When  $DT < t < T$  switch open and diode on.

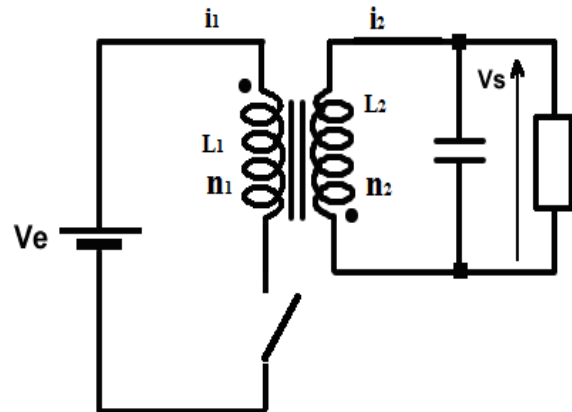
$$V_1 = \frac{-n_1}{n_2} \cdot V_s$$

$$V_s(t) = V_2(t) = -L_2 \cdot \frac{di_2}{dt}(t)$$

$$V_2(t) = V_s(t) = cst$$

so

$$i_2(t) = \frac{-V_s}{L_2} \cdot t + K$$



## Chapter 7: Synthesis of Static Converters

---

At  $t=DT$

$$i_2(D.T) = i_{2\max}$$

$$i_2(t) = \frac{-V_s}{L_2} \cdot (t - D.T) + i_{2\max}$$

At  $t=T$   $i_2(T) = i_{2\min}$

$$i_2(T) = \frac{-V_s}{L_2} \cdot (T - D.T) + i_{2\max} = i_{2\min}$$

If we consider that the converter has reached its steady state, the average voltage across the transformer windings is zero. If we consider in particular the average voltage across the secondary winding:

$$V_{2\text{avg}} = \frac{1}{T} \int_0^T V_2(t) dt = \frac{1}{T} \int_0^{DT} \frac{-n_2}{n_1} V_e dt + \frac{1}{T} \int_{DT}^T V_s dt = \frac{1}{T} \left( \frac{-n_2}{n_1} \cdot D.T.V_e + (1-D).T.V_s \right) = 0$$

$$V_s = \frac{n_2}{n_1} \cdot \frac{D}{(1-D)} V_e$$

$$V_s = m \cdot \frac{D}{(1-D)} V_e$$

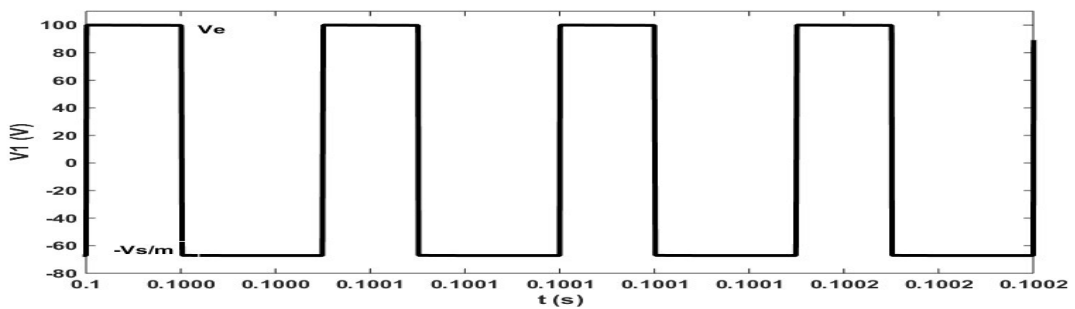
with  $m = \frac{n_2}{n_1}$

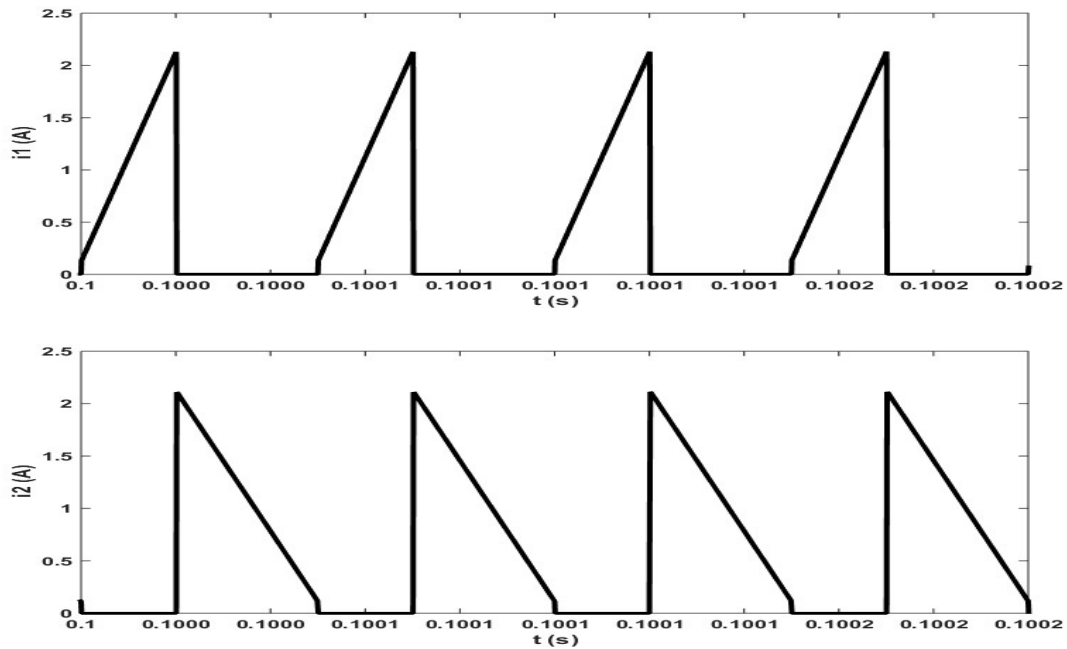
$$i_2 = i_s$$

On the conservation of power

$$p_1 = p_2 \Rightarrow V_e \cdot i_1 = V_s \cdot i_2 \Rightarrow i_1 = \frac{V_s}{V_e} \cdot i_2$$

$$i_1 = m \cdot \frac{D}{(1-D)} \cdot i_2$$





**Figure 7.4** Voltage and current timing diagrams of a FLYBACK switching power supply

### 7.3.2. Advantages [7, 8, 11, 12]

- Simplicity and cost-effectiveness: It is easy to design and implement, making it a cost-effective solution.
- Few components: The architecture requires a reduced number of components.
- Single wound component: A single transformer is used, simplifying the design.
- Low-power architecture: It is particularly suitable for low-power applications (<150W), offering a cost-effective solution for these needs.

### 7.3.3. Disadvantages of the flyback power supply:

- Switch sizing: The selection and sizing of the switch are crucial to ensure reliable operation.
- Overvoltage due to transformer leakage inductance: This can lead to overvoltages, requiring the addition of a surge suppressor to limit these voltage spikes.
- Transformer coupling: Transformer coupling can present challenges, particularly in terms of regulation.
- Filtering (discontinuous conduction): The nature of discontinuous conduction can make filtering more complex.

## Chapter 7: Synthesis of Static Converters

- Bulky for powers greater than 200W: Since the energy is stored in the coupled inductor and the output capacitor, these components become bulky for powers greater than 200W, making the flyback power supply less advantageous in these cases.
- Risk of overvoltage during no-load operation: During the demagnetization phase, the energy stored in the transformer can be transmitted to the capacitor, dangerously increasing its voltage. This can cause an overvoltage and potentially destroy the capacitor if the protection is not properly implemented.

### 7.4. FORWARD switching power supply

The circuit is derived from the series chopper. The need to generate a purely AC voltage across the transformer requires the addition of additional components:

- $D_m$ : This component, in combination with  $V_3$ , demagnetizes the transformer after  $T_p$  has been switched on. This ensures optimal operation by preventing the transformer from remaining magnetized after each switching cycle [7, 8, 11, 12].
- $D_{TR}$ : This diode is used to isolate the output stage, consisting of the freewheeling diode and the filter, when the negative voltage occurs across the transformer terminals, which is unavoidable during the demagnetization phase. The negative voltage is caused by the interaction between  $D_m$  and  $V_3$ .  $D_{TR}$  thus protects the rest of the circuit from this unwanted voltage, allowing stable and safe operation.

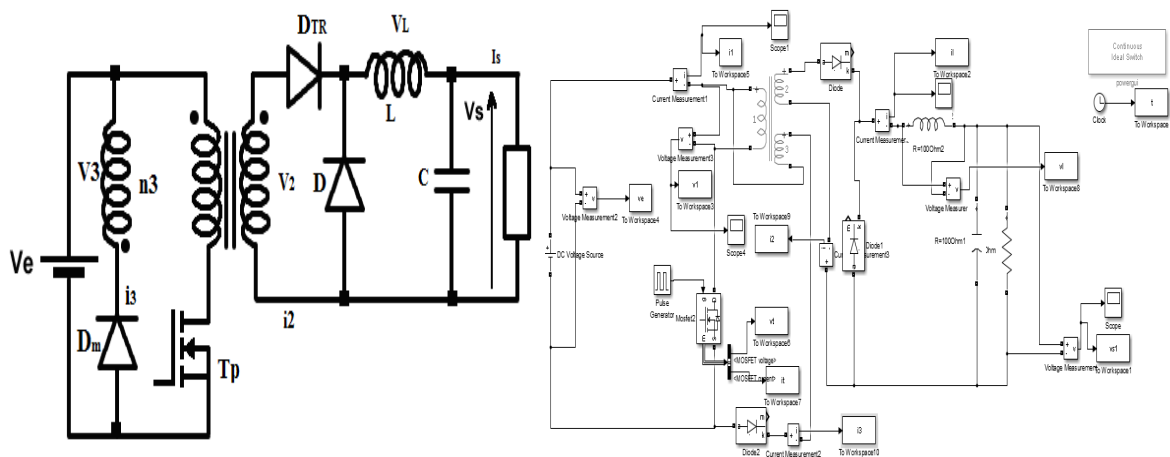


Figure 7.5 FORWARD switching power supply

#### 7.4.1. Continuous mode operating principle [7, 8, 11, 12]

Let , the reluctance of the core (linear magnetic behavior, no saturation).

## Chapter 7: Synthesis of Static Converters

Let , the common flux in the core.

$$-n_2.i_2 + n_1.i_1 + n_3.i_3 = \mathfrak{R}.\phi$$

$$v_1 = n_1 \frac{d\phi}{dt} \text{ et } \frac{1}{\mathfrak{R}} = \frac{L_1}{n_1^2} = \frac{L_2}{n_2^2} = \frac{L_3}{n_3^2}$$

$$m = \frac{n_2}{n_1} \text{ and } m' = \frac{n_3}{n_1}$$

**During the closure of  $T_p$  ( $0 < t < DT$ )**

$$v_1 = V_e \Rightarrow v_2 = \frac{n_2}{n_1}.V_e = mV_e$$

$$v_d = -mV_e$$

$$v_{dm} = -V_e - \frac{n_3}{n_1}.V_e = -(1+m').V_e$$

$$i_3 = 0 \text{ and } i_2 = i_L$$

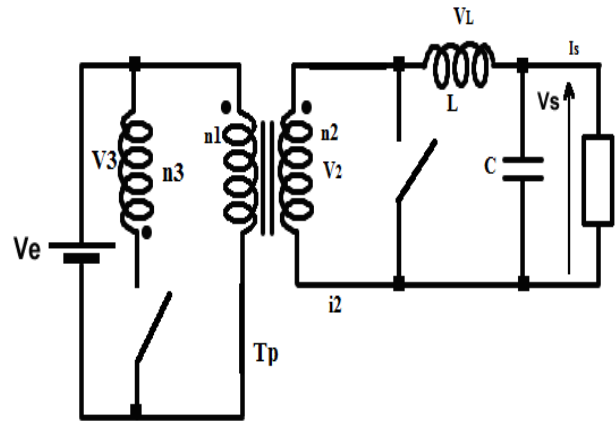
$$-n_2.i_2 + n_1.i_1 = \mathfrak{R}.\phi$$

$$v_1 = n_1 \frac{d\phi}{dt} = V_e \Rightarrow \phi = \frac{V_e}{n_1}.t$$

$$i_1 = \frac{n_2}{n_1}.i_2 + \frac{1}{n_1}.\mathfrak{R}.\phi = \frac{n_2}{n_1}.i_2 + \frac{1}{n_1}.\mathfrak{R}.\frac{V_e}{n_1}.t$$

$$i_1 = \frac{n_2}{n_1}.i_L + \frac{1}{n_1}.\mathfrak{R}.\frac{V_e}{n_1}.t$$

$$\phi_{\max} = \frac{V_e.D.T}{n_1}$$



**Demagnetization phase during opening of  $T_p$  ( $DT < t < 2DT$ )**

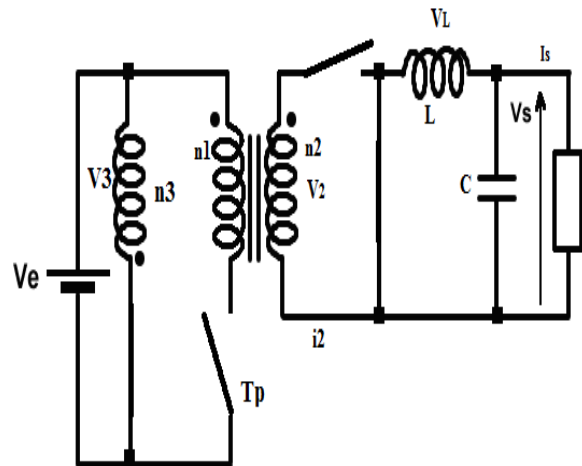
When  $T_p$  opens, the continuity of the ampere turns is ensured by the conduction of the winding  $n_3$  by the diode  $D_m$ .

$$v_3 = -V_e \Rightarrow v_1 = \frac{n_1}{n_3}.V_3 = \frac{-1}{m'}V_e$$

$$v_T = V_e - v_1 = (1 + \frac{1}{m'})V_e$$

$$v_2 = m.v_1 = -\frac{m}{m'}.V_e$$

$$i_1 = i_2 = 0 \text{ et } n_3.i_3 = \mathfrak{R}.\phi$$



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$$v_1 = n_1 \frac{d\phi}{dt} = -\frac{1}{m'} V_e \Rightarrow \phi = \frac{-V_e}{m' n_1} t$$

$$n_3 \cdot i_3 = \mathfrak{R} \cdot \phi = \mathfrak{R} \cdot \phi_{\max} - \frac{\mathfrak{R} \cdot V_e}{n_3} t$$

$$i_3 = \mathfrak{R} \cdot \phi = \frac{\mathfrak{R}}{n_3} \cdot \phi_{\max} - \frac{\mathfrak{R} \cdot V_e}{n_3^2} t = \frac{\mathfrak{R}}{n_3} \cdot \phi_{\max} - \frac{\mathfrak{R} \cdot V_e}{L_3} t$$

