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Thème

Path Planning and Control of a Drone Fleet

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Dedication

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Abstract

In recent years, air pollution has damaged our environment and caused a serious threat to human and animal life. Therefore, it is necessary to locate and identify the sources of pollution. This thesis describes the planning and control of a fleet of drones whose purpose is to gather at the location with the highest pollution concentration. A decentralized structure is employed to control the drone fleet; controllers are used to control each drone independently. Quadrotor's trajectory planning is performed by the metaheuristic algorithm, which is particle swarm optimization by maximizing the air pollution dispersion function and avoiding collisions among the fleet members.
Keywords: Unmanned Aerial Vehicle, Fleet of drones, Air Pollution, Synergetic control theory, Particle Swarm Optimization.

Résumé

Ces dernières années, la pollution de l'air a endommagé notre environnement et constitué une menace sérieuse pour la vie humaine et animale. Par conséquent, il est nécessaire de localiser et d'identifier les sources de pollution. Cette thèse décrit la planification et le contrôle d'une flotte de drones dont le but est de se rassembler à l'endroit où la concentration de pollution est la plus élevée. Une structure décentralisée est employée pour contrôler la flotte de drones; des contrôleurs sont utilisés pour contrôler chaque drone indépendamment. La planification de la trajectoire du quadrotor est effectuée par l'algorithme métaheuristique, qui est une optimisation par essaims de particules en maximisant la fonction de dispersion de la pollution atmosphérique et en évitant les collisions entre les membres de la flotte.
Mots clés: Véhicule aérien sans pilote, Flotte de drones, Pollution atmosphérique, Théorie du contrôle synergétique, Optimisation par essaims de particules.

ملخص

في السنوات الأخيرة، أضر تلوث الهواء ببيئتنا وشكل تهديدًا خطيرًا لحياة الإنسان والحيوان. لذلك، من الضروري تتبع وتحديد مصادر التلوث. تقدم هذه الأطروحة تخطيط مسار وتحكم في أسطول من الطائرات بدون طيار بهدف التجمع في الموقع الذي يحتوي على أعلى تركيز للتلوث. يتم استخدام هيكل لامركزي للسيطرة على أسطول الطائرات بدون طيار ؛ يتم استخدام controllers للتحكم بشكل مستقل في كل طائرة بدون طيار. يتم تخطيط المسار للاسطول من خلال الخوارزمية من خلالها ايجاد مركز تلوث الهواء وتجنب الاصطدامات بين أعضاء الأسطول.

كلمات مفتاحية:

طائرة بدون طيار ، أسطول طائرات بدون طيار ، تلوث الغلاف الجوي ، Particle swarm optimization ، Synergetic control theory .

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Acronyms

ABC Artificial Bee Colony

ACAR Analytical Aggregated Controller Construction

AQMs AIR Quality Monitoring Systems

COG Center Of Gravity

DOF degrees of freedom

LED Light-Emitting Diode

LQ Linear Quadratic

PD Proportional Derivative

PID Proportional Integral Derivative

PSO Particle Swarm Optimization

SCT Synergetic Control Theory

SI Swarm Intelligence

UAS Unmanned Aerial system

UAV Unmanned Aerial Vehicle

GENERAL INTRODUCTION

With a growing population, the globe is changing rapidly, which increases the number of factories and exhaust-gassing automobiles that pollute the air and destroy the environment. To prevent any pollution-related impacts, this calls for immediately finding polluting sources.

Currently, thanks to technological advances, researchers have developed the use of Unmanned Aerial Vehicle (UAV) swarms for a various purposes, such as military operations, environmental service delivery, and surveillance, as well as the capacity to localize polluted areas, thanks to their great agility and speed, and the ability to explore large areas in a reasonable amount of time.

The problem addressed in our thesis is to control and plan the trajectories of a fleet of quadrotors whose goal is to converge on the most polluting source. We start by defining UAVs, the history of UAVs in training, their applications, and by citing the different approaches and methods for controlling fleets of UAVs in the scientific literature. Next, based on the decentralized approach to control the overall system, we develop a control strategy for each UAV through mathematical modeling for the quadcopter using the Newton-Euler formalism, then we synthesize the control laws through synergetic control and derived proportional controllers to achieve the desired positions. Finally, we plan the fleet trajectories by maximizing the Gaussian plume function of air pollution dispersion by the particle swarm optimization algorithm while preventing collisions.

This thesis will be divided into four chapters, each of which is briefly described in the following paragraph:

- **Chapter 1:** introduces UAVs, the history of the UAVs fleet, their application, the different approaches of UAV fleet and the strategies used to control multiple UAVs, and the definition of the problem statement.
- **Chapter 2:** covers the control of the quadrotor fleet, using a decentralized structure and elaborating the mathematical model of each drone individually by the Newton-Euler formalism and the synthesis of the control laws by the synergetic control theory.
- **Chapter 3:** describes the planning of the UAV trajectories, modeling the dispersion of the air pollution by the Gaussian plume function, and the implementation of the particle swarm optimization algorithm to maximize the cost function and avoid collisions between drones.
- **Chapter 4:** presents the simulation results of the control laws as well as the PSO algorithm.

CHAPTER

1

STATE OF THE ART

Introduction

Unmanned aerial vehicle swarm (UAVs) is a technology in constant development, attracting more attention due to the variety of applications they can provide (photography, delivery, and primarily for military purposes). Therefore, every nation is trying to take the lead in the development and marketing of this futuristic technology due to its time-saving and cost-reducing properties, but what makes a swarm of UAV special and different from the rest is the command used to control the swarm and the way the UAVs communicate to achieve a purpose.

This chapter will define Unmanned Aerial Vehicles and the structure of UAV fleets with a brief history enumerating its different advantages and applications, followed by a literature review to compare our research with similar studies and ending with our problematic.

1.1 Definition of unmanned aerial vehicle (UAV)

An unmanned aerial vehicle (UAV) is an aircraft with no pilot on board used for a variety of applications (military, industrial or personal), they are mostly known for their (speed, agility, and efficiency), but the structure of the UAV depends on the function it needs to achieve and being part of the unmanned aerial system (UAS) family, they are either controlled remotely or autonomously [1].

1.2 Historical background

Swarm of drones is a group of UAVs that work together with the sole purpose of accomplishing a task as a cohesive unit.

In 1989, Gerardo Beni and Jing Wang first created the term Swarm Intelligence, marking the beginning of swarm robotics [2].

The first drone show was in 2012 by using 49 LED-equipped UAVs, called SPAXELS (short for "Space Elements") shown at the world premiere by Ars Electronica Futurelab. And every year more performances were executed for example in 2021 up to 15 performances in the spent of one year.

The development of UAV swarms for military purposes dates all the way to 1990, and the effective results only appeared from a study made by the United States military group in 2018, where they found that using swarms made weapons significantly more powerful. In that simulation, a swarm of 800 drones destroyed more targets in two hours than 1000 drones acting individually, showing the potential of UAV swarms.

1.3 UAV flight formation

In the past, where areal warfare was ravaging wars and airplanes became one of the strongest military powers, a new principle of mutual protection and support was created, named formation flight, executed with two or more aerial vehicles flying in complete harmony in a predetermined path to fulfill a pre-established mission as a cohesive unit.

Formation flying has been used since (WWI) World War One, where they discovered that sending multiple aircraft working together raised their chances of success immensely and lowered the losses, but on the other hand, more members in a flight formation meant complex organization and harder maneuvers [3].

After the 1920s, pilots were able to develop and create the most effective patterns for formation flying, and the formation differed from one situation to the other (scouting, bombarding, or even tracking enemy airplanes), each flight formation had a specific use, but flaying formations was not only used for military services, at the end of the war the aviation department decided that creating airplane shows could be a way to entertain people and making them feel safer [4].

FIGURE 1.1: UAVs formation flight

1.4 Advantages and applications

With the increasing technological advances, drones have become a center of interest for many people, mainly for aerial photography and infrastructure inspection. However, they have more potential to be exploited just by upgrading from single drone control to a fleet or swarm of drones; the possibilities become endless by creating an infrastructure composed of multiple drones communicating with one another. They can change our world, simplifying our jobs and adding safety to our daily lives.

- Researchers found a way to use a swarm of drones in rescue operations by scanning areas where help signals were located and then sending fleets of drones to search faster and more efficiently, increasing the speed and success rate of this kind of mission by saving more human lives in-time [5].

- In the case of Wildfires or forest fires, well-controlled UAV swarms can provide a real-time map of the fire progression and the spreading directions. Taking into consideration the wind and the terrain, they can help firefighters do their job safer and easier.

- On the other hand, military services use a swarm of drones for surveillance and tracking in dangerous areas or country borders to lower the risks of losing soldiers and increase tactical maneuvers protecting regions from internal or external threats [6].

- Drone swarm applications do not stop there. They also can be used for commercial usage, similar to the UAV swarm performance presented each new years event with LED-equipped acrobatic drones.

FIGURE 1.2: End year 2021 performance

- Even agriculture can use UAV swarms for automatic plant watering [7] by spreading several drones over a field, even performing agricultural treatment of crops with pesticides.

FIGURE 1.3: UAV swarms used in agriculture

- Big delivery corporations like Amazon expressed their interest in UAV swarms, particularly Unmanned Aircraft Systems (UAS), insinuating that they have high potential in future package delivery by using intercommunication between drones and swarm coordination. For a (fast, effective, reliable, and autonomous) delivery system.

- One potential swarm application that could have a bright future and a big impact in the environmental and technological field would be air quality monitoring swarm of drones [8]. As we developed already the concept of one AQM system using a quad-rotor UAV with a payload containing the air quality measuring sensors that communicate the concentration of each atmospheric pollutant using telemetry system supported by a gateway that provides the data to our analyzing center. The data will be treated using artificial

intelligence AI, mainly using fuzzy logic to determine the levels of pollution.
The same concept can be applied to swarm of UAVs, to get more precise and specific data providing critical feedback of our environment, particularly in urban and industrial areas.

- In short, a swarm of UAVs can reduce the number of hand workers and dangers or risks for human lives needed to execute challenging tasks and save time and resources, Thus lowering the costs for any of the previously mentioned operations and applications.

1.5 Approaches and structure of fleet of UAVs

In the scientific literature, there are two common approaches to using multiple UAVs in training. The first centralized approach, which is a direct extension of single-vehicle control, is based on the idea that a central station controls a group of vehicles. The second approach, known as a distributed strategy, has been studied since it does not require a central station for control and allows for better adaptation to numerous physical restrictions such limited resources, short wireless communication chains, and autonomous issues.

To solve the control challenge, the training control strategy is critical. For the formation control problem, there are three well-known structural approaches. Leader-follower structure [9]-[10], in which some vehicles are configured as leaders and others as followers,but this structure is not robust cause because it may result in UAVs collisions and poor disturbance rejection properties. In the virtual structure or virtual leader strategy [11], The entire formation is treated as a single virtual rigid body structure, and all formation entities receive the mission trajectory, which is considered the virtual leader itself. As a result, guiding a group is easier and more efficient than the leader-follower structure, however avoiding obstacles is difficult. the third structure is behavioral method where several desired behaviors are specified for each vehicle, including formation keeping, goal seeking, collision and obstacle avoidance. It is appropriate for uncertain situations, but it needs a strong theoretical knowledge [12]- [13].

1.6 Literature Review

Many scholars throughout the world are researching drones in formation control because of their use and versatility in accomplishing jobs. Several studies have been suggested. Classical control methods: Lyapunov functional approach and algebraic Riccati equation technique

were used to design the formation protocol [14]- [15], potential method [16], constraint forces [17], adaptive output feedback approach [18], sliding mode approach [19], and consensus-based method [20]. The key issue is that they cannot take explicitly account constraints like fixed-wing UAV velocity and angular rotation rate constraints.
Other researchers employed optimization strategies to achieve the aim and avoid collisions and impediments, such as predictive control [21]- [22] which is a feedback control scheme in which a trajectory optimization is solved at each time step. However, due to the lack of state information over a finite temporal horizon, receding horizon control has some intrinsic limitations. In [23]- [24]- [25], they proposed a general framework to pose the collision avoidance problem of remotely piloted aircraft as an optimal control problem using a stochastic estimator with a particle filter to minimize the effects of uncertainty caused by pop-up circumstances and to allow real-time implementation. Other methods for controlling UAV fleets are proposed include consensus synchronization [26]- [27] and rendezvous [19]- [28], flocking [29]- [13]- [30], and proportion integration differentiation (PID) approaches [31]. On the other hand, studies on fuzzy systems have also shown good results for navigation, guidance and control of autonomous vehicles and mobile robots [32]- [33]- [34].