### Dead phase during $T_p$ opening ( $2DT < t < T$ )

Only the freewheel diode D is conducting. The transformer is therefore virtually disconnected and the voltages across these windings are zero.

$$v_{L_{avg}} = 0 \Rightarrow (m \cdot V_e - V_s) D \cdot T = V_s (1 - D) \cdot T \\ \Rightarrow V_s = m \cdot D \cdot V_e$$

During  $DT$

$$v_L(t) = V_2 - V_s = m V_e - V_s$$

so

$$i_L(t) = \frac{m \cdot V_e - V_s}{L} t + i_{L_{min}}$$

At  $DT$  we have :

$$i_L(D \cdot T) = \frac{m \cdot V_e - V_s}{L} \cdot D \cdot T + i_{L_{min}} = i_{L_{max}}$$

so

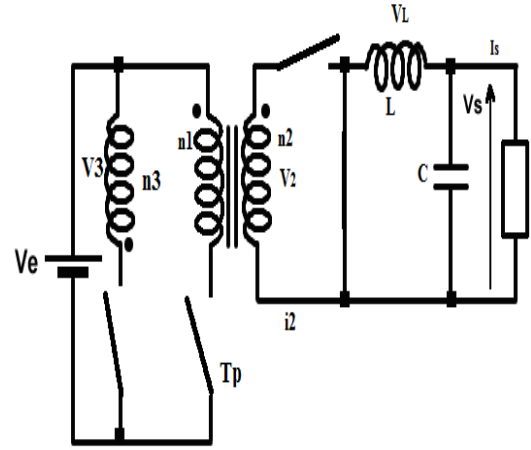
$$\Delta i_L = i_{L_{max}} - i_{L_{min}} = \frac{m \cdot V_e - V_s}{L} \cdot D \cdot T = \frac{m \cdot V_e - m \cdot D \cdot V_e}{L} \cdot D \cdot T \\ = \frac{D(1-D)}{L \cdot f} \cdot m \cdot V_e$$

$$\text{and } \Delta V_s = \frac{D(1-D)}{8 \cdot L \cdot C \cdot f^2} \cdot m \cdot V_e$$

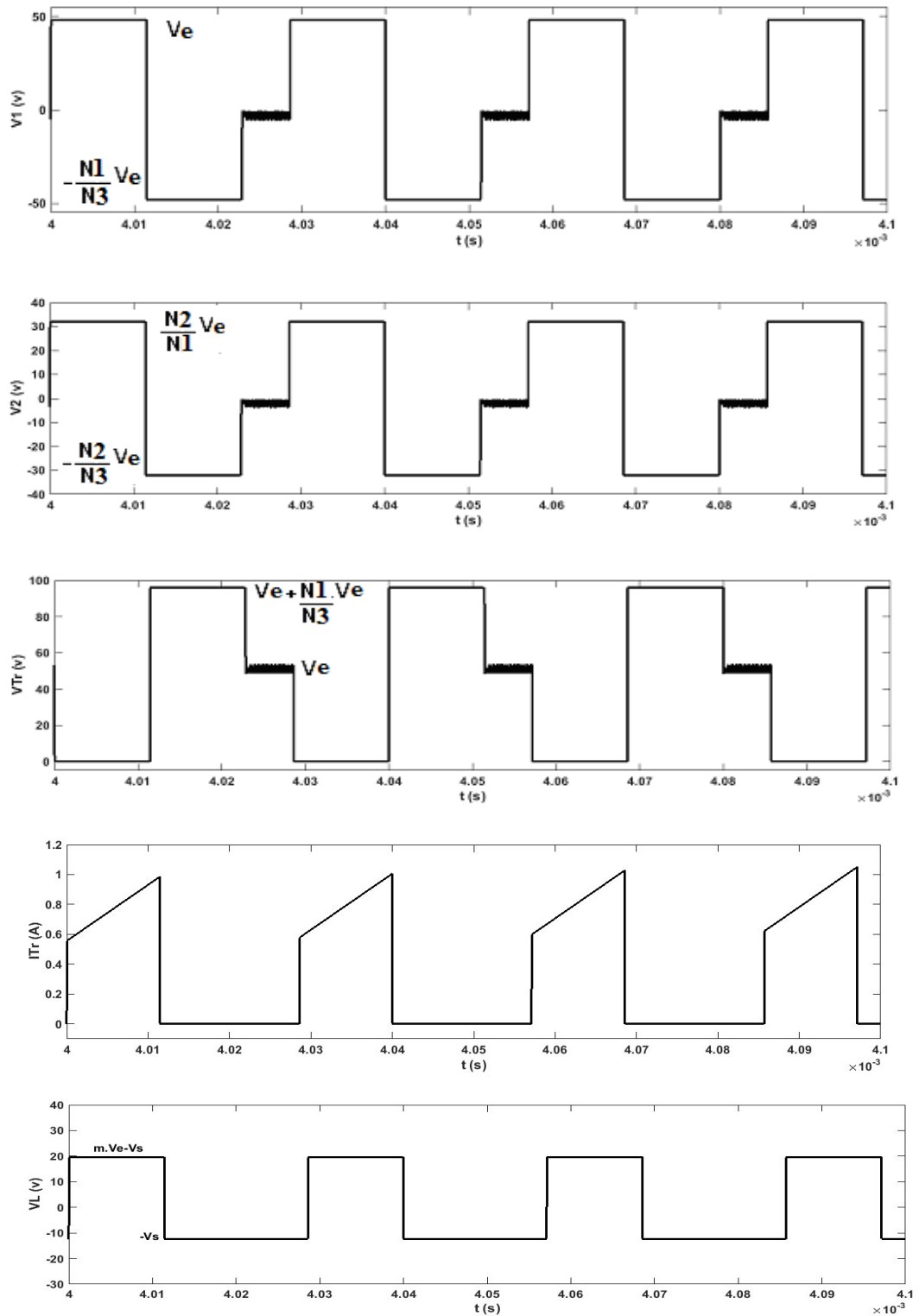
When the converter reaches its steady state, the average voltage across the transformer windings becomes zero. More precisely, regarding the average voltage across the secondary winding:

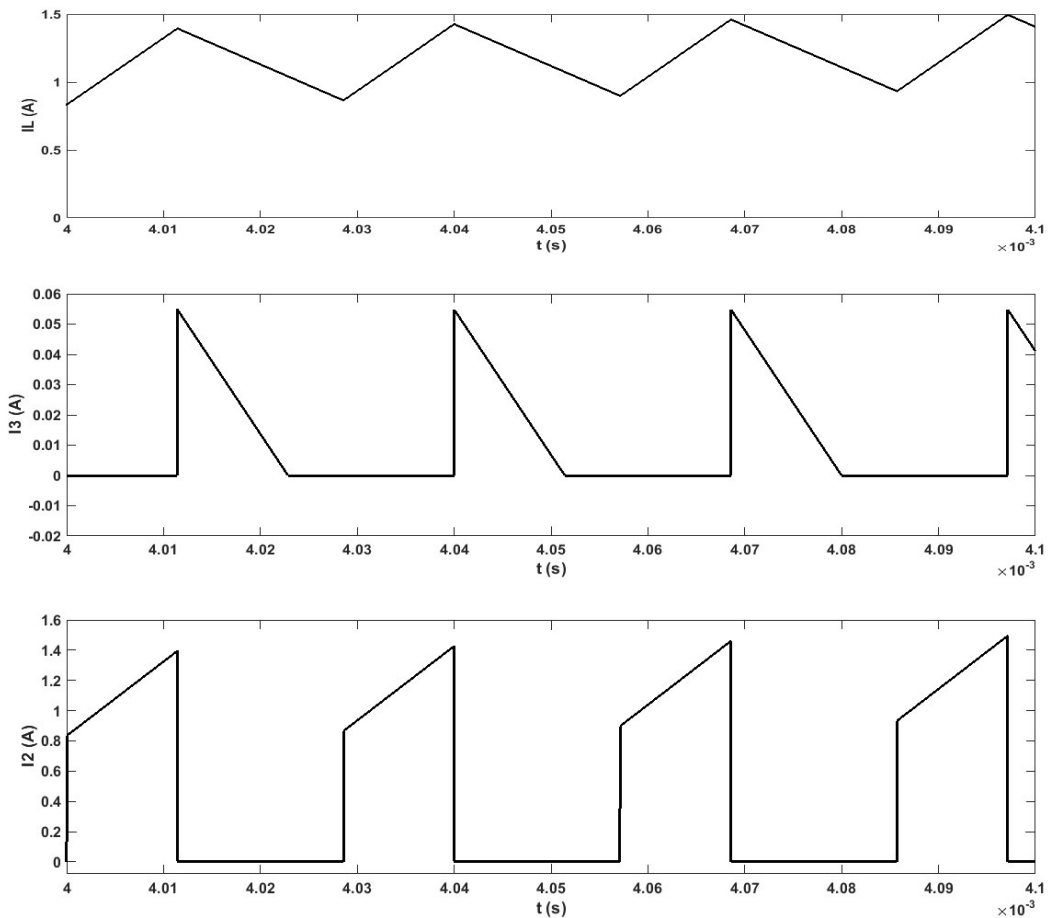
$$V_{2_{moy}} = \frac{1}{T} \int_0^T V_{2_{moy}}(t) dt = \frac{1}{T} \int_0^{DT} \frac{n_2}{n_1} \cdot (V_e - V_s) dt + \frac{1}{T} \int_{DT}^T -V_s dt = \frac{1}{T} \left( \frac{n_2}{n_1} \cdot (V_e - V_s) D \cdot T - (1-D) \cdot T \cdot V_s \right) = 0$$

$$V_s = \frac{n_2}{n_1} \cdot D \cdot V_e = m \cdot D \cdot V_e$$



## Chapter 7: Synthesis of Static Converters





**Figure 7.6** Voltage and current timing diagrams for a FORWARD switching power supply

### 7.4.2. Advantages/Disadvantages [7, 8, 11, 12]

**Advantages:** This architecture is particularly suited to low-voltage, high-current outputs, as the output filtering is relatively simple to implement, making it ideal for power outputs ranging from approximately 100 to 500W.

**Disadvantages:** However, it has certain structural drawbacks:

- Presence of two magnetic components: The architecture requires the use of two magnetic components, which can increase the complexity and cost of the system.
- Need for a transformer demagnetization system: It is essential to provide a transformer demagnetization mechanism, which adds an additional step to the design.
- Improper use of the transformer magnetic circuit: The transformer magnetic circuit is used only in one magnetic quadrant (from 0 to  $B_m$ ), which limits the efficiency of using the magnetic flux, which does not change sign.

7.5. Symmetrical switching power supply, PUSH-PULL [7, 8, 11, 12]

This type of power supply was designed to overcome one of the drawbacks of the forward power supply, namely the use of the transformer in a single magnetic quadrant. To allow the transformer to be used in two quadrants of the magnetic circuit (with  $B > 0$  and  $B < 0$ ), it must be able to magnetize it successively under a positive voltage, then under a negative voltage. Among the various available designs, the one based on the use of a bridge chopper is the simplest to design.

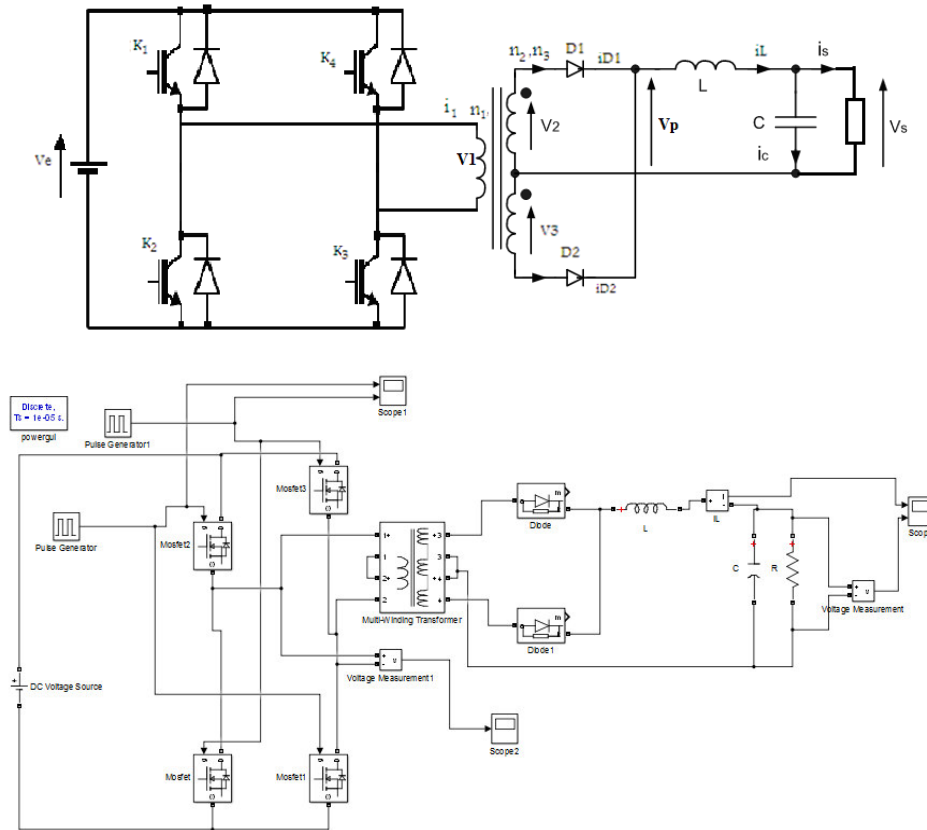


Figure 7.7 Push-pull switching power supply

The switches on the same bridge arm are controlled in a complementary manner, with a conduction time of half a cycle. The phase shift between the controls of the two bridge arms is denoted  $DT$ , with  $D < 1/2$ .

7.5.1. Operational Analysis:

First Interval:  $0 \leq t < DT$

During this conduction phase, switches  $K1$  and  $K3$ , as well as diode  $D1$ , are closed. However, switches  $K2$  and  $K4$ , as well as diode  $D2$ , are open. The equivalent diagram for this phase is shown in figure 7.8.