Recently, scientific use meta-heuristics to control flight formation cause they are inspired by animal compression, insects, and social behaviors. Using particle swarm optimization to avoid detected static and pop-up obstacles [35]- [36]- [37]. A path planning system for numerous UAVs with limited sensor and communication ranges was designed [38]. In addition, scientists have integrated particle swarm optimization with a predictive model to operate UAVs for static and mobility threat avoidance while on reconnaissance missions [39]. The Artificial Bee Colony (ABC) optimization meta-heuristic optimization method is also utilized, which is based on the honeybee swarm's intelligent behavior. is used to compute velocity profiles that avoid obstacles and collisions between UAVs while ensuring fleet formation control and target tracking, and is implemented on each UAV individually [40]-[41].

1.7 Problem statement

In this thesis, we consider a system composed of several quadcopters representing a fleet of drones. The main objective is to fly a group of UAVs in two dimensions converging towards the pollution source, to avoid collisions using particle swarm optimization, and to ensure the stability of each UAV in order to localize sites with high air pollution using control laws.

Conclusion

In the first chapter, we explained shortly what defines a UAV and swarm of UAVs with a brief history then we showed the purpose of formation flaying and the different approaches and structures to control UAV's in formation. Afterwards we briefly mentioned the different studies and researches proposed by the scientific literature to control the fleets of drones. Finally, we define the problem handled by our thesis as well as the control objectives.

CHAPTER

2

MODELLING AND CONTROL OF QUADROTOR

Introduction

The derivation of the quadrotor model is covered in this chapter. The result is critical because it explains how the helicopter moves in response to its inputs. By examining only the four motor speeds, these equations can be used to define, predict, and control the positions attained by the helicopter. To ensure that each UAV in the fleet is stable and converges to a predetermined place using decentralized structure control.

In this chapter, we have used the decentralized structure to control the UAV fleet. Beginning with the development of the mathematical model for the UAVs using Euler and Newton-Euler angels to define the dynamic and kinematic model for the quadcopters, we then focused on the control of the quadcopter fleet using the synergetic control theory followed by an explanation of the control procedure.

2.1 Structure of swarm control

We employ the decentralized strategy to control each agent separately to reach the desired position because it is difficult for a single controller to control a fleet of drones and ensure the stability of the system. As shown in Figure 2.1 [42], each drone has a controller, which comprises an altitude and attitude controller and a position controller.

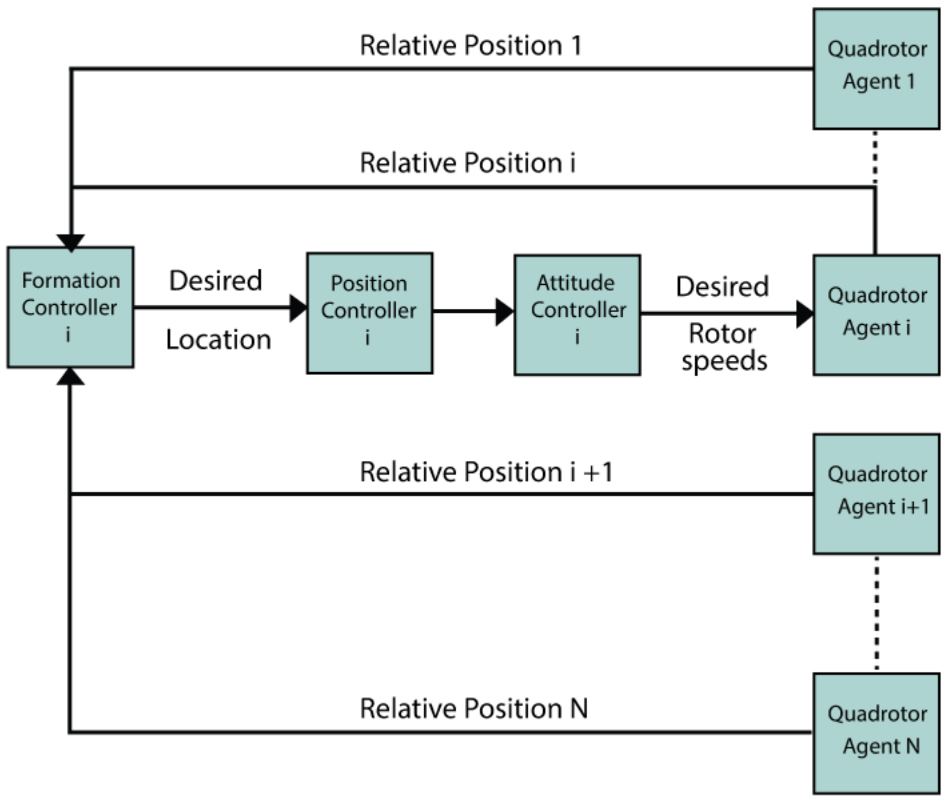


Figure 2.1: Decentralized formation controller

2.2 Basic concepts

The four motors' speeds can be changed to control the quadrotor's attitude and position to the desired values. The thrust produced by rotor rotation, the pitch moment, and the roll moment produced by the difference in four rotor thrusts, gravity, the gyroscopic effect, and the yaw moment can all be applied to the quadrotor [43]. The gyroscopic effect is only seen in the quadrotor's lightweight structure. The unbalanced rotational speeds of the four rotors create the yaw moment.

Reduction gears connect each propeller to the motor, and they are symmetrically on the crossbar. They are separated into two groups, with two diametrically opposed motors in each group; front and rear rotating counterclockwise. Right and left propellers, on the other hand, rotate clockwise. Figure 2.2 describes the direction of the propeller's rotations where the front, rear, right and left propellers are numbered from one to four, respectively.

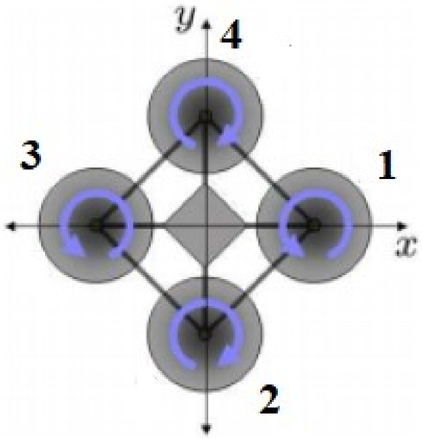


FIGURE 2.2: Direction of propeller's rotations

Even though the quadrotor has six degrees of freedom, it only has four propellers, making it impossible to achieve the desired set-point for all six degrees of freedom. The quadrotor performs the four movements of throttle, pitch, roll, and yaw by adjusting the speed propellers. The following figures depict the different movements.

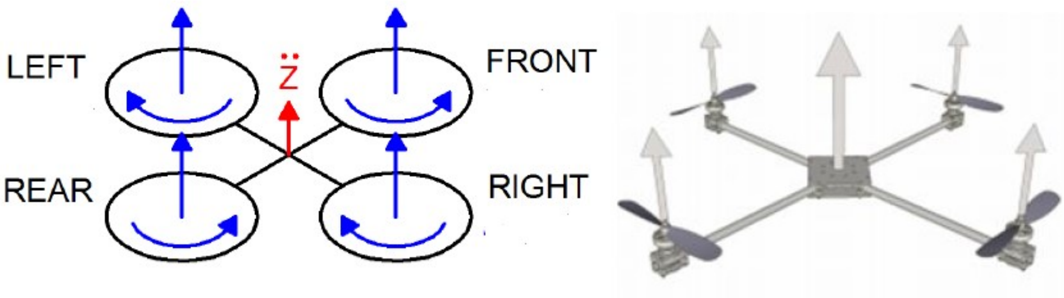


FIGURE 2.3: Throttle movement

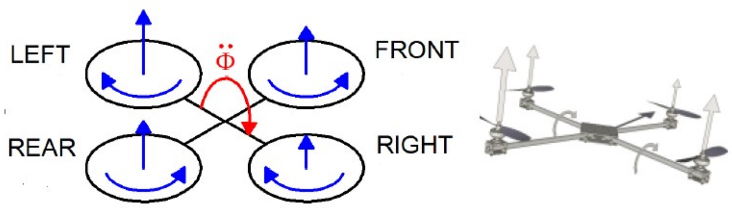


FIGURE 2.4: Roll movement

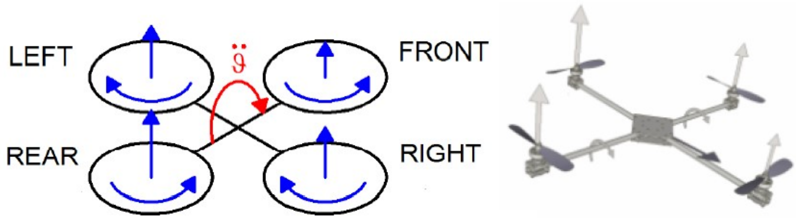


FIGURE 2.5: Pitch movement

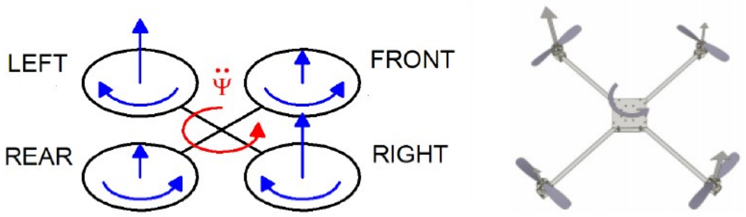


FIGURE 2.6: Yaw movement

2.3 Mathematical model

A quadcopter is a flying aircraft having four rotating motors on each of its four extremities [44]. The speeds of the four rotors control the distinct movements of roll, pitch, and yaw, since each rotor or motor creates a vertical force and a moment of rotation. It has six degrees of freedom (DOF), with three translations and three rotational motions along three axes [45]. The quadrotor is an under-actuated system with just four input forces and six outputs, making it complex and difficult to mathematically model [46]. Important effects from the domains of aerodynamics and mechanics, such as gyroscopic and gravity effects, must be included in the mathematical model [47].

To model the quadrotor dynamics, it's necessary to define the reference frames of the system. Quadcopters are typically specified spatially using two reference frame systems [48]- [49]:

- **Body or mobile frame:** It called B-frame and is attached to the barycenter of the quadcopter.

- **The earth inertial frame:** It is known as the E-frame and is fixed to the earth.

Figure 2.7 shows the two references frame system and the quadrotor schematic.

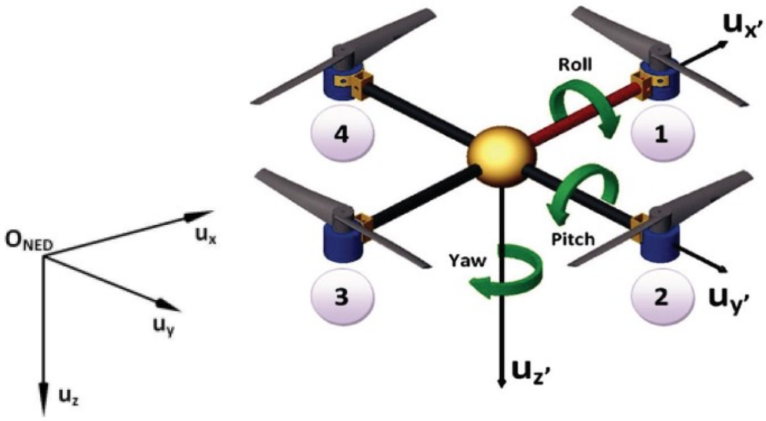


FIGURE 2.7: Quadcopter configuration

2.3.1 Euler angles

Leonhard Euler introduced the three Euler angles to describe the orientation of a rigid body from the body frame to the inertial frame. The roll, pitch, and yaw orientations are represented by the three angles ϕ, θ, ψ respectively [50]. As a result, the Euler angles ZYX are employed to describe the quadcopter's orientation in three-dimensional Euclidean space.

2.3.2 Rotation matrices

The transformation of the coordinates of a vector from the body frame to the inertial frame is given by the following rotation matrices.