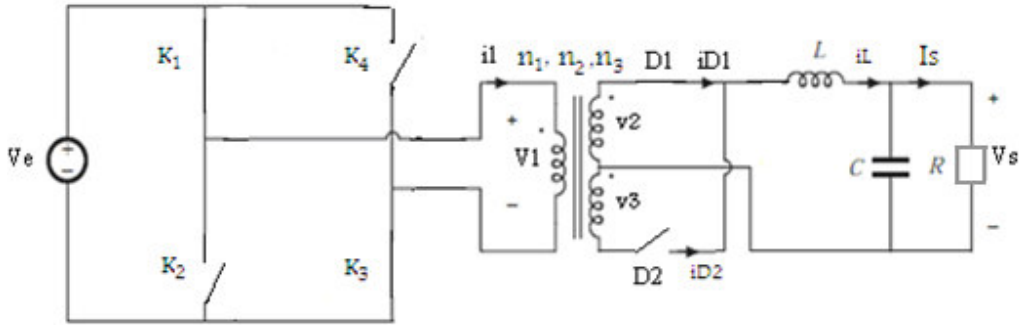


Figure 7.8 Push-pull power supply: switches K1, K3, and diode D1 closed.

The voltages across the switches are:

$$v_{K2} = v_{K4} = V_e$$

The voltage across the transformer primary is:

$$v_1 = V_e = L_1 \frac{di_1}{dt}$$

So the current in the primary of the transformer is:

$$i_1 = \frac{V_e}{L_1} t + I_{1\min}$$

$I_{1\min}$  : is the initial value of current  $i_1$

The voltages across the transformer secondary are:

$$v_2 = v_3 = \frac{v_1 \cdot n_2}{n_1} = \frac{n_2}{n_1} V_e$$

The voltage across diode D2 is:

$$v_{D2} = -v_3 - v_2 = -2 \cdot v_3 = -2 \frac{n_2}{n_1} V_e$$

$V_{D2} < 0$ , so D2 is blocked.

The voltage across the inductance L is:

$$v_L = v_2 - v_s = \frac{n_2}{n_1} V_e - V_s = L \frac{di_L}{dt}$$

So the current  $i_L$  becomes:

$$i_L = \frac{\frac{n_2}{n_1} V_e - V_s}{L} \cdot t + i_L(0)$$

## Chapter 7: Synthesis of Static Converters

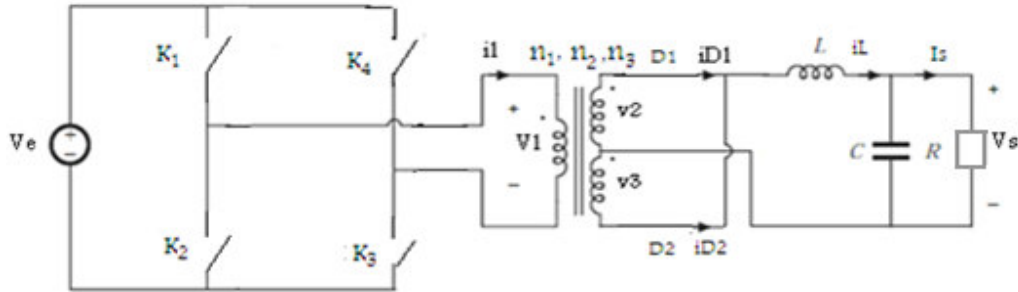
with :  $i_{D1} = i_L = i_2$

And the current through the switches is:

$$i_{K2} = i_{K3} = i_1 = \frac{V_e}{L_1} t + I_{1\min}$$

**Second interval:  $DT \leq t < T/2$  :**

During this phase, all switches are open, allowing the flux to remain constant  $v_2 = v_3 = 0$ . As a result, diodes D1 and D2 become conductive, because the voltage across inductance L is reversed, thus allowing the stored energy to be released. The equivalent diagram of this phase is shown in figure 7.9.



**Figure 7.9** Push-pull power supply: switches open and diodes conducting.

The opening voltages are the same for all switches, therefore:

$$v_{K1} = v_{K2} = v_{K3} = v_{K4} = \frac{V_e}{2}$$

The voltage across the transformer primary is:

$$v_1 = 0$$

so the current  $i_1 = 0$

The voltage across the secondary terminals of the transformer is:

$$v_2 = v_3 = 0$$

The voltage across the inductance L is:

$$v_L = -V_s = L \frac{di_L}{dt}$$

So, the inductance current  $i_L$  is defined as:

$$i_L = \frac{-V_s}{L} \cdot (t - DT) + i_L(DT)$$

## Chapter 7: Synthesis of Static Converters

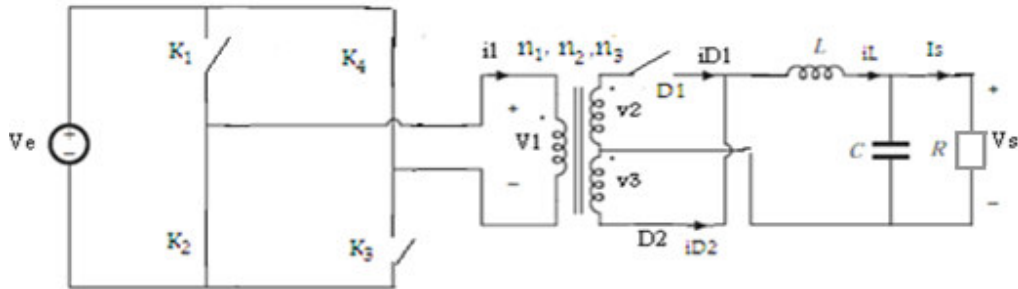
The mid-point rectifier is symmetrical, the inductor current is divided between the two diodes:

$$i_{D1} = i_{D2} = \frac{i_L}{2} = \frac{-V_s}{2L} \cdot (t - D \cdot T) + i_L(DT)$$

**Third interval:  $T/2 \leq t < T/2 + DT$  :**

During this phase, switches K1 and K3, as well as diode D1, are open.

On the other hand, switches K2 and K4, and diode D2, are closed. The equivalent circuit diagram for this phase is shown in figure 7.10:



**Figure 7.10** Push-pull power supply: switches K2, K4, and diode D2 closed.

The voltages across the switches are:

$$v_{K1} = v_{K3} = V_e$$

The voltage across the transformer primary is:

$$v_1 = -V_e = L_1 \frac{di_1}{dt}$$

The voltage across the secondary terminals of the transformer is:

$$v_2 = v_3 = \frac{v_1 \cdot n_2}{n_1} = -\frac{n_2}{n_1} V_e$$

The voltage across diode D1 is:

$$v_{D1} = v_2 - (-v_3) = -2 \frac{n_2}{n_1} V_e$$

The voltage across the inductance L is:

$$v_L = -v_3 - v_s = \frac{n_2}{n_1} V_e - V_s = L \frac{di_L}{dt}$$

The current in the third winding of the transformer, diode D2 and inductor is:

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$$i_3 = -i_{D2} = i_L = \frac{\frac{n_2}{n_1} V_e - V_s}{L} \cdot \left(t - \frac{T}{2}\right) + i_L \left(\frac{T}{2}\right)$$

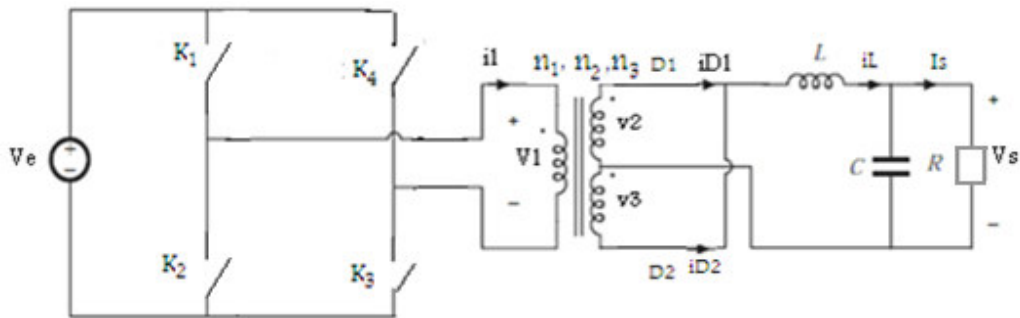
So the current in the primary of the transformer is:

$$i_1 = i_3 \frac{n_2}{n_1} = -i_{D2} \cdot \frac{n_2}{n_1} = \frac{n_2}{n_1} \cdot \frac{\frac{n_2}{n_1} V_e - V_s}{L} \cdot \left(t - \frac{T}{2}\right) + \frac{n_2}{n_1} i_L \left(\frac{T}{2}\right)$$

$$i_1 = \frac{-V_e}{L_1} t - I_{1\min}$$

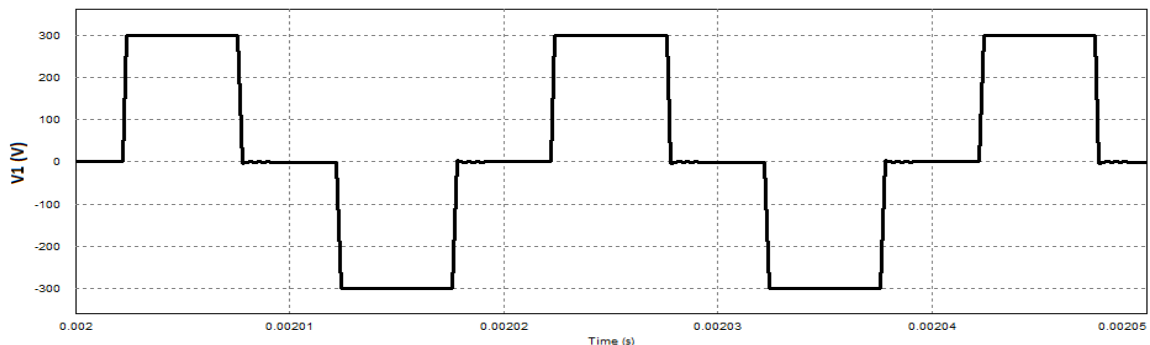
**Fourth interval:  $T/2 + DT \leq t < T$  :**

The electrical diagram of this interval is shown in figure 7.11, where all switches are open and both diodes are closed. In this situation, the analysis of the operation of this phase is identical to that of the interval " $DT \leq t < T/2$ ", that is, the behavior of the circuit remains similar, with dynamics corresponding to the switching conditions specific to this time range.

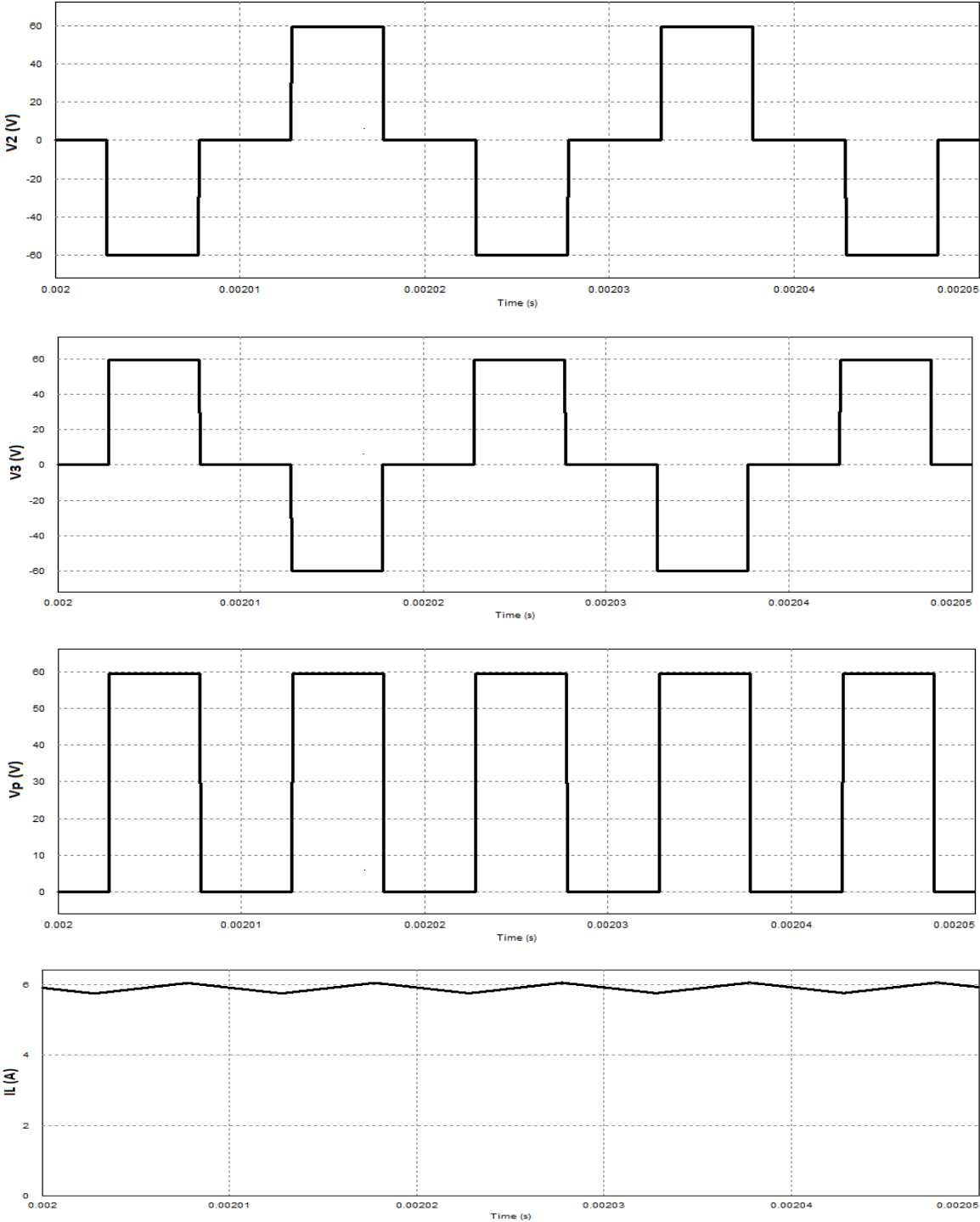


**Figure 7.11** Push-pull power supply: switches open and diodes conducting.

Figure 7.12 shows the voltage and current waveforms for continuous conduction mode (CCM):



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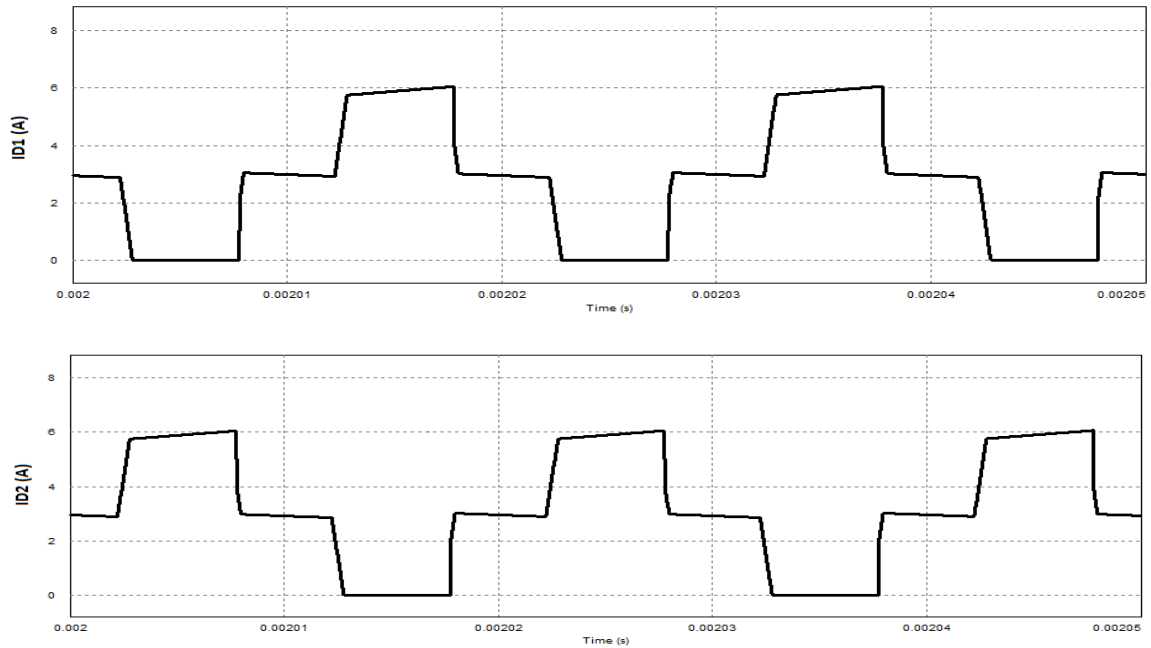


Figure 7.12 Voltage and current timing diagrams of a push-pull switching power supply

By calculating the average value of voltage  $V_p$ , the relationship is easily demonstrated:

$$V_{savg} = \frac{1}{T} \int_0^T V_2(t) dt = \frac{1}{T} \int_0^{DT} \frac{n_2}{n_1} V_e dt + \frac{1}{T} \int_{\frac{T}{2}}^{\frac{T}{2}+DT} -\frac{n_2}{n_1} V_s dt = \frac{1}{T} \left( \frac{n_2}{n_1} V_e D T - \left( \frac{1}{2} - D - \frac{1}{2} \right) T V_e \right)$$

$$V_s = 2 \cdot \frac{n_2}{n_1} \cdot D \cdot V_e$$

$$V_s = 2 \cdot m \cdot D \cdot V_e$$

with  $m = \frac{n_2}{n_1}$

There are also switching power supplies that operate in two magnetic quadrants, but have only two static switches at the primary level. In this case, it is necessary to provide two windings at the primary level to allow the sign of the magnetizing voltage to change. This allows the transformer to be magnetized in both directions of the magnetic flux, thus ensuring efficient operation in both quadrants.

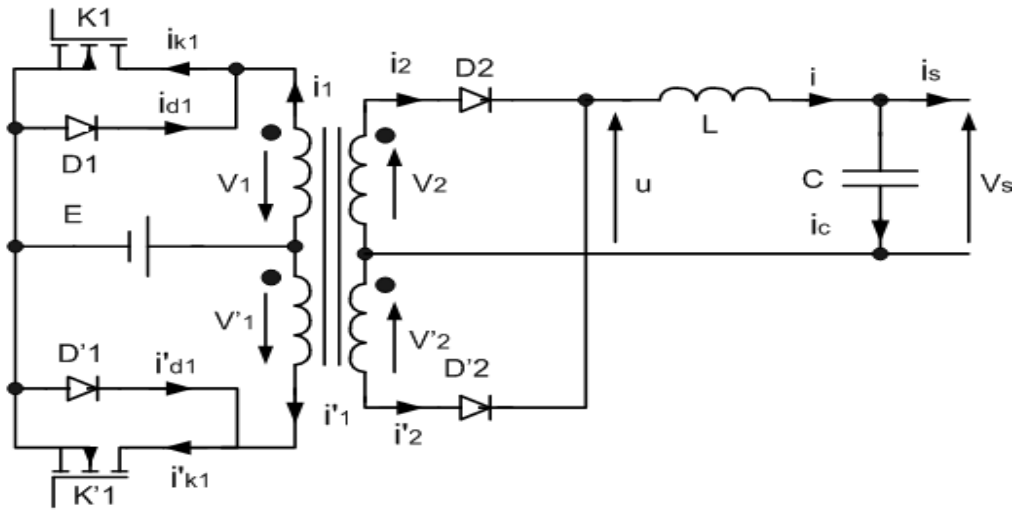


Figure 7.13 Symmetrical PUSH-PULL Switching Power Supply

### 7.6. Introduction to Static Multilevel Converters:

The origins of static multilevel converter structures date back several decades. In the early 1970s, a converter topology called "polygonal" was the first to allow the creation of multilevel waveforms. Since then, several other topologies have been proposed, covering diverse applications such as active filtering, reactive power compensation, uninterruptible power supplies, audio amplifiers, and, of course, actuators for variable speed.

In the 1990s, renewed interest in this static converter structure emerged, both in the research field (with the development of new topologies and control laws, DC voltage source balancing, dynamic performance analysis, etc.) and in industry, where several pilot projects have been implemented [7, 8, 11, 12].

Generally speaking, regardless of the topology, multilevel converter structures offer considerable advantages over conventional solutions based on two-level converters. These advantages are evident from both a technological and functional point of view.

#### 7.6.1. Multi-Level DC Power Conversion Structures

Multi-level DC power conversion structures are converter topologies that use multiple voltage levels to transform a DC input into a DC output, while minimizing losses and distortion. These multi-level converters are particularly suitable for applications where efficient power conversion is required, such as renewable energy systems, industrial power supplies, and energy storage systems [7,8,11,12].

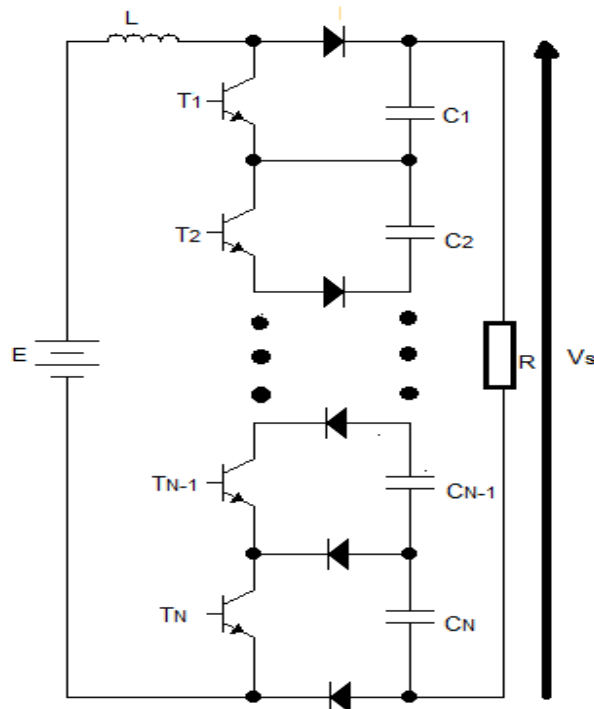
Here is an overview of the main multi-level DC converter structures:

### 7.6.2. Applications of Multi-Level DC Converters

1. Energy Storage Systems: Multi-level DC converters are used in battery and energy storage systems to efficiently manage power flows and optimize energy conversion.
2. Uninterruptible Power Supply (UPS) Systems: In UPS applications, multi-level converters provide stable power with low harmonic distortion, ideal for sensitive equipment.
3. Renewable Energy: Multi-level converters are used in photovoltaic and wind power systems to optimize DC energy conversion and improve overall system efficiency.
4. High-power Motor Drive Systems: Continuously variable multi-level converters are used in industrial motors and variable-speed motor drives, enabling more stable and precise speed and torque control [7, 8, 11, 12].

### 7.6.3. Basic Multilevel Choppers

There are two topologies that represent multilevel boost choppers. (Figures 7.14 and 7.15) represent the basic diagram of this type of converter.



**Figure 7.14** Basic schematic of an N+1-level boost converter

The second structure is represented by figure 7.15 as follows:

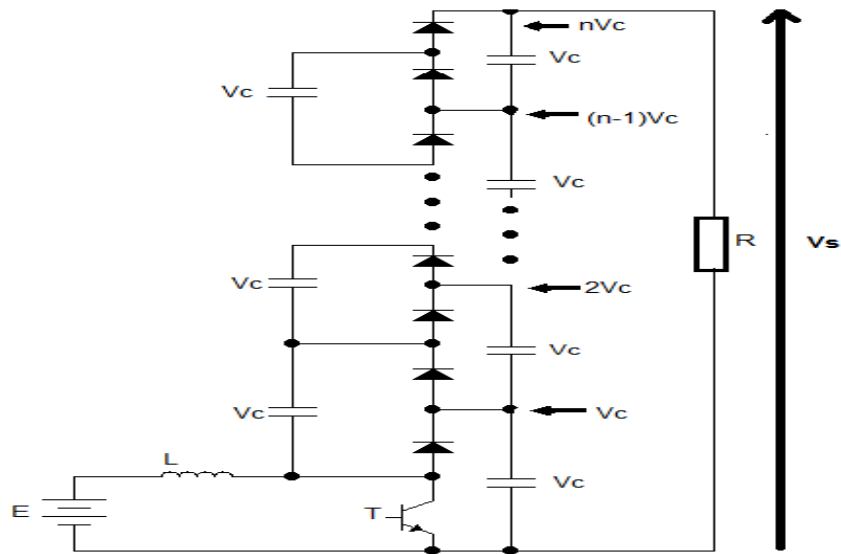


Figure 7.15 Basic schematic of an N+1-level boost converter

The output voltage is given by:

$$V_s = \frac{N}{1-D} \cdot E$$

To avoid the high duty ratio operation of conventional boost converters, a series combination of the converters can be used. The series combination of the conventional boost converter is presented in figure 7.16.

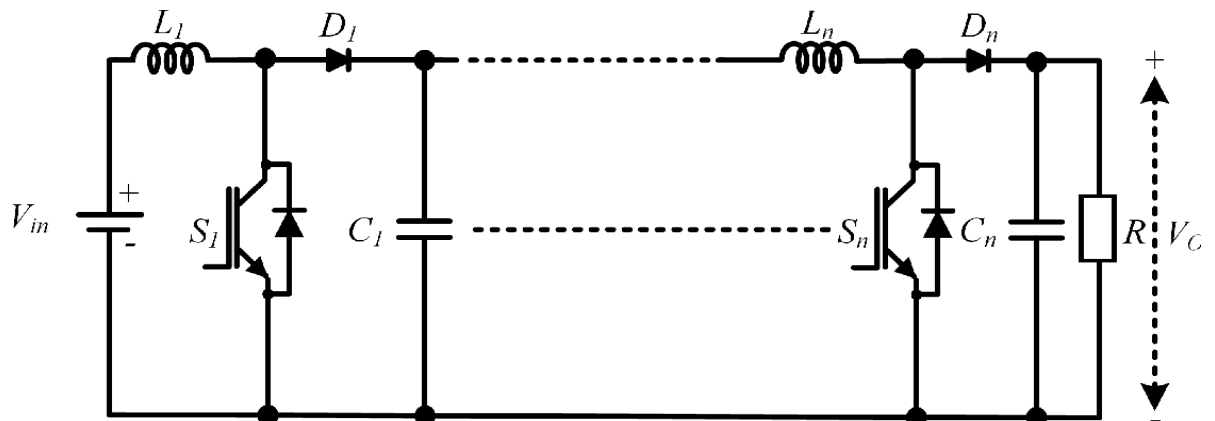


Figure 7.16 Multistage cascaded boost converter

The output voltage is given by:

$$V_c = \frac{1}{(1-D)^n} \cdot V_{in}$$

### 7.6.4. Multi-level power conversion structures in AC

Multi-level power conversion structures refer to converter topologies that use multiple voltage levels to produce an output waveform closer to a sinusoid, thereby improving power quality and optimizing system efficiency. These structures are particularly useful in high-power, low-distortion applications, such as motor drive systems, uninterruptible power supplies (UPS), renewable energy, and power transmission converters.

#### 7.6.4.1. Technological Advantages

Multilevel converters offer several technological advantages over traditional two-level solutions, including:

1. **Reduced harmonics:** By increasing the number of voltage levels, multilevel converters produce output waveforms that are closer to a perfect sine wave. This significantly reduces high-frequency harmonics and decreases distortion in powered systems.
2. **Improved power quality:** By better approximating sinusoidal waveforms, multilevel converters deliver superior power quality, with less electromagnetic noise and less interference to other connected equipment [7, 8, 11, 12].
3. **Reduced switching losses:** Switching occurs at lower voltages in a multilevel converter, which reduces switching losses (conduction and switching losses) compared to traditional converters. This leads to greater energy efficiency, particularly in high-power applications.
4. **Power component optimization:** By distributing voltage across multiple levels, each power component (such as switches and diodes) is subjected to a lower voltage, enabling the use of lower-capacity power components and thus improving system reliability.
5. **Improved thermal management:** Using multiple levels allows the thermal load to be distributed across a greater number of components, reducing the operating temperature of each individual component and improving system lifespan.
6. **Extended modulation range:** Multilevel converters allow for a wider modulation range, providing more flexibility in adjusting the frequency and shape of output voltages.
7. **Reduced component stress:** The voltages applied to the components of the multilevel converter are lower, reducing the electrical and thermal stress on these components, thus increasing their lifespan. These advantages make multilevel converters a preferred solution for high power applications, such as motor drives, uninterruptible power supply (UPS) systems, and reactive power compensation.