- The angle ϕ represents the rotation around the y axis or the roll motion where $-\frac{\pi}{2} < \phi < \frac{\pi}{2}$. The rotation matrix is:

$$R_X(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \tag{2.1}$$

- The angle θ represents the rotation along the Y axis or the pitch motion where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. The rotation matrix is:

$$R_Y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \tag{2.2}$$

- The angle ψ represents the rotation along the z axis or the yaw motion where $-\pi < \psi < \pi$. The rotation matrix is:

$$R_Z(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.3}$$

From the equations (2.1),(2.2) and (2.3), we find the rotation matrix that relates the body frame reference to the inertial frame reference. The matrix of rotation $R_{X,Y,Z}(\phi, \theta, \psi) \in SO(3)$ is given by the following relation:

$$R_{X,Y,Z}(\phi, \theta, \psi) = R_X(\psi) * R_Y(\theta) * R_Z(\phi) \tag{2.4}$$

$$= \begin{bmatrix} \cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\theta\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix}$$

2.4 Quadrotor model with Newton-Euler Formalism

Different ways of mathematically modeling the quadcopter have been proposed in the literature review. The Newton-Euler approach is based on forces and moments acting on a rigid body, and it is first formulated in the body frame reference before being expressed in the inertial frame reference by kinematic and rotation matrices [50], whereas the Euler-Lagrange approach is based on energy assumptions, and the dynamics is expressed directly in the inertial frame reference, resulting in much more complex mathematical expressions. Therefore, the approach of Newton Euler is elaborated due to the simplicity to find the dynamic model of the quadrotor [51].

2.4.1 Dynamic model assumptions

The dynamic model is derived using Newton-Euler formalism under the following assumptions [52]:

- The structure of the quadrotor must be rigid: The body part and the rotor part are respectively rigid.
- The structure of the quadrotor is symmetrical, so the inertia matrix will be diagonal.
- Thrust and drag constants are proportional to the square of the propeller speed of motors.

- The dynamics of the actuator are identical.
- The center of gravity (COG) coincides with the origin of the fixed coordinates of the body.

The purpose of using the Newton-Euler approach is to obtain a reliable dynamic model for further simulation and control. In the inertial reference frame, the vector $[x \quad y \quad z \quad \phi \quad \theta \quad \psi]^T$ describes the linear and the angular positioning and the vector $[u \quad v \quad w \quad p \quad q \quad r]^T$ contains the translational and rotational velocities in the body frame reference.

The relation between the angular velocities expressed in the earth frame $\omega = [\dot{\phi} \quad \dot{\theta} \quad \dot{\psi}]^T$and the angular velocities expressed in the body frame $\omega_B = [p \quad q \quad r]^T$is given by the relation:

$$\omega_B = \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + R_x(\phi)^{-1} \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + (R_y(\theta).R_x(\phi))^{-1} \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} \tag{2.5}$$

Therefore:

$$\omega_B = \begin{bmatrix} 1 & 1 & -\sin\theta \\ 0 & \cos\phi & \cos\theta\sin\phi \\ 0 & -\sin\phi & \cos\theta\cos\phi \end{bmatrix} \omega \tag{2.6}$$

and

$$\omega = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{bmatrix} \omega_B \tag{2.7}$$

The two reference frames are linked by the following relations:

$$\mathbf{v} = R.\mathbf{v}_B, \tag{2.8}$$

$$\omega = T.\omega_B, \tag{2.9}$$

where: $\mathbf{v} = [\dot{x} \quad \dot{y} \quad \dot{z}]^T \in \mathbb{R}^3, \omega = [\dot{\phi} \quad \dot{\theta} \quad \dot{\psi}]^T \in \mathbb{R}^3, \mathbf{v}_B = [u \quad v \quad w]^T \in \mathbb{R}^3, \quad \omega_B = [p \quad q \quad r]^T \in \mathbb{R}^3$, and T is a the angular transformations matrix from (2.7)

$$T = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{bmatrix} \tag{2.10}$$

2.4.2 Kinematic model of the quadrotor

From (2.8) and (2.9) the kinematic model of the quadrotor is:

$$\begin{cases} \dot{x} = w[\sin\phi\sin\psi + \cos\phi\cos\psi\sin\theta] - v[\cos\phi\sin\psi - \cos\psi\sin\phi\sin\theta] + u[\cos\psi\cos\theta] \\ \dot{y} = v[\cos\phi\cos\psi + \sin\phi\sin\psi\sin\theta] - w[\cos\psi\sin\phi - \cos\phi\sin\psi\sin\theta] + u[\cos\theta\sin\psi] \\ \dot{z} = w[\cos\phi\cos\theta] - u\sin\theta + v\cos\theta\sin\phi \\ \dot{\phi} = p + r[\cos\phi\tan\theta] + q[\sin\phi\tan\theta] \\ \dot{\theta} = q\cos\phi - r\sin\phi \\ \dot{\psi} = r\frac{\cos\phi}{\cos\theta} + q\frac{\sin\phi}{\cos\theta} \end{cases} \tag{2.11}$$

The matrix relation for the total force acting on the quadrotor according to Newton's law is the following:

$$m(w_B \wedge v_B + \dot{v}_B) = f_B \tag{2.12}$$

where:

- m: The mass of the quadrotor.
- $f_B = [f_x \quad f_y \quad f_z]^T \in \mathbb{R}^3$: The total force.
- $\wedge$: is the cross product operator.

On the other hand, the total torque applied to the quadrotor is given by Euler's equation:

$$I.\dot{\omega}_B + \omega_B \wedge (I.\omega_B) = m_B \tag{2.13}$$

where:

- $m_B = [m_x \quad m_y \quad m_z]^T \in \mathbb{R}^3$: The total torque or the external moments in the body frame.
- I : Represents the symmetric diagonal matrix of inertia.

$$\begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \in \mathbb{R}^{3\times3}$$

Therefore, we find the the dynamic model of the quadrotor in the body frame:

$$\begin{cases} f_x = m(\dot{u} + qw - rv) \\ f_y = m(\dot{v} - pw + ru) \\ f_z = m(\dot{w} + pv - qu) \\ m_x = \dot{p}I_x - qrI_y + qrI_z \\ m_y = \dot{q}I_y + prI_x - prI_z \\ m_y = \dot{r}I_z - pqI_x + pqI_y \end{cases} \tag{2.14}$$

2.4.3 External force and moments

The following relation describes the external forces in the B-frame:

$$f_B = mg\mathbf{R}^T.\hat{e}_Z - f_t.\hat{e}_3 + \mathbf{f}_w \tag{2.15}$$

where:

- $\hat{e}_Z$ and $\hat{e}_3$: The unit vector in the inertial z axis and the unit vector in the body z axis, respectively.
- g: The gravitational acceleration
- f_t: The total thrust generated by rotors.
- $f_w = [f_{wx} \quad f_{wy} \quad f_{wz}]^T \in \mathbb{R}^3$: The forces produced by wind on the quadrotors.

The external moments in the body frame are also are given by the relation (2.16):

$$m_B = \tau_B - g_a + \tau_w \tag{2.16}$$

where:

- m_b: The torques produced by differences in the rotor speeds of motor's.
- $\tau_B = [\tau_x \quad \tau_y \quad \tau_z]^T \in \mathbb{R}^3$: The torques produced by wind on the quadrotors.
- $\tau_w = [\tau_{wx} \quad \tau_{wy} \quad \tau_{wz}]^T \in \mathbb{R}^3$ describes the gyroscopic moments caused by the combined rotation of the four rotors and the vehicle body and its relation is given by the following equation.

$$g_a = \sum_{n=1}^{4} J_p(\omega_B \wedge \hat{e}_3)(-1)^{i+1}\Omega_i \tag{2.17}$$

where:

J_p and are Ω_i are the inertia of each rotor and the angular speed of each motor respectively. The value of the inertia of each rotor J_p is negligible. So, the gyroscopic moments are not taken into account in the formulation of the controller.

By substituting the external forces and moments relations in the dynamic model expression (2.14). We get the expression:

$$\begin{cases} -mg\sin\theta + f_{wx} = m(\dot{u} + qw - rv) \\ mg[\cos\theta\sin\phi] + f_{wy} = m(\dot{v} - pw + ru) \\ mg[\cos\theta\cos\phi] + f_{wz} - f_t = m(\dot{w} + pv - qu) \\ \tau_x + \tau_{wx} = \dot{p}I_x - qrI_y + qrI_z \\ \tau_y + \tau_{wy} = \dot{q}I_y + prI_x - prI_z \\ \tau_z + \tau_{wz} = \dot{r}I_z - pqI_x + pqI_y \end{cases} \tag{2.18}$$

2.4.4 Actuator dynamics of quadrotor

According to [53], the inputs that control the behavior of the quadrotor are one for the vertical thrust and one for each of the angular motions, and the values of the input forces and torques proportional to the square of the speeds of the rotors. They can be defined as follows:

$$\begin{cases} f_t = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ \tau_x = bl(\Omega_4^2 - \Omega_2^2) \\ \tau_y = bl(\Omega_3^2 - \Omega_1^2) \\ \tau_z = d(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2) \end{cases} \tag{2.19}$$

where

- l: The distance between any rotor and the center of the drone
- b: Thrust factor
- d: The drag factor

By replacing (2.19) in (2.18), the new expression of the quadrotor dynamic model in B-frame is:

$$\begin{cases} -mg\sin\theta + f_{wx} = m(\dot{u} + qw - rv) \\ mg[\cos\theta\sin\phi] + f_{wy} = m(\dot{v} - pw + ru) \\ mg[\cos\theta\cos\phi] + f_{wz} - b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) = m(\dot{w} + pv - qu) \\ bl(\Omega_4^2 - \Omega_2^2) + \tau_{wx} = \dot{p}I_x - qrI_y + qrI_z \\ bl(\Omega_3^2 - \Omega_1^2) + \tau_{wy} = \dot{q}I_y + prI_x - prI_z \\ d(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2) + \tau_{wz} = \dot{r}I_z - pqI_x + pqI_y \end{cases} \tag{2.20}$$

2.5 State space model

From the previous sections, the quadcopter dynamic model has 12 states, six attitude states, and six position and linear velocity states. We denote X as the state-space vector:

$$X = [x \quad y \quad z \quad u \quad v \quad w \quad \phi \quad \theta \quad \psi \quad p \quad q \quad r]^T \in \mathbb{R}^{12} \tag{2.21}$$

From (2.20) and (2.11) the new equations of the dynamic model are:

$$\begin{cases} \dot{\phi} = p + r[\cos\phi\tan\theta] + q[\sin\phi\tan\theta] \\ \dot{\theta} = q\cos\phi - r\sin\phi \\ \dot{\psi} = r\frac{\cos\phi}{\cos\theta} + q\frac{\sin\phi}{\cos\theta} \\ \dot{p} = \frac{I_y - I_z}{I_x}rq + \frac{\tau_x + \tau_{wx}}{I_x} \\ \dot{q} = \frac{I_z - I_x}{I_y}pr + \frac{\tau_y + \tau_{wy}}{I_y} \\ \dot{r} = \frac{I_x - I_y}{I_z}pq + \frac{\tau_z + \tau_{wz}}{I_z} \\ \dot{u} = rv - qw - g\sin\theta + \frac{f_{wx}}{m} \\ \dot{v} = pw - ru + g[\sin\phi\cos\theta] + \frac{f_{wy}}{m} \\ \dot{w} = qu - pv + g[\cos\theta\cos\phi] + \frac{f_{wz} - f_t}{m} \\ \dot{x} = w[\sin\phi\sin\psi + \cos\phi\cos\psi\sin\theta] - v[\cos\phi\sin\psi - \cos\psi\sin\phi\sin\theta] + u[\cos\psi\cos\theta] \\ \dot{y} = v[\cos\phi\cos\psi + \sin\phi\sin\psi\sin\theta] - w[\cos\psi\sin\phi - \cos\phi\sin\psi\sin\theta] + u[\cos\theta\sin\psi] \\ \dot{z} = w[\cos\phi\cos\theta] - u\sin\theta + v[\cos\theta\sin\phi] \end{cases} \tag{2.22}$$

From Newton's law, we also can write:

$$m\dot{\mathbf{v}} = \mathbf{R}.\mathbf{f}_B = mg\hat{e}_{\mathbf{z}} - f_t\mathbf{R}.\hat{e}_3 \tag{2.23}$$

Thus,

$$\begin{cases} \ddot{x} = -\frac{f_t}{m}[\sin\phi\sin\psi + \cos\phi\cos\psi\sin\theta] \\ \ddot{y} = -\frac{f_t}{m}[-\cos\psi\sin\phi + \cos\phi\sin\psi\sin\theta] \\ \ddot{z} = g - \frac{f_t}{m}[\cos\phi\cos\theta] \end{cases} \tag{2.24}$$

For reasons of simplification and for small angles of motion we can consider that $[\dot{\phi} \quad \dot{\theta} \quad \dot{\psi}]^T = [p \quad q \quad r]^T$ [54]. The final dynamic model of quadrotor in the E-frame is:

$$\begin{cases} \ddot{x} = -\frac{f_t}{m}[\sin\phi\sin\psi + \cos\phi\cos\psi\sin\theta] \\ \ddot{y} = -\frac{f_t}{m}[-\cos\psi\sin\phi + \cos\phi\sin\psi\sin\theta] \\ \ddot{z} = g - \frac{f_t}{m}[\cos\phi\cos\theta] \\ \ddot{\phi} = \frac{I_y - I_z}{I_x}\dot{\psi}\dot{\theta} + \frac{\tau_x}{I_x} \\ \ddot{\theta} = \frac{I_z - I_x}{I_y}\dot{\phi}\dot{\psi} + \frac{\tau_y}{I_y} \\ \ddot{\psi} = \frac{I_x - I_y}{I_z}\dot{\phi}\dot{\theta} + \frac{\tau_z}{I_z} \end{cases} \tag{2.25}$$

The input forces and torques are the control inputs for the system. Therefore, we can define the vector $U = [U_1 \quad U_2 \quad U_3 \quad U_4]^T$:

$$U = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} f_t \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ bl(\Omega_4^2 - \Omega_2^2) \\ bl(\Omega_3^2 - \Omega_1^2) \\ d(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2) \end{bmatrix} \tag{2.26}$$

We redefine the state's vector as:

$$X = [x \quad y \quad z \quad \phi \quad \theta \quad \psi \quad \dot{x} \quad \dot{y} \quad \dot{z} \quad p \quad q \quad r]^T \in \mathbb{R}^{12} \tag{2.27}$$

The dynamic model can be written in the affine form in control state-space:

$$\dot{\mathbf{x}} = f(x) + \sum_{n=1}^{4} g_i(x)u_i \tag{2.28}$$

where: f(x)=

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ q\frac{\sin\phi}{\cos\theta} + r\frac{\cos\phi}{\cos\theta} \\ q\cos\phi - r\sin\phi \\ p + q[\sin\phi\tan\theta] + r[\cos\phi\tan\theta] \\ 0 \\ 0 \\ g \\ \frac{I_y - I_z}{I_x}qr \\ \frac{I_z - I_y}{I_y}pr \\ \frac{I_x - I_y}{I_z}pq \end{bmatrix} \tag{2.29}$$

and:

$$g_1(x) = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad g_1^7 \quad g_1^8 \quad g_1^9 \quad 0 \quad 0 \quad 0]^T \tag{2.30}$$

$$g_2(x) = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \frac{1}{I_x} \quad 0 \quad 0]^T \tag{2.31}$$

$$g_3(x) = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \frac{1}{I_y} \quad 0]^T \tag{2.32}$$

$$g_4(x) = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad \frac{1}{I_z}]^T \tag{2.33}$$

where:

$$g_1^7 = -\frac{1}{m}(\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi) \tag{2.34}$$

$$g_1^8 = \frac{1}{m}(\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi) \tag{2.35}$$

$$g_1^9 = -\frac{1}{m}(\cos\theta\cos\phi) \tag{2.36}$$

2.6 Control of quadcopter

Quadrotors are underactuated systems, which makes them difficult to control. In the literature review, there are several proposed methods for controlling them. A number of these studies can be briefly described as follows: Researchers have proposed to use classical methods such as proportional-integral-differential (PID) controllers and PID controllers augmented with angular acceleration feedback and linear quadratic (LQ) controllers [55]. There are also nonlinear control approaches such as the use of sliding mode controllers [56], feedback methods, and

predictive and robust controllers to synthesize control laws [57]- [58]- [59].
There are also techniques using computer vision and fuzzy logic to control quadrotor system including us Euler-Newton method and worked on vision based stabilization and output tracking control and image based visual servo control for quadrotors [60]- [61]- [48].

2.6.1 Synergetic control theory

Synergetic control theory (SCT) is more appropriate for synthesizing controls for complex nonlinear systems [62]. Professor A. Kolesnikov developed this theory to study and synthesize control laws for high degree nonlinear systems without the need of linearization or simplification [63]. The controller's synergetic synthesis principle starts with the creation of a macro-variables, which is a user-defined functions of system state variables. They can theoretically be chosen as a specific combination of system state variables.Using the method of analytical aggregated controller construction (ACAR), the goal then is to drive the system so that it can operate on the manifold [64].
The control rule laws ensure that the closed-loop system is around invariant manifolds, which ensure that the control object operates in the desired state. As a result, the adoption of SCT principles and methods, particularly the ACAR method, which is ideally suited for dealing with high degree nonlinear issues [65].

2.7 Control procedure of the quadcopter

We employ the cascade control structure to control the quadcopter, where the inner loop regulates the altitude and z-position while the external loop regulates the x- and y-positions. To detect disturbances before they reach the main loop's output, the inner loop must to be faster than the outer loop. The global controller is composed of two controllers: a PD controller for the external loop and a synergetic controller to control the internal loop of the system. Figure 2.8 describes the global structure of a quadrotor strategy control [66].

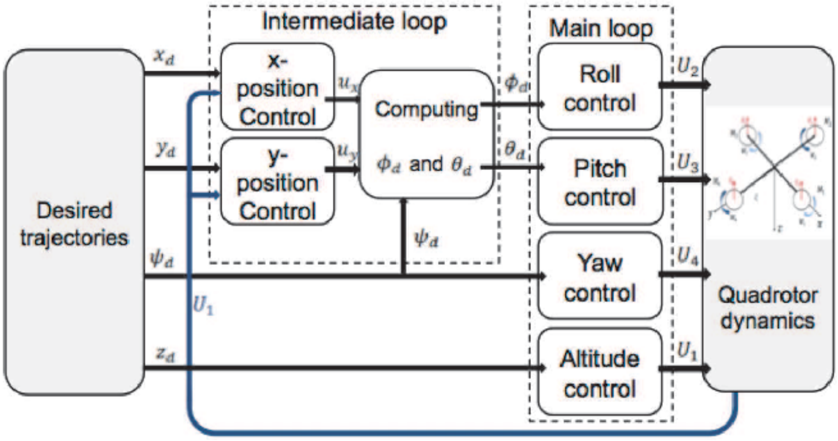


FIGURE 2.8: Structure of a quadrotor strategy control

2.7.1 Control objectives

The control objective is that the quadrotor converges to the desired coordinates x_0, y_0 and z_0 , as well as maintaining the yaw angle in the given direction ψ_0. These outputs constitute the set of invariants. The set of objectives can be written as follows:

$$\sum_{1} = \{x = x_0, \quad y = y_0, \quad z = z_0, \psi = \psi_0\} \tag{2.37}$$

The main purpose of the control law synthesis is to ensure the stability of the UAV under different disturbances. The principle of integral adaptation in the SCT can be used for disturbance rejection. By adding the external disturbance estimation equations to the previous quadrotor dynamic model, the extended mathematical model of the quadcopter is as below:

$$\dot{x} = V_x; \dot{y} = V_y; \dot{z} = V_z; \dot{\phi} = \omega_\phi; \dot{\theta} = \omega_\theta; \dot{\psi} = \omega_\psi;$$

$$\begin{cases} \dot{V}_x = -\frac{U_1}{m}[\sin\phi\sin\psi + \cos\phi\cos\psi\sin\theta] + \gamma_x \\ \dot{V}_y = -\frac{U_1}{m}[-\cos\psi\sin\phi + \cos\phi\sin\psi\sin\theta] + \gamma_y \\ \dot{V}_z = g - \frac{U_1}{m}[\cos\phi\cos\theta] + \gamma_z \\ \dot{\omega}_\phi = [\frac{I_y - I_z}{I_x}\omega_\psi\omega_\theta + \frac{U_2}{I_x}] + \gamma_\phi \\ \dot{\omega}_\theta = [\frac{I_z - I_x}{I_y}\omega_\phi\omega_\psi + \frac{U_3}{I_y}] + \gamma_\theta \\ \dot{\omega}_\psi = \frac{I_x - I_y}{I_z}\omega_\phi\omega_\theta + \frac{U_4}{I_z} + \gamma_\psi \end{cases} \tag{2.38}$$

and:

$$\begin{cases} \dot{\gamma}_x = \eta_1(x_0 - x); \dot{\gamma}_y = \eta_2(y_0 - y); \dot{\gamma}_z = \eta_3(z_0 - z); \\ \dot{\gamma}_\phi = \eta_4(\phi_0 - \phi); \quad \dot{\gamma}_\theta = \eta_5(\theta_0 - \theta); \quad \dot{\gamma}_\psi = \eta_6(\psi_0 - \psi); \end{cases} \tag{2.39}$$

where:

- $\dot{\gamma}; \dot{\gamma}_y; \dot{\gamma}_z; \dot{\gamma}_\phi; \dot{\gamma}_\theta; \dot{\gamma}_\psi$: The estimates of external disturbances.
- $\eta_1; \eta_2; \eta_3; \eta_4; \eta_5; \eta_6$: are positive constants.

2.7.2 Control law synthesis

Position controller

A quadrotor's x and y axis positions are controlled by a conventional method called position control. By manipulating the roll and pitch angles, a quadrotor can rotate in either the x or y axes. For example, to move the quadrotor on the y-axis, one should regulate the roll angle, which will cause the quadrotor to tilt.

From the dynamic model expressed in (2.24), we have [67]:

$$\begin{cases} \ddot{x} = -\frac{U_1}{m}[\sin\phi\sin\psi + \cos\phi\cos\psi\sin\theta] \\ \ddot{y} = -\frac{U_1}{m}[-\cos\psi\sin\phi + \cos\phi\sin\psi\sin\theta] \end{cases} \tag{2.40}$$

The pitch angle is used to make a rotation around the x axis, we can consider $\phi = \psi \approx 0$. Therefore, we can write:

$$\ddot{x} = -\frac{U_1}{m}\sin\theta \tag{2.41}$$

We can design the PD control as:

$$\theta = \arcsin\left(K_{px}(x_d - x) + k_{dx}(\dot{x}_d - \dot{x}) * \frac{-m}{U1}\right) \tag{2.42}$$

In the same way, the roll angle is used to make a rotation around the y axis, we can consider $\theta = \psi \approx 0$.

$$\ddot{y} = -\frac{U_1}{m}\sin\phi \tag{2.43}$$

$$\phi = \arcsin\left(K_{py}(y_d - y) + k_{dy}(\dot{y}_d - \dot{y}) * \frac{-m}{U1}\right) \tag{2.44}$$

where k_{px}, k_{py}, and k_{dx}, k_{dy} stand for proportional and derivative control gain, respectively, and xd and yd indicate the required references.

Attitude and altitude controller

The Analytical construction of aggregate regulators (ACAR) method is the most used method for control law synthesis utilizing the SCT approach. To begin, define a set of control criteria for the system [63]. The system is then expressed as a system of invariants, with the invariants acting to achieve the control goals. As a result, the SCT procedure is reduced to the search for control laws to implement these invariants. Based on synergetic control theory and ACAR method, we define the following set of macrovariables:

$$\begin{cases} \Psi_1 = V_z - K_1(z_0 - z) - \gamma_z; \\ \Psi_2 = V_\phi - K_2(\phi_0 - \phi) - \gamma_\phi; \\ \Psi_3 = V_\theta - K_3(\theta_0 - \theta) - \gamma_\theta; \\ \Psi_4 = V_\psi - K_4(\psi_0 - \psi) - \gamma_\psi; \end{cases} \tag{2.45}$$

where:

$K_i \quad ; i = 1, .., 4$: are positive constants.

Ψ_1 serve to ensure the quadrotor motion converge to the desired position.

Ψ_2, Ψ_3, Ψ_4 serve to maintain the angles of orientation.

The control law force the system to exponentially converge the desired manifold $\Psi = 0$. The dynamic evolution of the manifold can be expressed as:

$$\dot{\Psi}_i + \lambda_i \Psi_i = 0; i = 1, ..4 \tag{2.46}$$

where:

λ_i are positive constants that describe the convergence rate of the system states.