### 7.6.4.2. Functional Advantages [7, 8, 11, 12]

The functional advantages of multilevel converters are numerous and varied, particularly in terms of performance, flexibility, and compatibility with different applications. The main advantages are:

1. **Better output voltage quality:** Multilevel converters generate voltage waveforms that are much closer to a perfect sine wave, which improves the quality of power delivered to sensitive loads (such as motors or electronic equipment), thus reducing distortion and noise.
2. **Suitability for high-power systems:** Multilevel converters are particularly suited to high-power applications (typically above 1 kW), where two-level converters would not be as effective. They can handle higher power while maintaining high efficiency and low power loss.
3. **Wide modulation range:** By increasing the number of levels, multilevel converters allow for more flexible and precise voltage modulation, improving the ability to adjust operating conditions to meet application needs.
4. **Reduced switching currents:** Using multiple voltage levels reduces switching currents, which reduces switching losses and enables more efficient power management. This results in better overall system efficiency.
5. **Improved speed control in motor applications:** In motor drive systems, multilevel converters allow for finer and more stable control of speed and torque, with less unwanted variation or oscillation. This is particularly beneficial for high-performance motors requiring precise speed variation.
6. **Less stress on components:** Because multilevel converters use multiple switches that are not subjected to as high voltages as a two-level converter, this reduces electrical and thermal stress on components, increasing their reliability and lifespan.
7. **Better dynamic response:** Due to their ability to generate waveforms closer to a sinusoid, multilevel converters can offer a better dynamic response to load variations or fluctuations in power demand, thus ensuring more precise and stable regulation.
8. **Reduced filtering requirements:** Thanks to the better quality of the waveform produced, multilevel converters reduce the need for noise suppression and harmonic reduction filters, simplifying system design and improving long-term profitability.
9. **Adaptability to multiple applications:** Multilevel converters can be used in a wide variety of applications, such as uninterruptible power supplies (UPS), renewable energy systems (such as solar inverters), high-

voltage power transmission systems, as well as reactive power compensation and active filtration devices.

### 7.6.4.3. Main Multilevel Converter Topologies

#### 1. Flying Capacitor Converter

This converter uses flying capacitors to create multiple voltage levels. These capacitors are charged and discharged sequentially through switches, generating an output voltage with multiple levels [7, 8, 11, 12].

This type of converter offers good voltage regulation and is capable of handling higher voltages while maintaining good output waveform quality.

#### 2. Diode-Clamped Multilevel Converter

This converter uses diodes to limit the voltage levels in the circuit, allowing voltages to be regulated to specific values. It is also known as a diode-clamped converter.

This is one of the most popular topologies, offering good performance, high stability, and low distortion. However, it requires a large number of switching components [7, 8, 11, 12].

#### 3. H-Bridge Multilevel Converter

This structure uses multiple switches and diodes to create multiple voltage levels. It is one of the simplest and most common topologies for multilevel applications.

The output voltage is divided into multiple levels using H-bridge switches for each level. This reduces harmonics and produces a waveform closer to a sinusoid.

#### 4. Cascading H-Bridge Converter

This structure uses multiple H-bridges connected in series, each with its own voltage source. By combining the voltage levels produced by each bridge, an output with a large number of voltage levels can be obtained.

One of the main advantages of this structure is its simplicity of implementation, with reduced power components at each stage. Furthermore, it allows precise output control while avoiding excessively high voltages in the switches [7, 8, 11, 12].

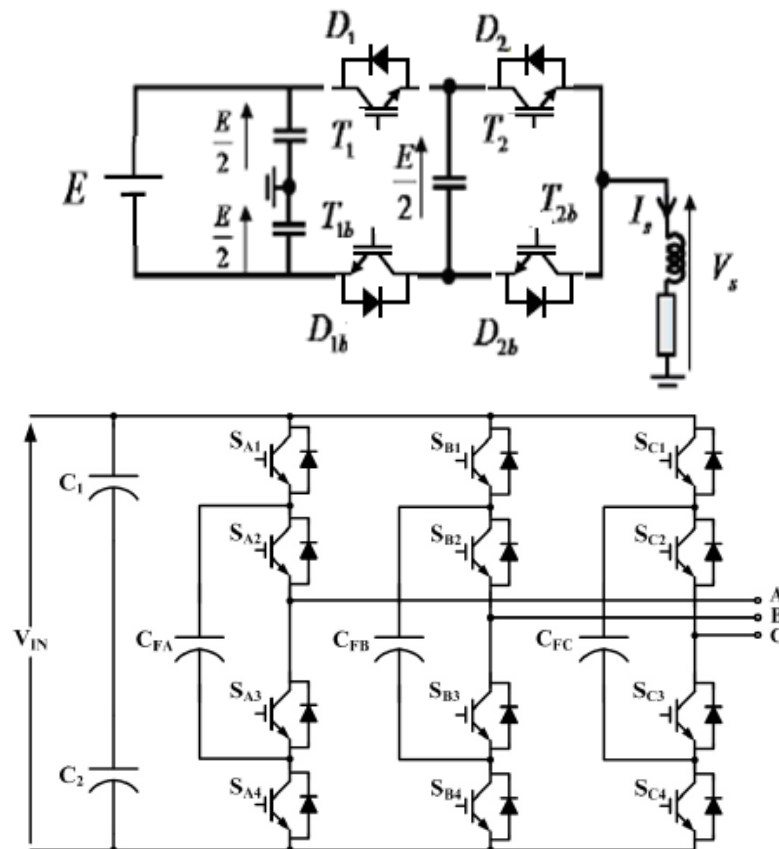
#### 7.6.4.3.a. Floating Capacitor Multilevel Inverter (FCMLI).

The floating capacitor converter, also known as a multi-cell converter, is a power conversion topology based on the series connection of controlled switches. This architecture, which appeared in the early 1990s, is the result of a patent filed by Meynard and Foch. It consists of cells with loop capacitors connected together.

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One of the main advantages of this topology lies in the absence of loop diodes, a characteristic of NPC-type inverters. Furthermore, the voltage constraints supported by the power components are naturally limited, with a particularly low  $dv/dt$  value across the switches [7, 8, 11, 12].

The presence of redundancies in the switching sequences allows the introduction of additional states, which can be used to balance the capacitor charge. Furthermore, only one DC source is required per phase, simplifying the system's power supply. Finally, FC inverters are capable of producing both even and odd number of voltage levels, as shown in figure 7.17.



## Chapter 7: Synthesis of Static Converters

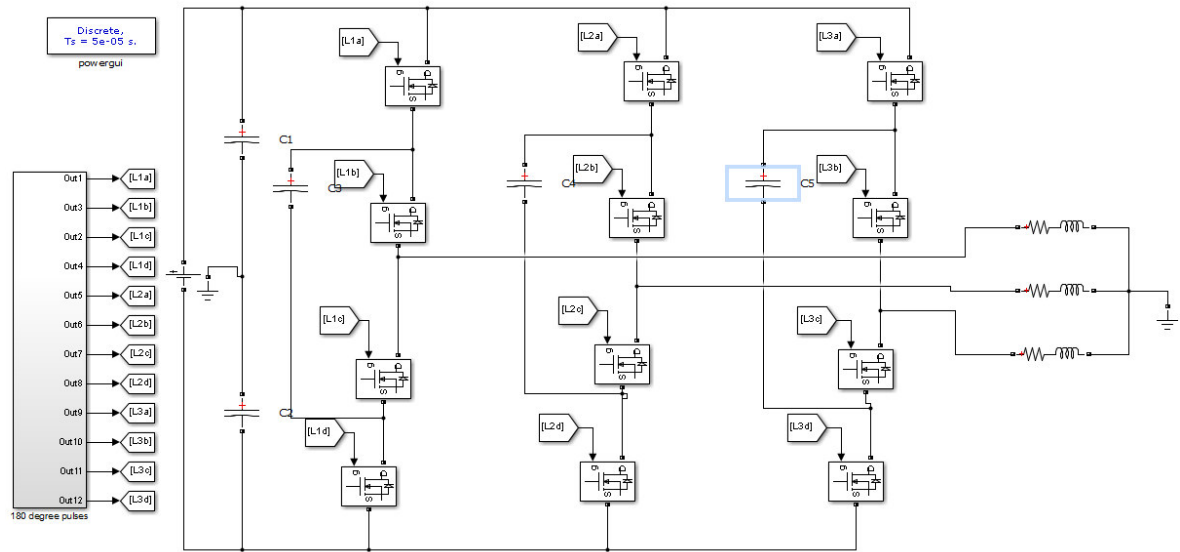
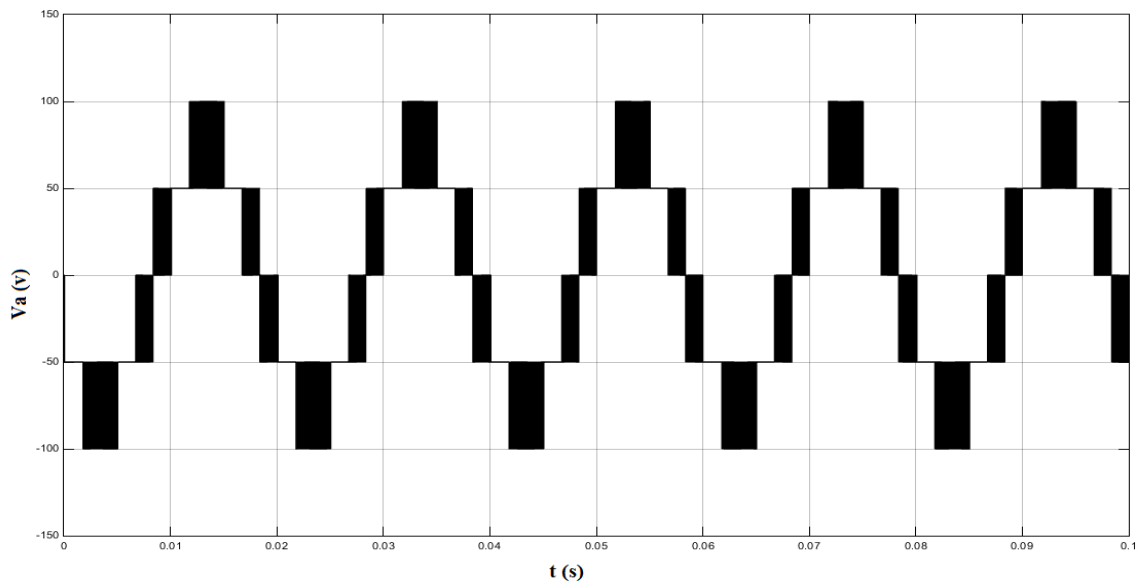
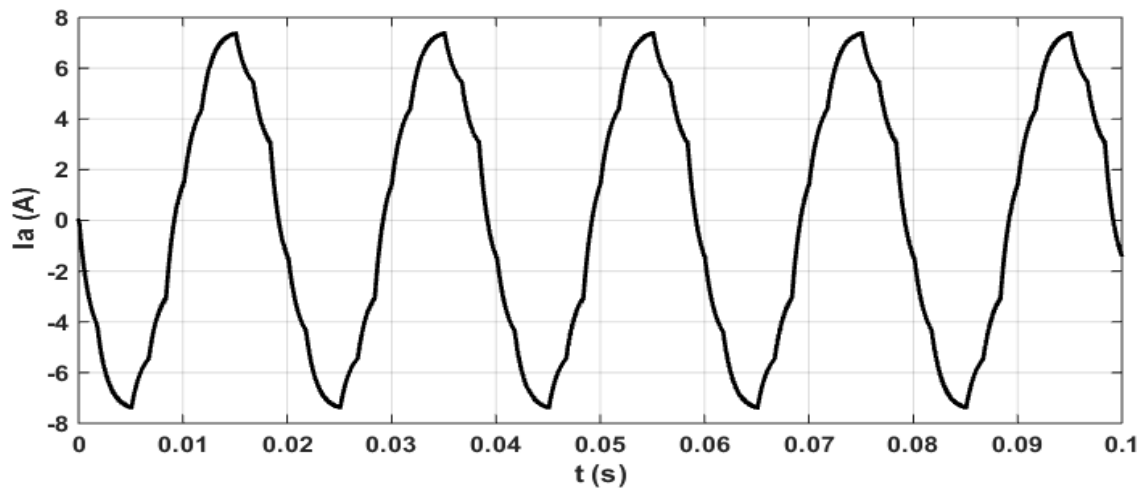


Figure 7.17 Three-Level FC Inverter

This operation is illustrated by the timing diagrams in figure 7.18.





**Figure 7.18** Voltage and current timing diagrams

The FC topology has several disadvantages, such as:

The capacitor charge controller increases the complexity of controlling the entire circuit;

It requires capacitors connected in parallel, which can cause high currents to flow through them;

There is a potential for parasitic resonance between the decoupled capacitors.

Despite its many advantages, the FC topology also has some disadvantages:

- Capacitor charge control complicates the control strategy for the entire converter.
- Using capacitors connected in parallel can cause high currents to flow through these components.
- There is a risk of parasitic resonance between the decoupled capacitors, which could affect system stability.

### 7.6.4.3.b. Floating Diode Multilevel Inverters (DCMI)

Floating diode inverters, also known as Diode Clamped Multilevel Inverters (DCMI), represent one of the classic multilevel inverter topologies. Their structure is based on a modular architecture where the DC input voltage is distributed among several levels using clamping diodes [7, 8, 11, 12].

#### ► Operating Principle

DCMI uses multiple voltage stages obtained by dividing the DC source using capacitors.

Clamping diodes are inserted in the switching paths to fix the voltage levels and evenly distribute

## Chapter 7: Synthesis of Static Converters

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the loads between the switches (IGBTs, MOSFETs, etc.). Each voltage level is obtained by combining several possible switching states.

### ► Advantages and Disadvantages

- Reduced harmonics: improved output voltage quality.
- Less stress on switches: the voltage applied to each component is a fraction of the total voltage.
- Structure well-suited to high-voltage applications (HVDC, industrial motor drives, etc.).

### Disadvantages

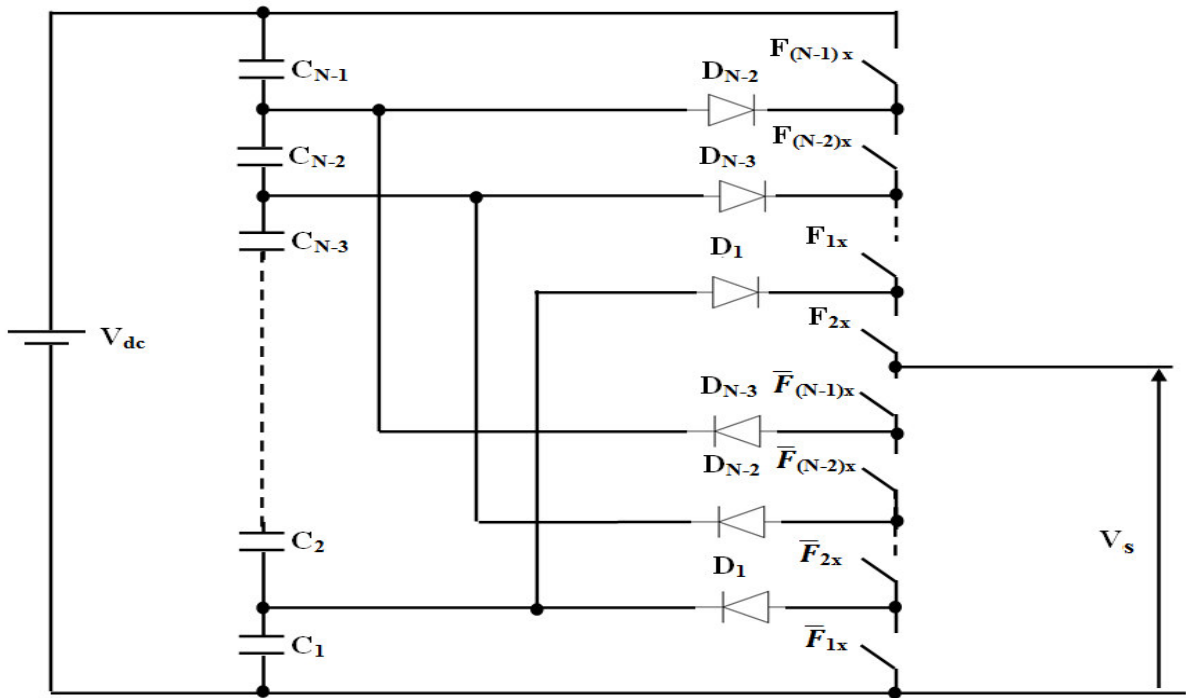
- Large number of passive components (clamping diodes), especially for a high number of levels, which increases complexity and volume.
- Complex balancing of voltages across capacitors, requiring sophisticated control techniques.
- Less modularity compared to other topologies such as cell-in-series inverters (CHB – Cascaded H-Bridge).

### ► Key Features

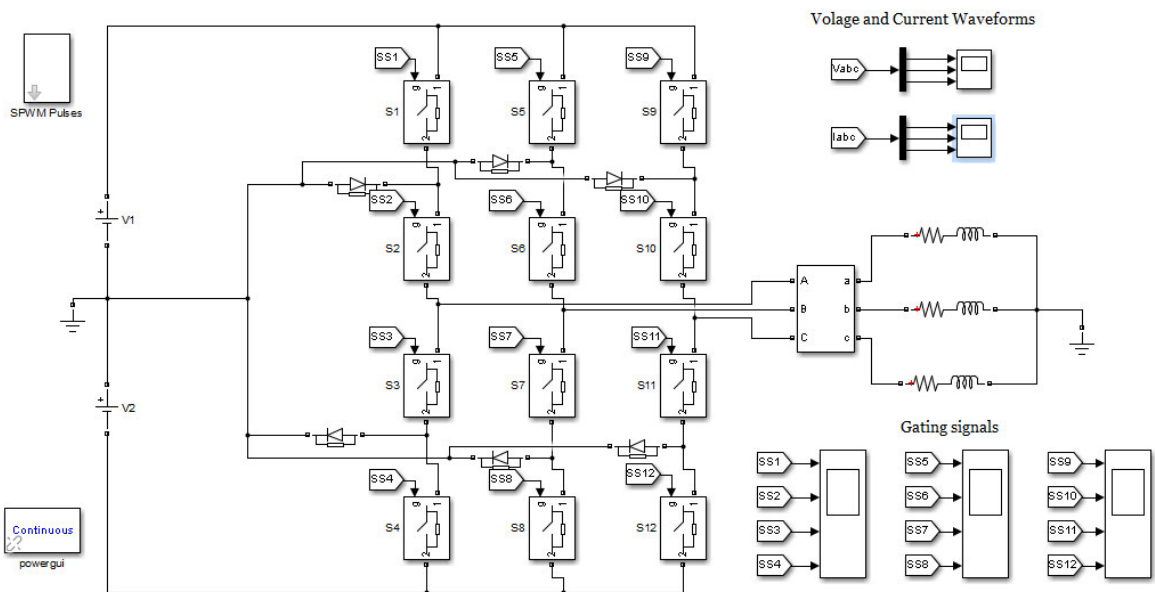
- The topology generates a stepped output voltage, with a waveform closer to a sine wave.
- Clamping diodes limit the voltages applied to each switch, thus reducing the electrical stresses on the semiconductors.
- The higher the number of levels, the smoother the output voltage, improving signal quality and reducing harmonics.

### ► Typical Applications

- Medium- and high-voltage motor drives
- HVDC systems
- Renewable energy production (including large-scale solar photovoltaic)
- Smart grids and microgrids



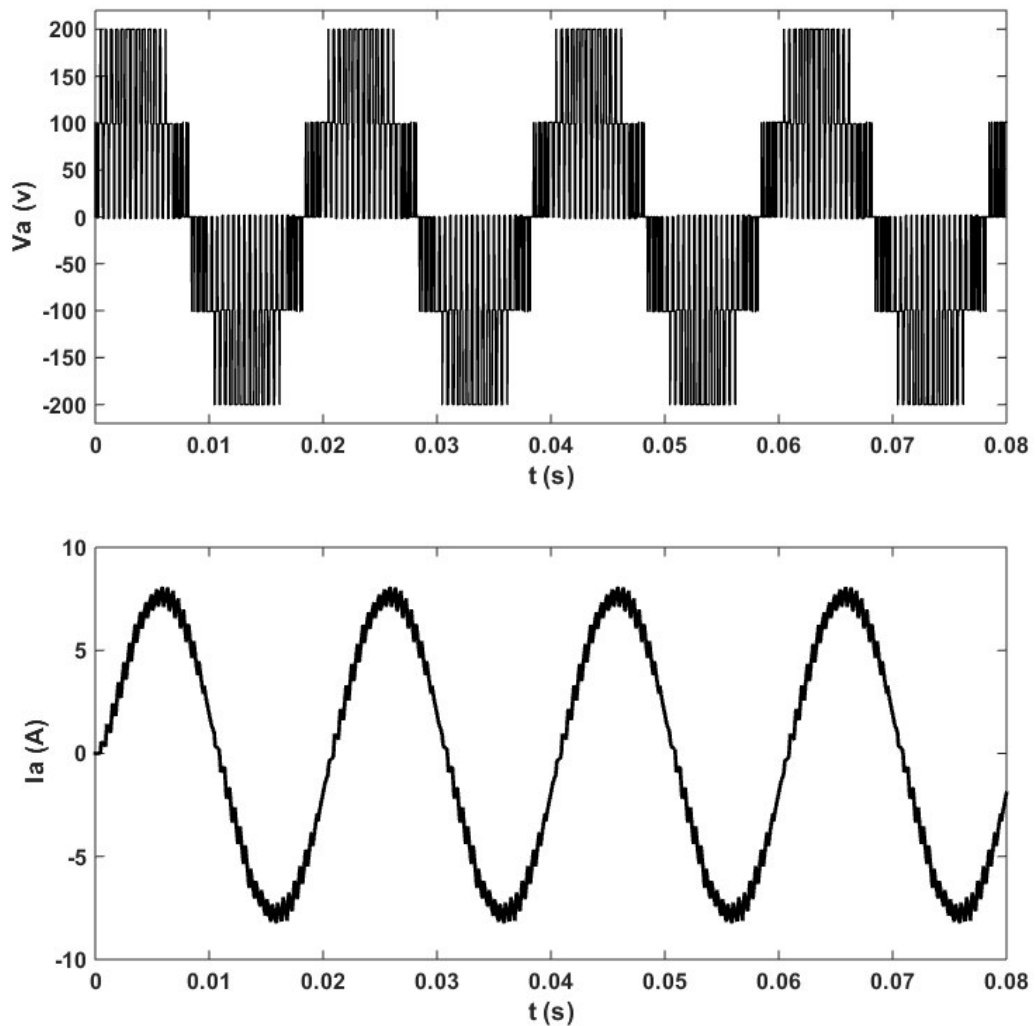
a



b

Figure 7.19 a) NPC inverter b) Three level NPC inverter simulink

This operation is illustrated by the timing diagrams in figure 7.20.



**Figure 7.20** Voltage and current timing diagrams

### 7.6.4.3.c. H-Bridge Multilevel Inverters

The first inverter model developed was the H-bridge, introduced in 1975. The major evolution of multilevel inverters was marked by the emergence of the series-connected H-bridge (CHB) topology. The first application of this structure was in 1988, in the context of plasma stabilization [7,8,11,12].

In this configuration, the outputs of the various H-bridges are connected in series, so that the resulting output voltage is the sum of the voltages delivered by each cell. One of the key advantages of this approach is the ability to increase the number of output voltage levels without adding new active components in each cell.

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The use of series-connected converter cells therefore makes it possible to increase both the output voltage and the overall power of the converter. However, this topology has a major drawback: it requires a large number of isolated DC sources, one for each H-bridge, which can complicate practical implementation, particularly in terms of power supply and isolation.

### ► Hybrid Multilevel Inverters

Three-phase hybrid multilevel inverters are designed for specific applications by connecting basic inverters in series or parallel. These hybrid models are particularly suitable for driving high-power medium-voltage synchronous and asynchronous motors. The combination of different topologies is achieved using methods derived from graph theory, allowing for the creation of optimized configurations [7, 8, 11, 12].

Hybrid topologies improve power quality and increase the number of voltage levels, while reducing the number of DC voltage sources required at the input and limiting the number of switching operations. This offers a compromise between performance, cost, and efficiency.

In industry, three types of hybridization are commonly developed:

1. Bridge hybridization, generally between inverters of the same type (see Fig. 7.21).

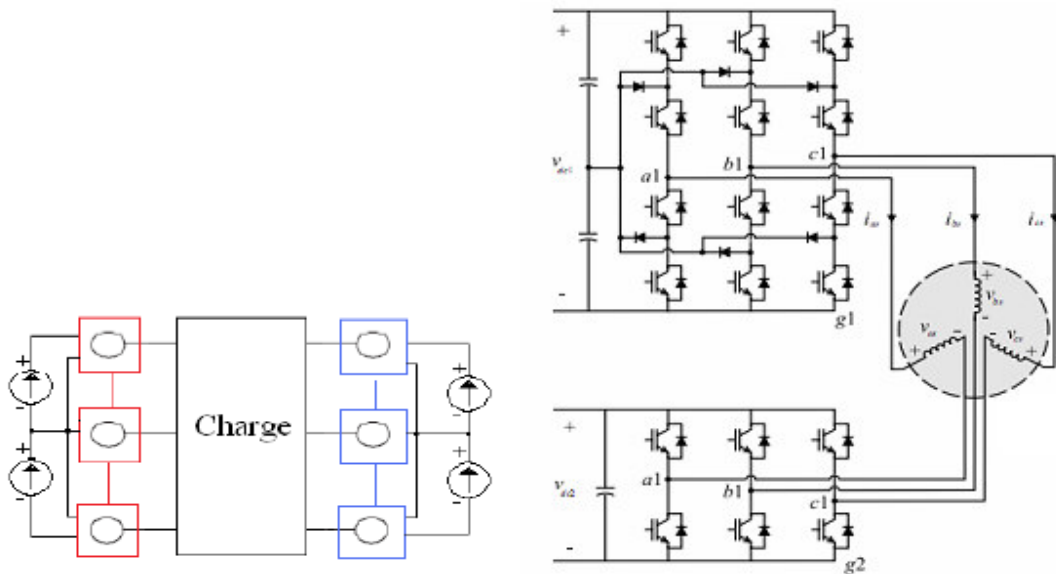


Figure 7.21 Bridge hybridization of 2 multilevel inverters.

► **Cascaded Hybridization:** This approach allows topologies to provide a multiplier effect on the number of voltage levels, depending on the structuring model.

### ► Cascade Hybridization in Multilevel Inverters

Cascade hybridization refers to the connection of multiple inverters of different types (or even the same type) in series, thus multiplying the number of voltage levels. This type of configuration is often used to improve output voltage quality while reducing component stress and switching complexity [7, 8, 11, 12].

### ► Cascade Hybridization Principle

- **Series Inverters:** In this configuration, each inverter is responsible for a portion of the total voltage. The inverter outputs are connected in series so that the overall output voltage is the sum of the voltages of each individual inverter.
- **Increased Voltage Levels:** By adding multiple inverters, a higher number of voltage levels can be achieved. For example, a cascaded hybrid system with two 5-level inverters can provide 10 output voltage levels.
- **Fewer DC voltage sources:** Unlike other configurations, cascaded hybridization allows for the use of fewer DC voltage sources while increasing the number of output voltage levels.

### ► Advantages of Cascade Hybridization

1. **Improved voltage quality:** Increasing voltage levels better approximates a sine wave, reducing harmonics and improving power quality.
2. **Reduced stress on switches:** Less voltage is applied to each switch, allowing them to be used more efficiently and reducing switching losses.
3. **Cost and resource optimization:** Using common voltage sources across multiple inverters reduces the number of independent DC sources, which is beneficial in terms of cost and space.

### ► Applications of Cascade Hybridization

- **Industrial motor drives:** Cascade inverters are widely used in applications where high power quality is required, such as driving high-power motors.
- **HVDC (High Voltage Direct Current) systems:** Cascade hybridization allows for high power handling while improving the reliability and quality of energy conversion.
- **Renewable energies:** In large-scale photovoltaic or wind power systems, the use of hybrid inverters maximizes efficiency and reduces installation costs.

► Schematic example of cascaded hybridization

• Fig. 7.22 (Illustration of a hybrid system): Consider a five-level hybrid system consisting of two two-level inverters. Each inverter generates a portion of the voltage, but their series summation produces an output with five levels, thus increasing signal quality [7, 8, 11, 12].