Therefore, The control syneregtic laws can be calculated from relations (2.7.1),(2.45) and (2.46):

$$\begin{cases} U_1 = \frac{m(-\lambda_1\gamma_z - \lambda_1 k_1 z_0 + k_1 V_z - \eta_3 z_0 + \eta_3 z + g + \lambda_1 k_1 z + \lambda_1 V_z + \gamma_z)}{\cos\theta\cos\phi}; \\ U_2 = -k_2\omega_\phi I_x - \omega_\theta\omega_\psi I_y + \omega_\theta\omega_\psi I_z - \lambda_2 I_x\omega_\phi - \gamma_\phi I_x + \eta_4 I_x\phi_0 - \eta_4 I_x\phi + \lambda_2 I_x k_2\phi_0 \\ -\lambda_2 I_x k_2\phi + \lambda_2 I_x\gamma_\phi; \\ U_3 = -k_3\omega_\theta I_y - \omega_\phi\omega_\psi I_z + \omega_\phi\omega_\psi I_x - \lambda_3 I_y\omega_\theta - \gamma_\theta I_y + \eta_5 I_y\theta_0 - \eta_5 I_y\theta + \lambda_3 I_y k_3\theta_0 \\ -\lambda_3 I_y k_3\theta + \lambda_3 I_y\gamma_\theta; \\ U_4 = -k_4\omega_\psi I_z - \omega_\phi\omega_\theta I_x + \omega_\phi\omega_\theta I_y - \lambda_4 I_z\omega_\psi - \gamma_\psi I_z + \eta_6 I_z\psi_0 - \eta_6 I_z\psi + \lambda_4 I_z k_4\psi_0 \\ -\lambda_4 I_z k_4\psi + \lambda_4 I_z\gamma_\psi; \end{cases} \tag{2.47}$$

From the relations of control inputs, we can the speed of rotation of the propellers according to the inputs:

$$\begin{aligned}\Omega_1 &= \sqrt{\tfrac{1}{4b}U_1 - \tfrac{1}{2bl}U_3 - \tfrac{1}{4d}U_4};\\ \Omega_2 &= \sqrt{\tfrac{1}{4b}U_1 - \tfrac{1}{2bl}U_2 + \tfrac{1}{4d}U_4};\\ \Omega_3 &= \sqrt{\tfrac{1}{4b}U_1 + \tfrac{1}{2bl}U_3 - \tfrac{1}{4d}U_4};\\ \Omega_4 &= \sqrt{\tfrac{1}{4b}U_1 + \tfrac{1}{2bl}U_3 + \tfrac{1}{4d}U_4};\end{aligned} \tag{2.48}$$

Therefore, the velocity control for the motors $\Omega_1, \Omega_2, \Omega_3$ and Ω_4 can be derived by taking into account the dynamic model of quadcopter (2.25), the control inputs (2.26), and equation of interrelations (2.48) to provides and improve a asymptotic stability in an environment with external disturbances [64].

To prove the asymptotic stability,we use Lyapunov method; the candidate Lyapunov function is chosen in terms of micro-variables is expressed as:

$$V = \frac{1}{2}(\Psi_1^T\Psi_1 + \Psi_2^T\Psi_2 + \Psi_3^T\Psi_3 + \Psi_4^T\Psi_4) \tag{2.49}$$

We derive the previous equation (2.49):

$$\dot{V} = (\Psi_1^T\dot{\Psi}_1 + \Psi_2^T\dot{\Psi}_2 + \Psi_3^T\dot{\Psi}_3 + \Psi_4^T\dot{\Psi}_4) \tag{2.50}$$

Substituting Equation (2.46) into(2.50):

$$\dot{V} = (\Psi_1^T(-\lambda_1\Psi_1) + \Psi_2^T(-\lambda_2\Psi_2) + \Psi_3^T(-\lambda_3\Psi_3) + \Psi_4^T(-\lambda_4\Psi_4)) \tag{2.51}$$

$$\dot{V} = -(\lambda_1\Psi_1^2 + \lambda_2\Psi_2^2 + \lambda_3\Psi_3^2 + \lambda_4\Psi_4^2) \tag{2.52}$$

Therefore, $\dot{V} \leq 0$. As a result, the stability of the Quadrotor system is asymptotic.

Conclusion

In the second chapter, we presented the structure of the fleet control and the mathematical model of quadcopter using Newton-Euler formalism. Then we synthesize a control laws using synergetic approach and PD controller's to provide the stability in an environment with external disturbances and to reach the desired positions.

CHAPTER

3

PATH PLANNING FOR FLEET OF DRONES

Introduction

Following the synthesis of the control law, each vehicle's stability is guaranteed. The key goal now is to develop a robust optimization algorithm for each UAV that knows its neighbors coordinates, with the intended configuration at each sample time, and finds the next optimal location at the following simple time by minimizing its own cost function while considering motion constraints.

This chapter will define and classify the different metaheuristic methods, explaining their multiple advantages and applications. Then, we used Particle Swarm Optimization (PSO) as our path planning algorithm, followed by the Gaussian plume function to show the spread of air pollution, ending with an implementation of the PSO algorithm into our system.

3.1 Metaheuristic

Optimization has become a big part of our everyday life. Applied everywhere, be it in (engineering, design, business, or even regular tasks) like planning a party or an event.

The use of optimization will always be present willingly or instinctively, mainly to find the best solution from a variety of options to our problems. That is why researchers created an algorithmic framework used in different optimization problems with relatively few to no modifications named metaheuristic algorithms, this type of methods are widely used and implemented due to their simplicity and robustness in exploring the search space to get efficiently optimal and near-optimal solutions.

A metaheuristic is defined commonly as an iterative process that guides a subordinate heuristic by combining different concepts to explore and exploit the search space and find solutions in affordable computational times.

3.1.1 Metaheuristics classification

Metaheuristic algorithms can vary from simple local search methods to complex learning processes based on global search, but different search strategies will give different results.

The most known local search method is the hill-climbing algorithm and its ability to find any local optimum without guarantee of finding the global optimum solution.

On the other hand, there are global search methods with many population-based metaheuristics, including (ant colony optimization, particle swarm optimization, and genetic algorithm) [68].

However, some metaheuristic algorithms can be part of local and global search methods, such as (iterated local search, variable neighborhood search, and GRASP).

Most of the well-known algorithms are mainly separated into single-solution methods and population-based methods. Either by developing one singular candidate solution, for example (iterated local search and variable neighborhood search) or multiple potential solutions, at the same time including (particle swarm optimization, evolutionary computation, and genetic algorithms).

Figure 3.1 shows an abstract of all the algorithms present in metaheuristics and the methods overlapping on multiple areas.

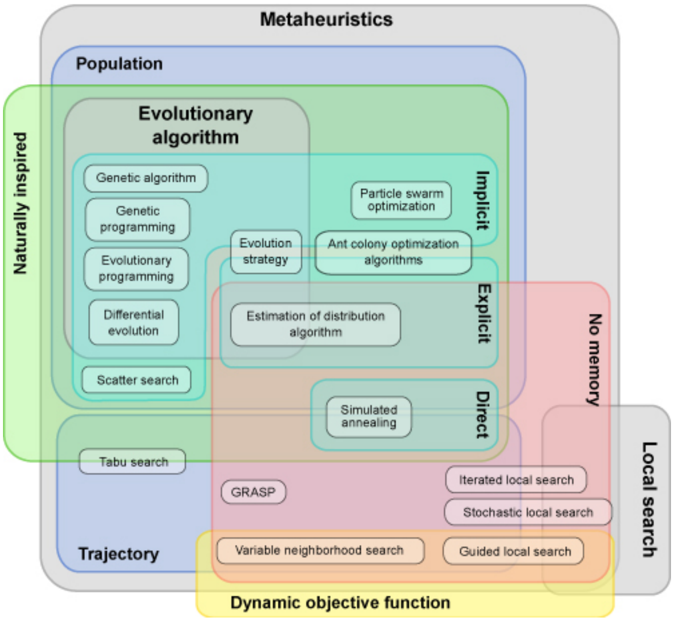


FIGURE 3.1: Metaheuristics methods

Many of the metaheuristics algorithms are inspired by natural systems, such as biology (evolutionary and genetic algorithms) or ethology (ant colony algorithms and bee colony algorithms).

To use a metaheuristic method there must be four criteria needed and pursued as a result, it should be an optimal solution followed with accurate, precise, and complete results and with a fast execution time.

- **Optimality:** The results needed must be optimal or near-optimal, even in the presence of multiple solution for the same problem.
- **Accuracy and precision:** The provided results by the heuristics must be in a trustful range, with a minor marge or error.
- **Completeness:** Needing all the possible solutions.
- **Execution time:** The problem and the applied method need to be compatible, because some heuristics algorithms can find the desired solution faster than others [69].

3.1.2 Advantages and applications

Metaheuristic algorithms can have a variety of advantages:

- Easily understandable and fast to implement.
- Very efficient; it can solve larger problems in short periods of time.

- Broad application: Used for optimization purposes it needs to be applied for a widespread range of problems.
- Good aspect of different methods can be exploited using hybridization or by combining them with traditional methods.
- It can be used to solve complex problem or multiple possible solutions problems.

They shine most when used in the right area of application. As we mentioned, different search strategies will give different results according to the problem.

Researchers explained how to find the right approach using this kind of method:

"A metaheuristic will be successful on a given optimization problem if it can provide a balance between the exploitation of the accumulated search experience and the exploration of the search space to identify regions with high-quality solutions in a problem-specific, near-optimal way." [70]

Figure 3.2 shows the different uses of metaheuristic algorithms in multiple areas of applications:

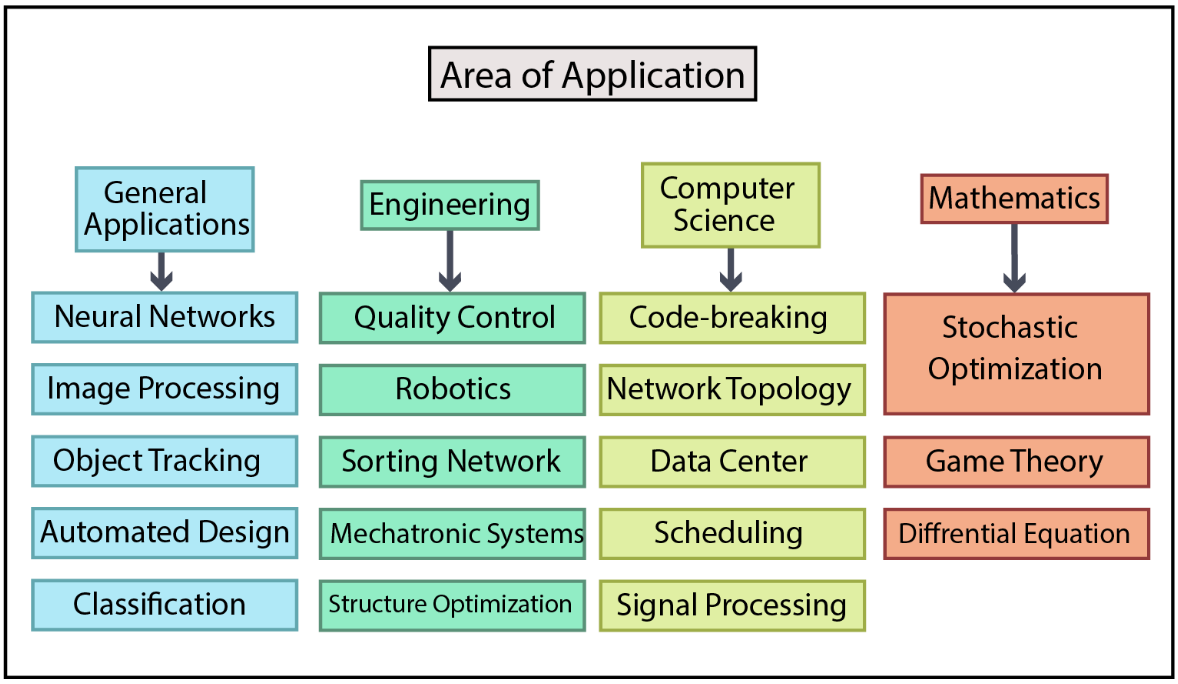


FIGURE 3.2: Metaheuristic applications

3.2 Particle swarm optimization (PSO)

3.2.1 Introduction

Particle swarm optimization (PSO) is a population-based metaheuristic method for the optimization of non-linear functions. It can navigate the search space seeking optimality using its population-based aspect and relying on the initial distribution of this group of particles.

PSO is a global search Swarm intelligence (SI) metaheuristic purely inspired by nature by mimicking the social behavior of birds when searching for food.

3.2.2 Operating principle

The particle swarm optimization (PSO) method is considered to be one of the most popular and used algorithms for swarm intelligence development [71]. By starting with a set of particles distributed in random locations and considered as potential solutions, the (PSO) algorithms can be used to change the positions of the particles around the search space in order to improve the last solution and to locate the global optimum thru particle inter-communications, using a predefined group of mathematical equations.

Using this set of mathematical expressions, it can predict the best possible positions for our swarm by anticipating each particles movement to the best available location.

Figure 3.3 will show the potential movement of each particle.