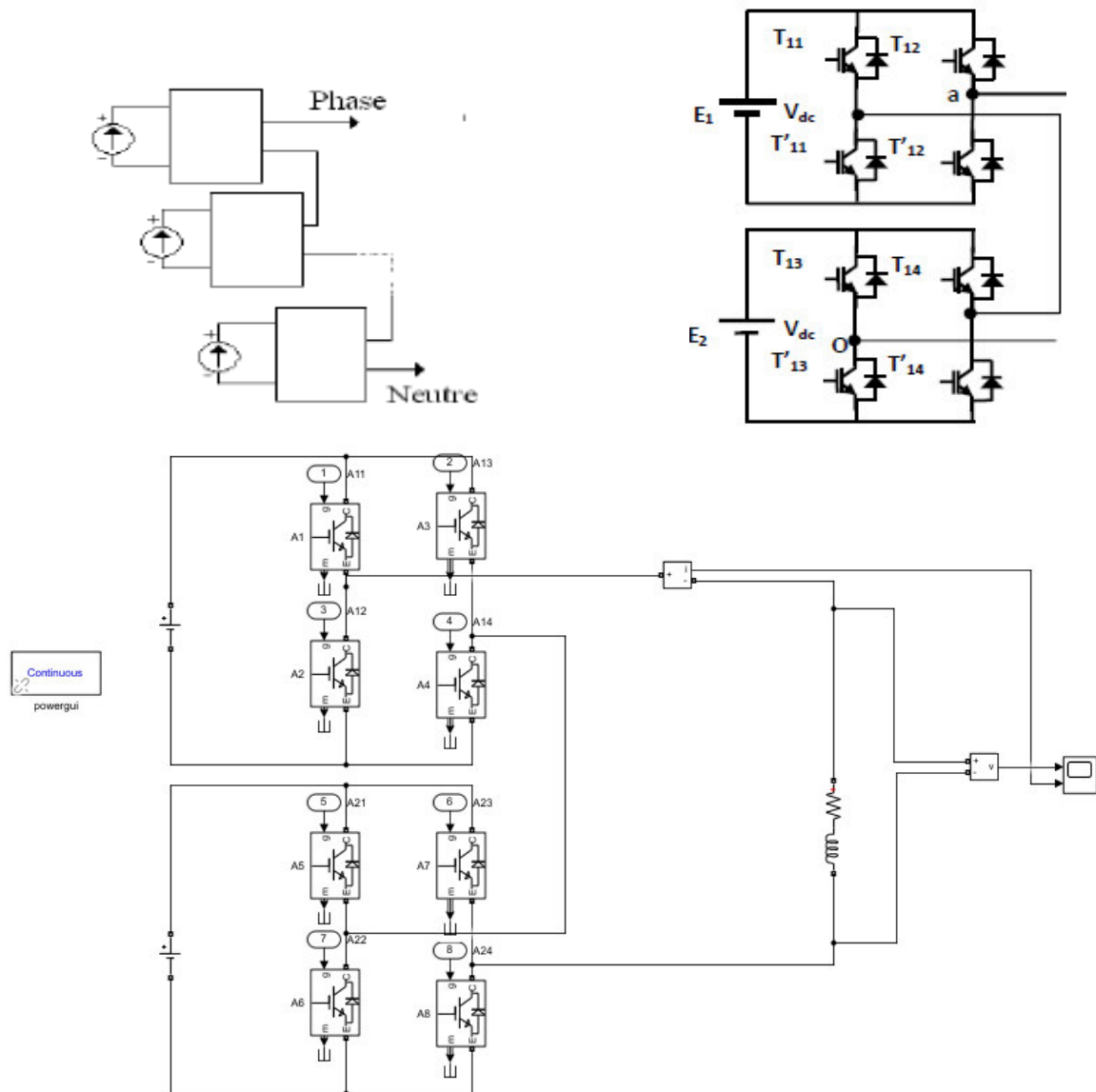
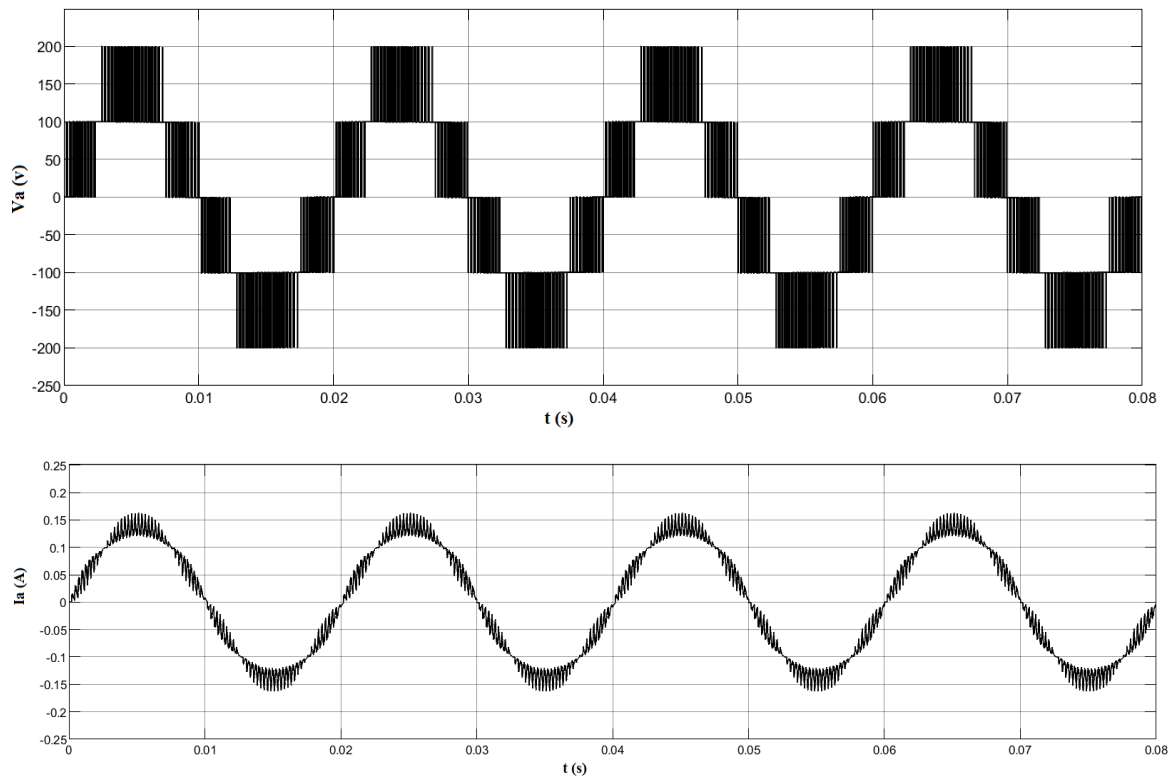


Figure 7.22 Cascaded hybridization of multilevel inverters.

This operation is illustrated by the timing diagrams in figure 7.23.



**Figure 7.23** Voltage and current timing diagrams

### ► Side-by-side hybridization to power two loads

Side-by-side hybridization is another method of combining inverters to meet the specific power needs of separate loads. Unlike cascade hybridization, which is used to increase the number of series voltage levels, side-by-side hybridization involves paralleling multiple inverters to power two separate loads while optimizing the use of voltage sources [7, 8, 11, 12].

### ► Side-by-side hybridization principle

- **Parallel inverter arrangement:** In this configuration, two or more inverters are connected in parallel, each powering a different load, but using a shared voltage source or an independent voltage source for each inverter [7, 8, 11, 12].
- **Independent power supply:** Each inverter in this topology provides the voltage required for a specific load, allowing for better control of the power supply to each device while distributing the total power required across the entire system.

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- **Performance Optimization:** Side-by-side hybridization helps reduce losses and optimize resource utilization by dividing power tasks. This also helps reduce potential disruptions between loads.

### ► Advantages of Side-by-Side Hybridization

1. **Powering Independent Loads:** This approach allows for the simultaneous management of different loads without interference, with the ability to control the power distributed to each load independently.
2. **Reduced Control Complexity:** Compared to cascade hybridization, side-by-side hybridization simplifies control since each inverter operates independently for each load, requiring less complex coordination.
3. **Modularity:** This configuration is easily modular, allowing new inverters to be added to meet increased power or supply needs without modifying the entire system.
4. **Load Balancing:** Each inverter distributes power to the loads in a balanced manner, contributing to even load distribution.

### ► Disadvantages of Side-by-Side Hybridization

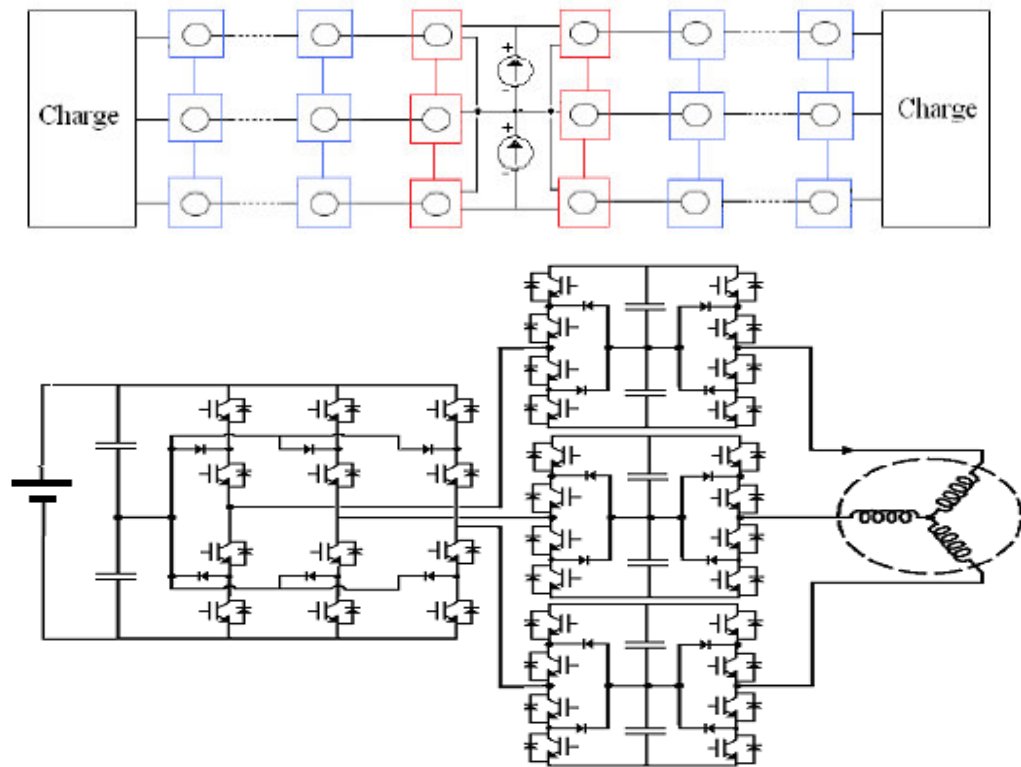
1. **Requires more components:** While side-by-side hybridization offers some modularity, it can increase the number of voltage sources and components required to manage power for each load.
2. **Shared voltage control:** If the inverters share a common DC source, voltage management becomes more complex, especially in the event of fluctuations or irregularities in power demand.

### ► Side-by-Side Hybridization Applications

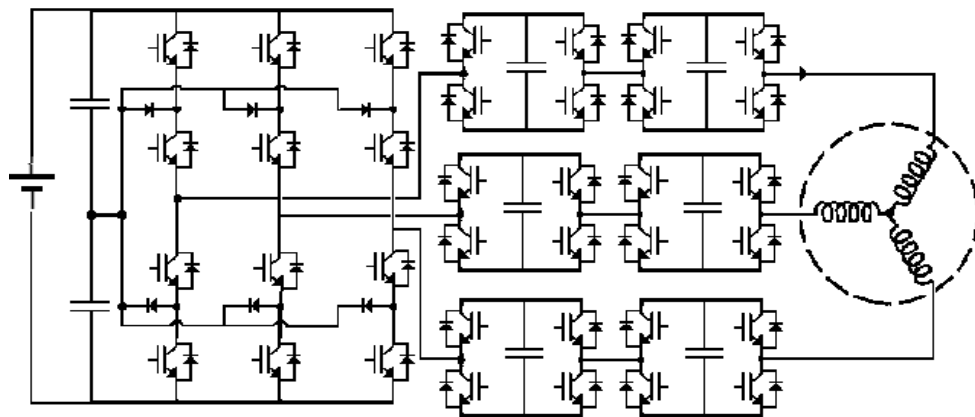
- **Industrial Power Systems:** This topology is particularly useful for applications where multiple loads must be powered independently, such as in industrial facilities with diverse equipment.
- **Power Distribution Applications:** Side-by-Side Hybridization is ideal for systems where multiple circuits or loads must be managed independently while maximizing energy conversion efficiency [7, 8, 11, 12].

### ► Illustration: Side-by-side hybridization

A diagram like figure 7.24.a would illustrate two inverters connected side-by-side, each supplying a separate voltage to different loads. Using two inverters would allow for efficient power distribution, ensuring optimal operation of both loads without interference (Fig. 7.24.b).



a) Side-by-side hybridization with 3-phase 3-level NPC and one 3-level NPC per line



b) Dual side-by-side hybridization with a three-phase, three-level NPC and two H-bridges in series per phase.

**Figure 7.24** Side-by-side hybridization of multilevel inverters.

### 7.6.5. Matrix Converters

Traditional AC–AC conversion typically relies on back-to-back voltage or current source converters, where an intermediate energy storage element—usually a DC-link capacitor or inductor—connects the front-end AC–DC rectifier to the back-end DC–AC inverter. In contrast,

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matrix converters (MCs) perform direct AC–AC conversion without the need for any intermediate energy storage. Conventional matrix converters, referred to as direct matrix converters (DMCs), are single-stage configurations that link an  $m$ -phase input source to an  $n$ -phase output load via an array of  $m \times n$  bidirectional switches. Alternatively, the indirect matrix converter (IMC) architecture separates the voltage and current conversion processes into two distinct stages [7, 8, 11, 12].

### 7.6.5.1. Fundamentals Of Matrix Converters

A matrix converter consists of an array of bidirectional power switches that directly connect each input phase to each output phase. In the case of a three-phase to three-phase matrix converter, the structure comprises nine bidirectional switches arranged in a  $3 \times 3$  configuration. Each bidirectional switch is typically implemented using two semiconductor devices—such as IGBTs or MOSFETs—connected in a back-to-back configuration with anti-parallel diodes to enable current conduction in both directions (as illustrated in figure 7.25).