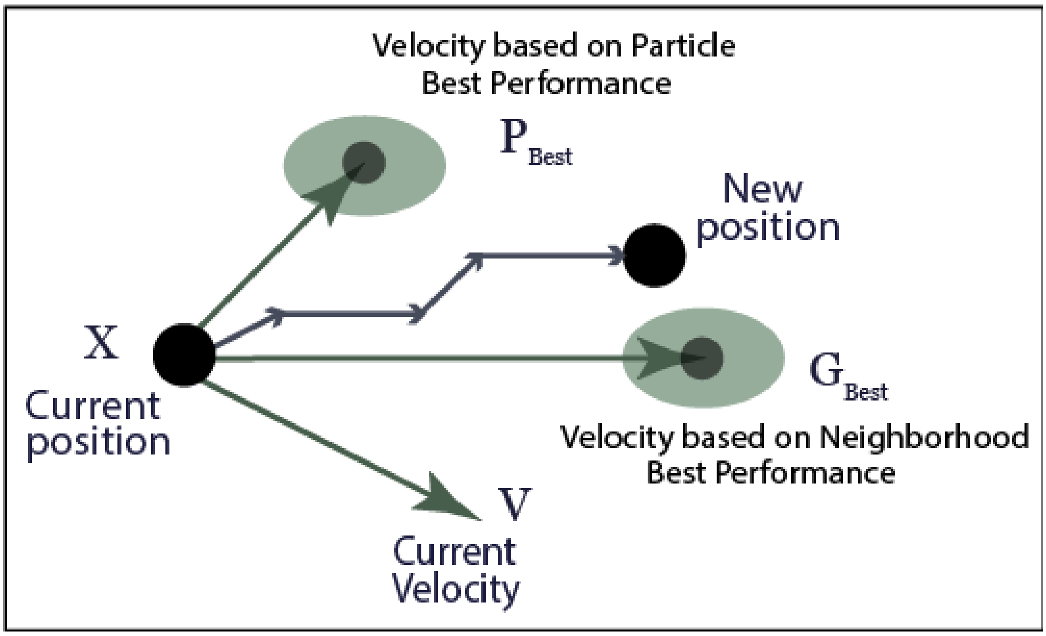


Figure 3.3: Movement of a particle

The particle swarm optimization algorithm depends greatly on two major phases named the

Exploitation and Exploration of the search space, creating a balance in the algorithm.

In general, Exploration represents the ability of the algorithm to search for a new solution different from the current one in the search space, and Exploitation searches the surrounding area around the current solution, similar to local search seeking for better solutions.

For the PSO algorithm, the two phases are different from each other, in the first phase the algorithm starts with the exploration to search in large space area because of its higher particle velocity. When the velocity of the particles lowers, it switches to the final phase, where it will start the exploitation of the search space by pushing the algorithm to focus on the best solutions.

3.2.3 Historic of PSO algorithm

Particle Swarm Optimization (PSO) have been developed by Russ Eberhart and Jim Kennedy in 1995 but it all started when William. T. Reeves tried to create a particle system predefined with a set of points and assigning a velocity vector to each one of them, then Robert Reynolds developed this system by granting it the ability of inter-communication between each particle, after that Kennedy and Eberhart came up with the idea to use the social behavior of birds to solve various problems of optimizations explaining that this kind of method had huge potential because it applied all the principles of swarm intelligence (proximity, quality, diverse responses, stability, and adaptability) all this principles were mentioned [72] when proving the importance of collective intelligence for swarms.

- **Principle of proximity:** They should be able to manage simple spaces with short time computations.
- **Principle of quality:** They should be able to detect quality factors in the environment in our case, by using a fitness function.
- **Principle of diverse responses:** they should be able to practice its activities in wide channels.
- **Principle of stability:** They should maintain the same behavior even in cases of environmental changes.
- **Principle of adaptability:** They should be able to change behavior if they can reduce the computational time.

In the end Kennedy and Eberhart agreed on the name particle swarm optimization for it inter particle communication aspect and it optimization seeking properties .

3.2.4 Advantages and applications

- The PSO algorithm has a lot of advantages many of them being:

- Simple to understand, to program and to use.

- It is efficient on a wide variety of problems, particularly on optimization problems with continues variables.

- Many algorithms in metaheuristics encourage survival of the fittest, but in PSO there is no such thing, in reality a currently poor particle can become the best solution eventually.

  In other words, the particles are cooperating in the search for the global optimum rather than in competition. Which means that the PSO algorithm is influenced by social behavior rather than survival of the fittest.

-The particle swarm optimization algorithm has multiple ares of application (electrical engineering, electronic engineering, mechanical engineering and computer engineering) with a lot of other areas of research proving the efficiency of this meta heuristic algorithm, for example:

- In electrical engineering, the Proportional-Integral-Derivative (PID) Controllers is present for multiple supervision applications. The PSO algorithm can provide optimal operations for the PID controller with proper tuning of (Kp, Ki, and Kd) parameters [73].

- It was also used in home energy management system by taking in consideration the different energy sources of the infrastructure, the cost rate and the multiple household appliances that consume the energy. Using all this information the PSO algorithm is applied to reduce the costs [74].

- On the other hand, in electronic engineering PSO algorithm was used in similar ways to our research for swarm intelligence development. Due to the fact that PSO method does not require a leader to coordinating the movement of the swarm [75].

- Also used for allocation tasks in wireless sensor networks, by designating the workload for each task to proper nodes efficiently and in shorter periods of time [76].

- Applied as well to supervise and detect faults in industrial machinery, in this case. PSO was used to find machine bearing faults by grouping the optimal inputs provided by the system [77].
- As for computer engineering the most known application was the Face Recognition systems based on videos. By combining fuzzy logic and artificial neural network many researches were able to use PSO algorithm to guide the classifiers which decides whether the taken image is negative or positive based on the desired output [78].

3.3 Path planning of fleet of drones using PSO

The main mission of the drone fleet is to converge on the point or location that manages the pollution. Otherwise, the drones converge on the point with the highest concentration of pollution.
With the decentralized approach for synthesizing a drone fleet control method, the goal is to have each drone self-organize around a rendezvous point while avoiding collisions between team members.
The synthesis of drone formation control is considered as a distributed optimization problem. Each drone is assigned an optimization algorithm that optimizes the cost function while allowing the fleet to reach the set objective.

3.4 Synthesis of path planning optimization

We assume a fleet of unmanned aerial vehicles (UAVs) made up of n homogeneous UAV agents. The goal is to get a group of UAVs to fly in two dimensions, x and y, while convergent around a rendezvous point that is either a pollution source or a place with the highest concentration of pollution, while avoiding collisions.

3.4.1 Quadrotor formation topology

In the Cartesian plane or the inertial frame reference, we describe the motion control of each UAV and their positions is denoted as:

$$q_i = [x_i, y_i]^T, q_i \in R^2 \tag{3.1}$$

where:

- x_i: The position of UAV in x axis.
- y_i: The coordinate of the UAV in y axis.

The graph theory can be used to define the interaction topologies of m agents in a multi-agent system, where the whole system can be represented by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, with V $\mathcal{V} = \{1, 2, .., n\}$ are the set of the nodes and $\mathcal{E} \subseteq \{(i, j) : i, j \in \mathcal{V}, i \neq j\}$ is the edges. Therefore, the quadrotor is represented by the node and the communication between two quadrotor is described by the edges.

Let's define $A \in \mathbb{R}^{n*n}$ the adjacency positive and symmetric matrix as $a_{ij} > 0$ if $(j, i) \in \varepsilon$. Cause we have a undirected graph, $\forall(i, j) \in \varepsilon \rightarrow \forall(i, j) \in \varepsilon$

The set of neighbors of each node i is defined as [41]:

$$\Xi_i(t) = \{j \in n : \|q_j(t) - q_i(t) < l\|\} \tag{3.2}$$

where:

- l: Scope of the neighborhood.
- Ξ: The metrical neighborhood of the UAV i.

The constraint of collision avoidance between agents can be described as follows:

$$\|q_i - q_j\| > r_s, r_s \in \mathbb{R}^{n*n} \tag{3.3}$$

where:

- r_s: The safety distance between two agents.

In order to avoid collisions between swarm members, a strategy based on repellent morse potential was developed. The method is defined by the control rule between the i'th and j'th agents as follows [79]- [80]:

$$U^r_{ij} = \begin{cases} \alpha(\exp\{-\beta\|r_{ij}\|\} - \exp\{-\beta * r_s\}) & \text{If } \|r_{ij}\| < r_s \\ 0 & \text{Otherwise} \end{cases} \tag{3.4}$$

where:

- $\alpha, \beta > 0$ are the control gain and the exponent scalar respectively.
- r_{ij} is the distance between two agents.

As a result, the total repellent force acting on each quadrotor i can be given as:

$$u^r_i = \sum_{j=1}^{n} u^r_{ij} \tag{3.5}$$

3.4.2 Cost function optimization

To model air dispersion pollution, many dispersion models have been developed and used to estimate the downwind ambient concentration of air pollutants from sources such as industrial facilities. The Gaussian model is considered the most widely utilized of these. The mathematical model was developed using the experimental campaign's boundary conditions, and the major benefit of Gaussian plume models is their extraordinarily quick, virtually instantaneous response time. For each receptor site, their calculation is dependent on the resolution of only one formula. The main assumptions to use this model are [81]:

- The emission rate of the source is constant.
- Horizontal meteorological conditions are homogeneous over the space modeled. The wind speed, wind direction, temperature mixing height are constant.
- The pollutants are non-reactive gases or aerosol.
- The plume is reflected at the surface with no deposition or reaction with the surface.
- The dispersion in the crosswind and vertical direction take the form of Gaussian distributions.

The Gaussian plume model is illustrated in the figure below:

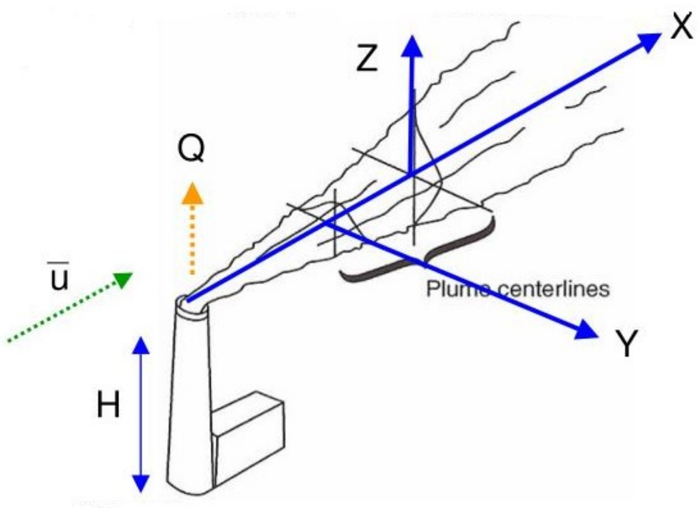


FIGURE 3.4: Gaussian plume model

The origin of an orthogonal Cartesian reference system is considered to correspond to the source's base position, with the x axis parallel to the wind direction. The y axis is horizontal and parallel to the x axis, whereas the z axis is vertical and corresponds to the distance from the ground. Equation (3.6) describes the concentration C(x,y,z) in any position [82].

$$C(x,y,z) = \frac{Q}{2\pi\sigma_y\sigma_z * U}.\exp\left\{-(\frac{y^2}{2\sigma_y^2} + \frac{(z-H)^2}{2\sigma_z^2})\right\} \tag{3.6}$$

where:

- $\sigma_y, \sigma_z[m]$ are horizontal and vertical dispersion coefficients. They depend on the distance x from the source.
- Q: The source strength or mass flow rate (mass/time).
- $U[m/s]$: The time-averaged wind speed at source height.
- $H[m]$: The height of the emitter or stack.

The main goal is to converge the UAV fleet to the point where the concentration of pollutants is maximum by maximizing the cost function in (3.6) by particle swarm optimization with collision avoidance between agents.

3.4.3 Implementation of particle swarm optimization algorithm

To start the PSO process, we create a random population of potential solutions x(t), understood as particles traveling in the search space with V(t) representing their speed. Each particle is drawn to its best known position in the past, P1, as well as the best known position of the particles in its immediate vicinity, P2. Several setup factors are included in the algorithm to act on the compromise Exploration-Exploitation [83].
The velocity and position of each particle are expressed as:

$$V(t+1) = a * V(t) + b_1 r_1 (P_1(t) - x(t)) + b_2 r_2 (P_2(t) - x(t)) \tag{3.7}$$

$$X(t+1) = X(t) + V(t+1) \tag{3.8}$$

where:

- a: The coefficient of inertia.
- b_1 and b_2: The intensity of attraction.
- r_1,$r_2 \in [0; 1]$: Random values.

Each drone in our model symbolizes a particle, and the number of drones in the fleet indicates the swarm's population. The method of maximizing the air dispersion pollution function using PSO metaheurstic with collision avoidance between agents is described by the following algorithm.

Algorithm 1 Particle swarm optimization with collision avoidance

```
for each drone in the fleet do
    Initialize its position and velocity
end for
while iteration < max_iterations do
    for each drone in the fleet do
        Evaluate the fitness function
        if the objective fitness value is better than the personal best objective fitness value P1 in history then
            current fitness value set as the new personal best P1
        end if
    end for
    From all the drones, choose the drone with best fitness value as P2
    for each drone in the fleet do
        Update the drone velocity according equation (3.7)
        Update the drone position according equation (3.8)
        if The distance between agents is shorter than the safety distance then
            Calculate the control rule using equation (3.3)
            Update drone position
        end if
    end for
end while
```

3.5 Conclusion

The third chapter covers the trajectory planning of a UAV fleet. Starting with the reformulation of the problem and the definition of the topology of the quadrotors formation. Next, we have modeled the dispersion of air pollution by the Gaussian plume function. Then, we have used particle swarm optimization metaheuristics for the implementation of the algorithm with the synthesis of the collision avoidance law between fleet members.

CHAPTER

4

SIMULATION AND EXPERIMENTAL RESULTS

4.1 Introduction

After presenting the dynamic model of each UAV in the second chapter and synthesizing the control law to reach the desired position in the second chapter, and planning the trajectories to follow for each UAV by optimizing the objective function using the PSO algorithm in the previous chapter, this chapter presents the simulation of each UAV to evaluate the performance of controllers and the simulation of the path planning algorithm.

4.2 Quad-copter control results

As we have seen, we have developed a cascade structure to control the quadrotor. Where the internal controller controls the attitude (ϕ, θ, ψ) as well as the altitude (z) using synergetic control and the external loop is responsible for the control of the position (x, y) by PD controller. In this section we will present the quadrotor parameters and the temporal responses.

4.2.1 Quadrotor parameters

The parameters of quadcopter are presented in table:

Parameter	Notation	Unit	Value
The quadcopter's mass	m	kg	1
The distance between the center of quadcopter and the center of the propeller	b	Ns^2	$26.5.10^{-6}$
The aerodynamic component of the resistance of the medium	d	Nm s^2	$0.6.10^{-6}$
The moment of inertia about the axis X	I_{xx}	Kg m^2	0.1
The moment of inertia about the axis Y	I_{yy}	kg m^2	0.1
The moment of inertia about the axis Z	I_{zz}	kg m^2	0.1

Table 4.1: Design parameters of the quadcopter

The parameter of the external and internal controller are summarized in Table.

Parameter	Value
K_p	2
K_d	1

Table 4.2: Design parameters of PD controller

The goal of employing a PD controller is to assure reaction stability and accuracy. Proportional gain makes the system fast and accurate, but it also puts it at risk of being unstable, while derivative gain improves the system's stability and speed while affecting the oscillations in the response. Because the outer loop's response time is considerably longer than the inner loop's, lower PD values are chosen.

Parameter	Notation	Value
Parameter of the 1st functional equation (2.46)	λ_1	8
Parameter of the 2nd functional equation (2.46)	λ_2	90
Parameter of the 3rd functional equation (2.46)	λ_3	90
Parameter of the 4th functional equation (2.46)	λ_4	80
Parameter of the macrovariable Ψ_1	k_1	50
Parameter of the macrovariable Ψ_2	k_2	25
Parameter of the macrovariable Ψ_3	k_3	25
Parameter of the macrovariable Ψ_4	k_4	80
Parameter of the estimation of the external disturbances for z	η_1	0.2
Parameter of the estimation of the external disturbances for ϕ	η_2	10
Parameter of the estimation of the external disturbances for θ	η_3	10
Parameter of the estimation of the external disturbances for ψ	η_4	10

TABLE 4.3: Design parameters of the internal controller

The choice of the values parameters of the internal controller is made by the observation of responses of the system for different values. The gain K influences the speed, the stability of the system. For small values of the gain the system response has oscillations. The value of the parameter λ influences the oscillations of the system as well as the overshoot.

4.2.2 Simulation results of a single quadcopter

Simulation experiments of the attitude control of a single quadcopter were performed to evaluate the efficiency of the synergetic control before implementing the control laws in all of the fleet's UAVs. The dynamic model of the quadcopter was simulated with both controllers without and with external disturbances using MATLAB software. The desired positions are {

$x_0 = 1, y_0 = 1, z_0 = 20\}$ with the yaw angle $\psi_0 = 0.3$. The simulation results are presented below:

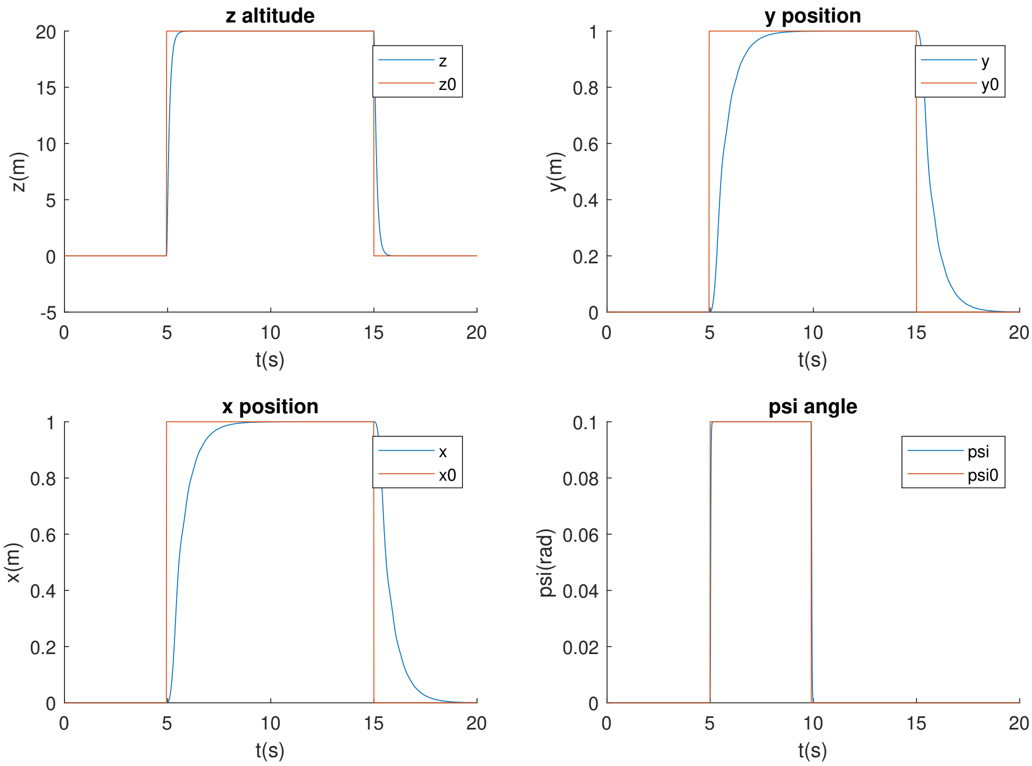


FIGURE 4.1: xyz positions and the yaw angle ψ

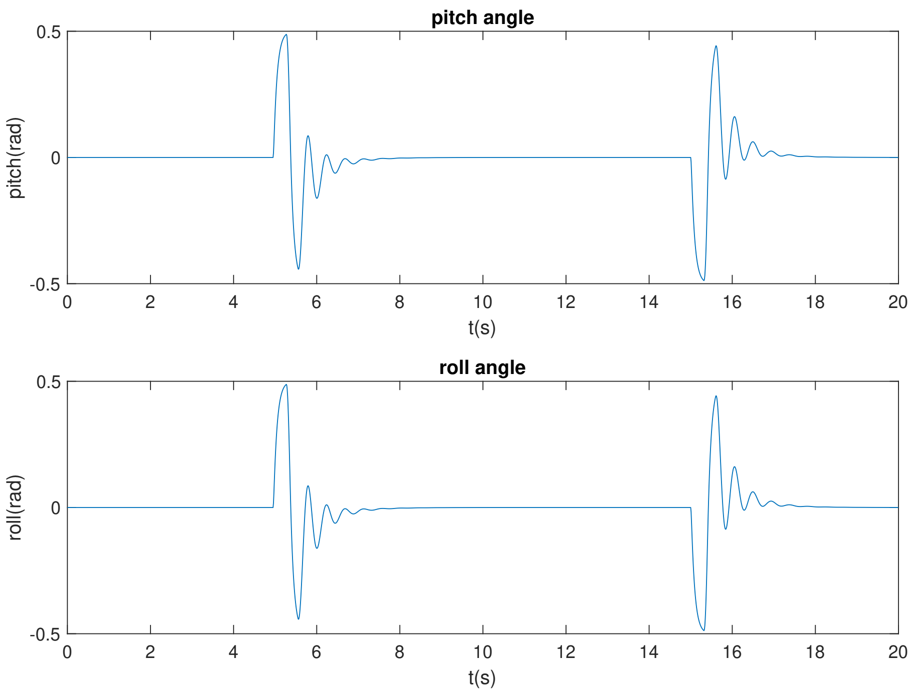


FIGURE 4.2: Pitch and roll angles

Figures 4.1 show the evolution of the translation and yaw positions. The quadcopter can follow the reference trajectories, as can be observed. The z position has a much faster response than the x and y positions. Therefore, the performances of the controllers are proven. The outer loop controller generated roll and pitch trajectories for the vehicle to follow the x and y reference trajectories, as shown in Figure 4.2. Because of the coupling of the quadcopter between translational and rotational dynamics, there are some peaks of considerable amplitude when the quadrotor changes its position.

To evaluate the performance of the synergetic control in the presence of disturbances, we introduce disturbance signals as square wave signals, as shown in Figure 4.3. Figures 4.4, and 4.5 present the simulations results.