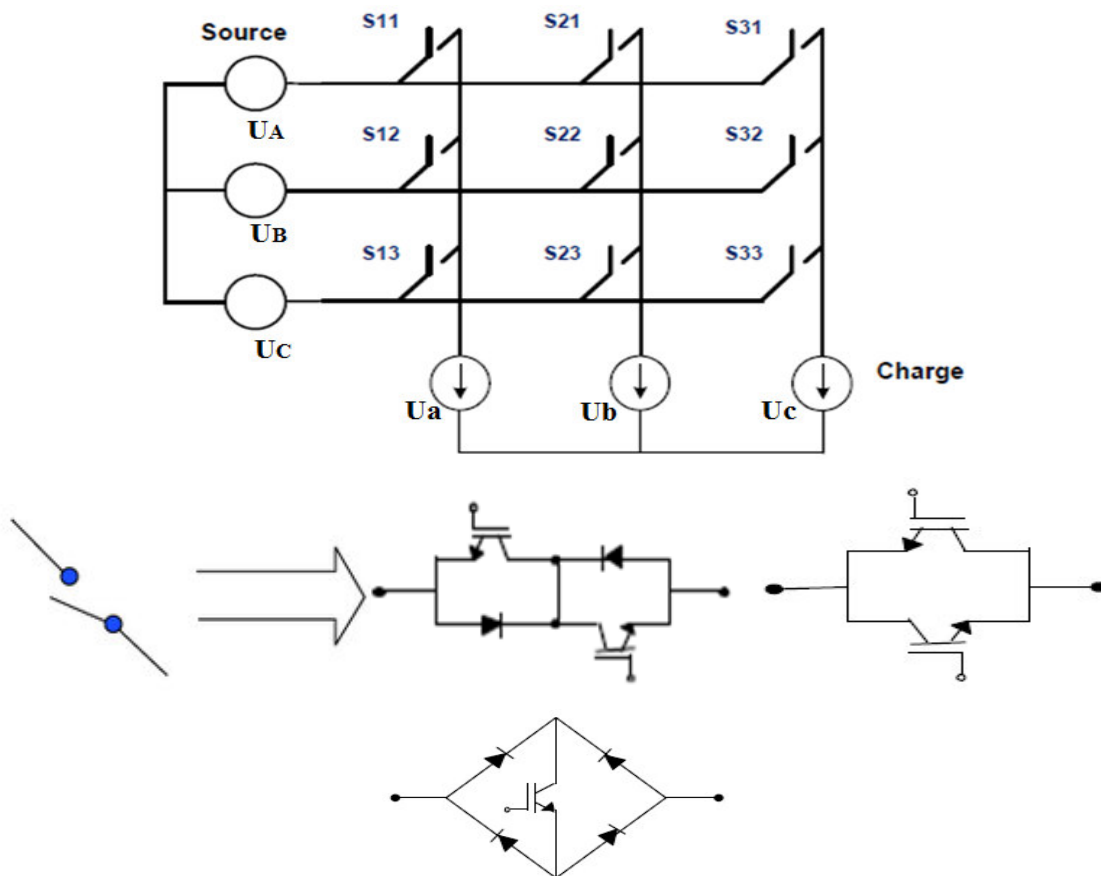


Figure 7.25 Matrix converter topology

### 7.6.5.2. Operation Of Matrix Converters

The matrix converter facilitates power transfer from the input phases to the output phases by selectively controlling the opening and closing of bidirectional switches. By precisely adjusting the switching timing and duty cycles, the converter can synthesize an output voltage waveform with the desired amplitude, frequency, and phase shift. Additionally, the switching strategy directly shapes the input current waveform, maintaining it sinusoidal and in phase with the input voltage, thereby achieving a high input power factor.

### 7.6.5.3. Advantages:

Direct conversion without a DC bus → compactness.

Fully reversible (four quadrants).

Good efficiency and low THD (if well modulated).

Less maintenance (no limited-life capacitors).

### 7.6.5.4. Disadvantages:

Complexity of control and modulation.

Output voltage limited to 86.6% of the input.

Difficulty in practical implementation of bidirectional switches.

Sensitivity to network disturbances (no decoupling via DC bus).

### 7.6.5.5. Operational Analysis

Generally, the relationship between the  $n$  output voltages ( $U_a, U_b, U_c, .. U_n$ ) and the  $M$  input voltages ( $U_A, U_B, U_C, .. U_M$ ) is determined by the states of the  $M \times n$  bidirectional switches ( $S_{i,j}$ ), where  $S_{i,j} = 1 =$  closed,  $S_{i,j} = 0 =$  open, according to:

$$\begin{bmatrix} U_a \\ U_b \\ \cdot \\ \cdot \\ U_n \end{bmatrix} = \begin{bmatrix} S_{11} & \cdot & \cdot & S_{M1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ S_{n1} & \cdot & \cdot & S_{Mn} \end{bmatrix} \begin{bmatrix} U_A \\ U_B \\ \cdot \\ \cdot \\ U_M \end{bmatrix}$$

and

$$\begin{bmatrix} I_A \\ I_B \\ \cdot \\ \cdot \\ I_M \end{bmatrix} = \begin{bmatrix} S_{11} & \cdot & \cdot & S_{M1} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ S_{n1} & \cdot & \cdot & S_{Mn} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ \cdot \\ \cdot \\ I_n \end{bmatrix}$$

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From Kirchhoff's voltage law, the number of switches on in each row must be either one or none, otherwise at least one input supply is shorted, that is ( $i$  refers to the input and  $j$  refers to the output)

$$\sum_{i=1}^n S_{ij} \leq 1 \quad \text{for any } j.$$

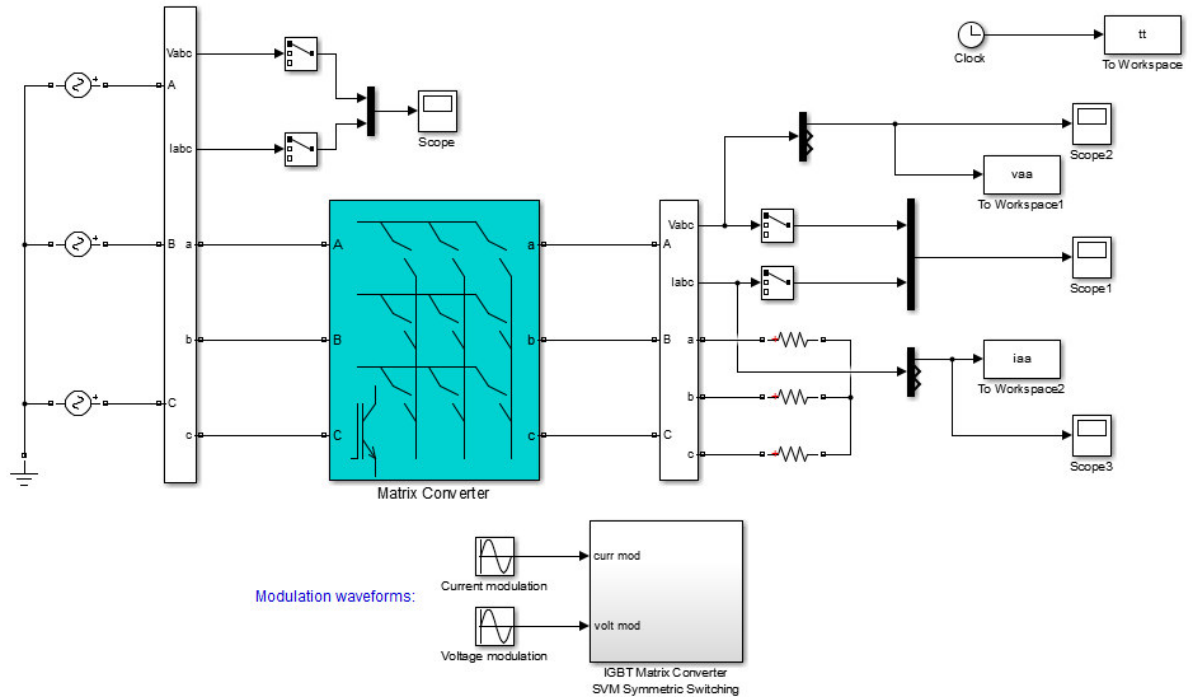
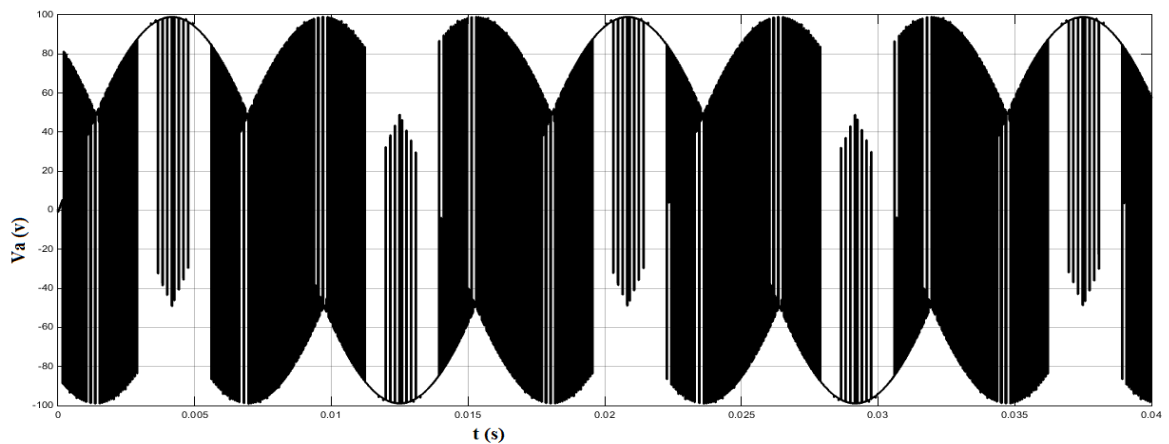
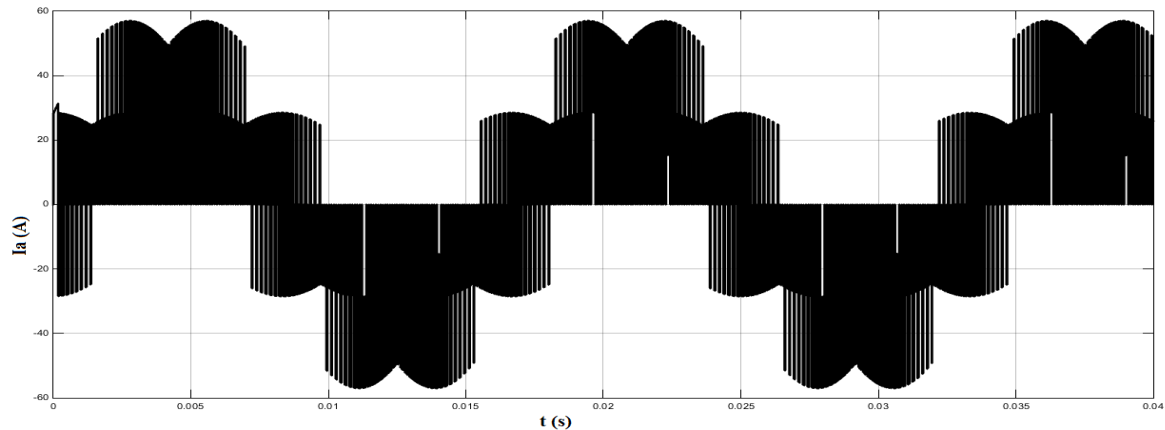


Figure 7.26 Matrix converter topology in matlab

This operation is illustrated by the timing diagrams in figure (7.27).





**Figure 7.27** Voltage and current timing diagrams

### 7.7. Conclusion

Switched-mode power supplies (SMPS) are DC-DC converters designed to supply electronic devices with a DC voltage, usually constant, whose behavior can be compared to that of a resistive load.

It should be noted that the function performed by a switched-mode power supply can be performed by a regulated linear power supply. In this case, a "ballast" transistor absorbs the difference between the supply voltage and the voltage applied to the load terminals. Controlling the transistor makes its behavior comparable to that of a variable resistor, with the corresponding losses. Efficiency is therefore always significantly lower than that of a switching-mode power supply.

Multilevel converters are devices used in power electronics to convert electrical energy at multiple voltage levels, resulting in improved signal quality and reduced harmonics. These systems are widely used in industrial applications such as renewable energy, smart grids, and motor drives. Their advanced design improves energy efficiency and thermal management, while offering modularity that facilitates their integration into diverse environments.

Multilevel DC-DC converters are used to transform direct current (DC) voltage from one level to another, typically with significant gains in energy efficiency and harmonic distortion reduction. They are valuable in systems where precise power management is required.

DC-to-AC multilevel converters are essential for converting direct current (DC) electricity to alternating current (AC), particularly in industrial applications and renewable energy systems. These devices enable energy transformation with increased efficiency while minimizing harmonic distortion.

## Chapter 7: Synthesis of Static Converters

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A matrix converter is a converter consisting of a matrix of bidirectional switches allowing the direct connection (without an intermediate storage element) of an  $m$ -phase AC source to an  $n$ -phase AC load. Using PWM in conjunction with filters, sinusoidal values can be obtained at both input and output. The three-phase-to-three-phase matrix converter is of significant industrial interest; it connects a three-phase network to a three-phase load, typically a motor, without a DC intermediate stage. It is a fully reversible structure that produces sinusoidal values adjustable in amplitude and frequency at the output, and balanced sinusoidal currents with a unity power factor at the input.

## Conclusion

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The use of semiconductor switches allows for static converters with numerous advantages over rotating units: reduced maintenance, shorter response times, and, above all, a much broader range of applications.

Power Electronics studies static converters at three levels:

- the component level (semiconductor switches and reactive elements);
- the power structure level (converter schematic);
- and the control level.

In our course, we have tried to dedicate the first chapter to a comprehensive overview of the basic concepts related to the electrical circuits of converters, containing the important theoretical and mathematical foundations and the second chapter is devoted to the study of the static and dynamic characteristics of the components used in power electronics. This includes the study of diodes, thyristors, transistors, and their derivatives. The third chapter is dedicated to the study of controlled and uncontrolled rectifiers. The fourth chapter is devoted to the study of DC/DC converters. The fifth chapter deals with DC/AC converters. The chapter six focuses on AC/AC converters. The synthesis of static converters is studied, the seventh chapter dealt with converters with recent configuration such as switching power supplies, matrix converters and modular multilevel converters.

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