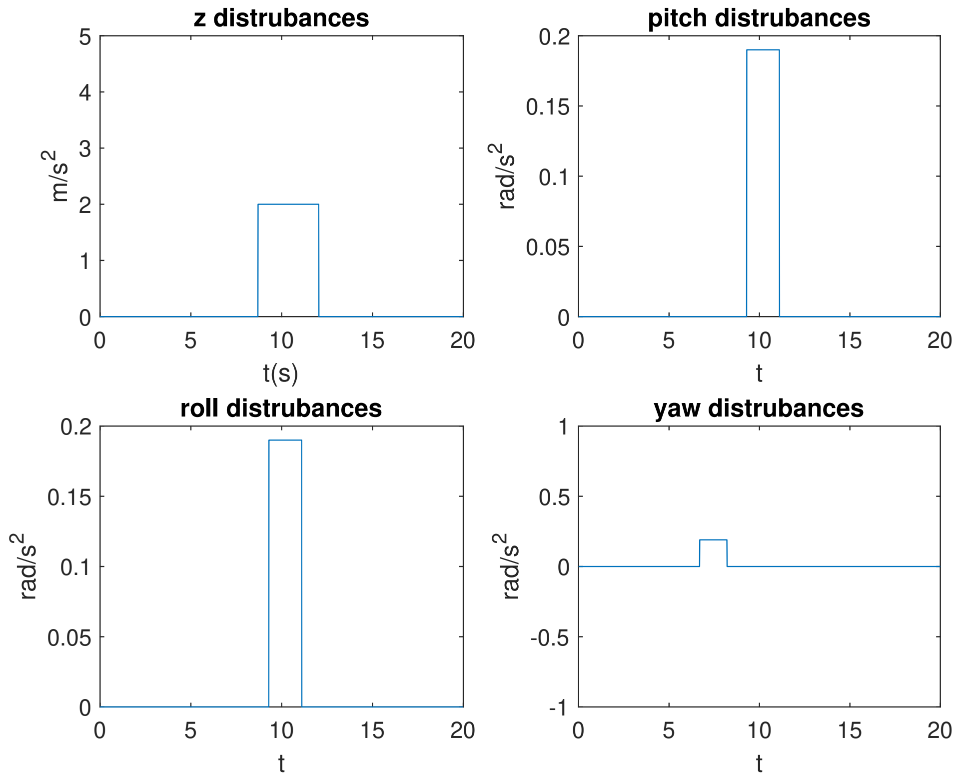


FIGURE 4.3: Disturbances signals

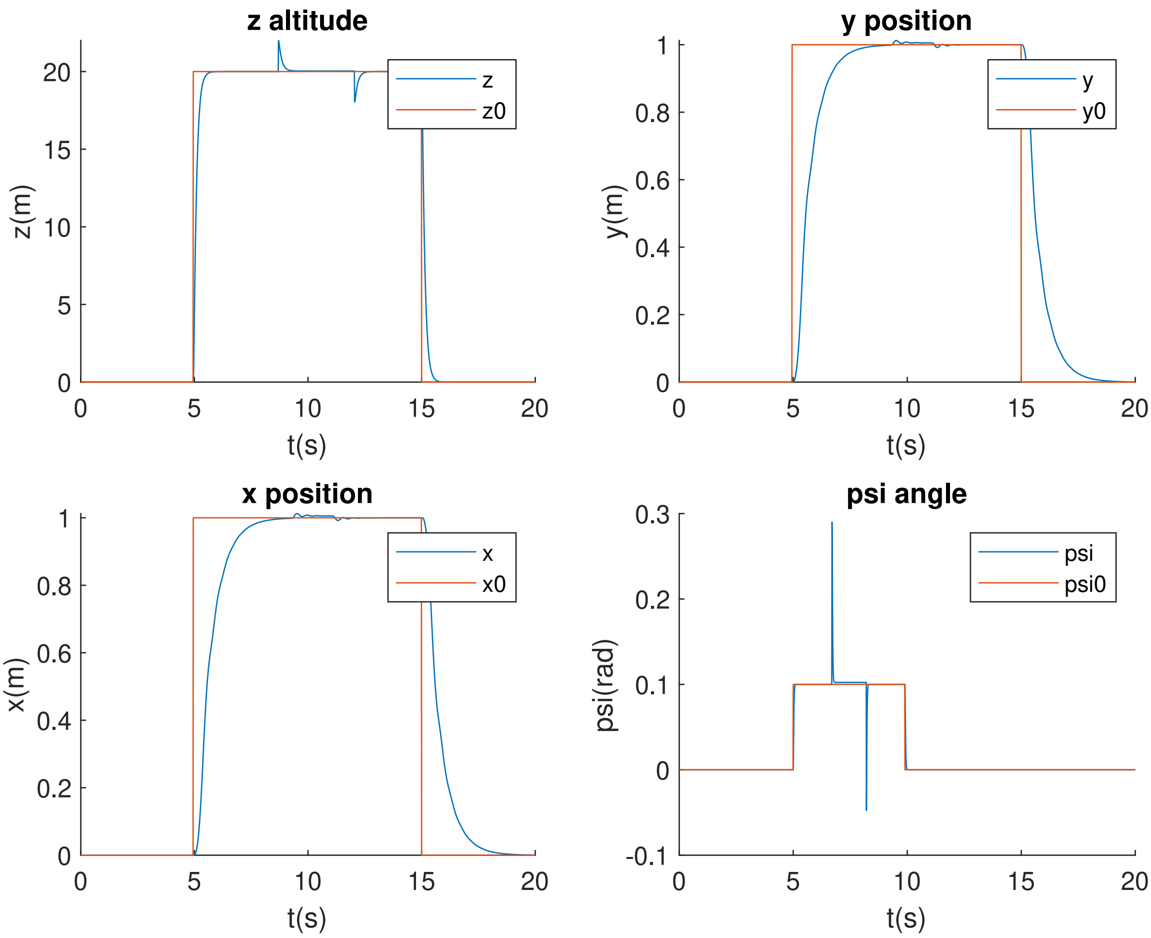


FIGURE 4.4: xyz positions and the yaw angle ψ with disturbances

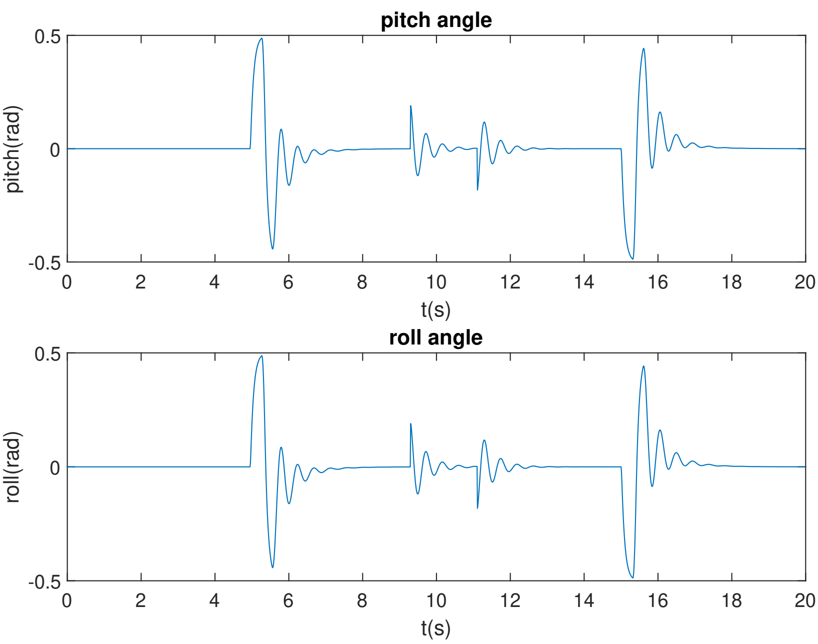


FIGURE 4.5: Pitch and roll angles ϕ, θ with disturbances

The simulation results confirm the effectiveness of the control laws developed for disturbance rejection without linearizing the dynamic system. In the trajectory of the z position and the ψ angle response, we also notice that the disturbance rejection time is short. There are slight oscillations in the roll and pitch angle responses when the disturbance is applied, but these dissipate after a few seconds.

4.3 Path planning simulation results

The proposed method for UAV fleet trajectory planning was evaluated in simulation for a group of 6 agents. The results presented in the following are given from multiple simulations with different initial conditions on drone positions. The main objective is to find the position that has the maximum concentration of pollutants and cluster around it. The time constant is set to 1 second. The position of the drones according to x to is set $x_0 = 1$ to have constant values of σ_y and σ_z. The initial positions and the simulation parameters are presented in table 1 and 2

	UAV1	UAV2	UAV3	UAV4	UAV5	UAV6
y_0 (m)	70	30	40	5	10	60
$z_0(m)$	25	50	40	45	15	40

Table 4.4: Initial positions of UAVs

k_p	k_c	α	β	c	max_{ite}	rs
1.1	0.85	4	0.3	[1.5 1.5]	200	1

Table 4.5: Simulation parameters

The parameters of the atmospheric pollution model are summarized in table:

H [m]	u[m/s]	$\sigma_y[m]$	$\sigma_z[m]$	Q [kg/s]
20	3	30	20	20

Table 4.6: Atmospheric pollution model parameters

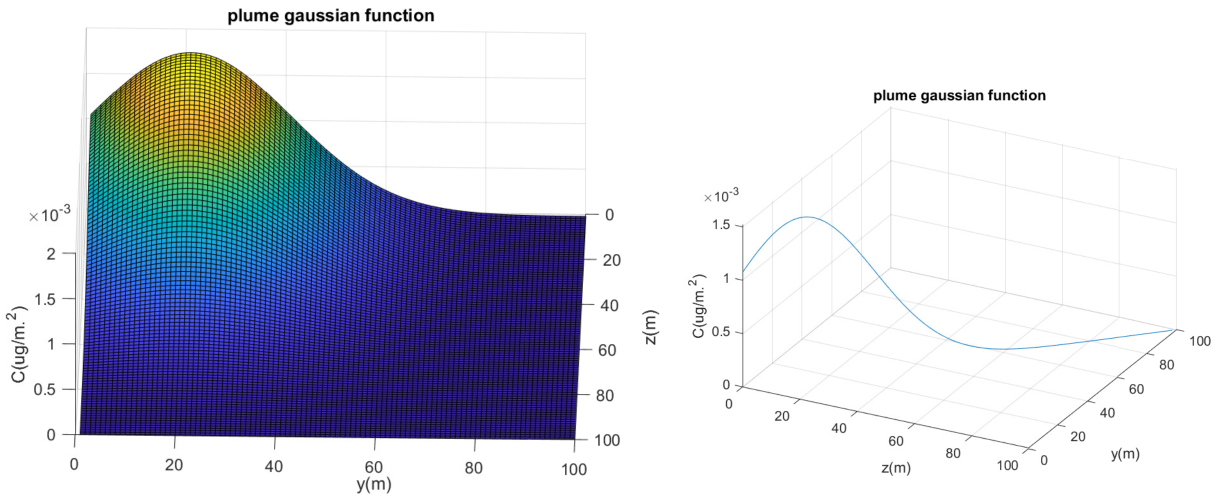


FIGURE 4.6: Air pollution dispersion plot

Figure 4.6 shows the pollution dispersion in an area of 100m^2. By a graphical observation, we can see that the pollution source is located at point (0,20) with a concentration of 0.00176 μg/m3.

Figure 4.8 shows the evolution of trajectories of each drone from the initial position to the desired location, where the pollution concentration is maximum.

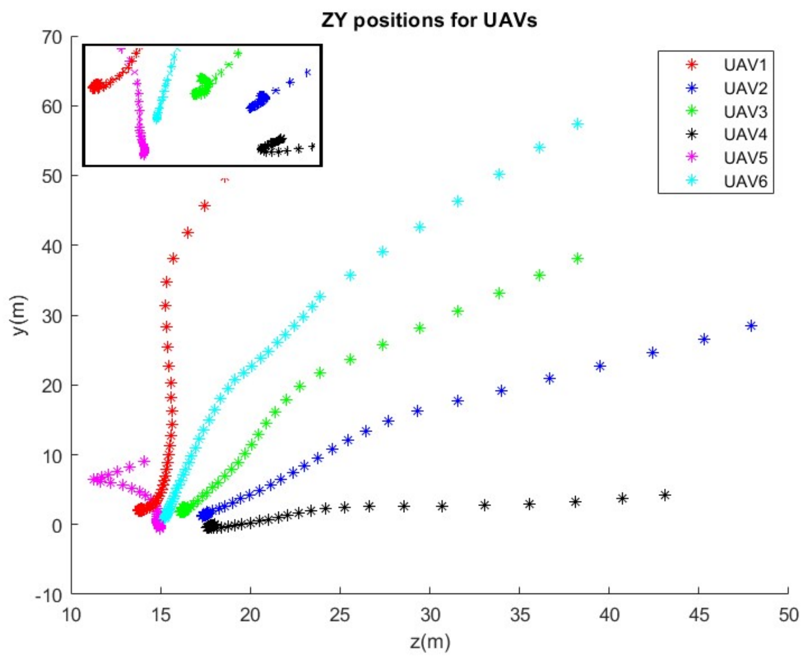


FIGURE 4.7: 2D targets and trajectory generated by PSO algorithm with collision avoidance

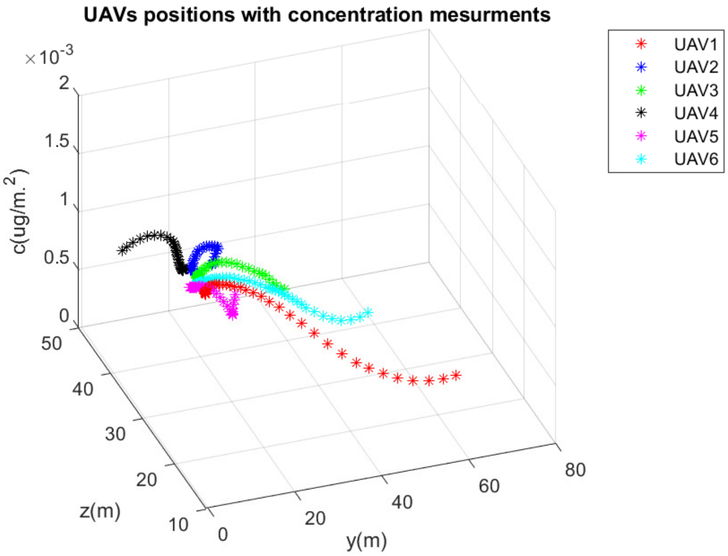


FIGURE 4.8: UAVs positions with concentrations measurement

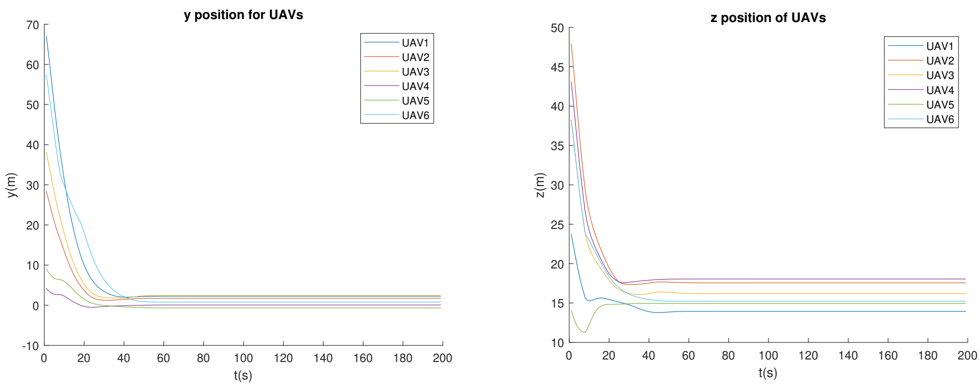
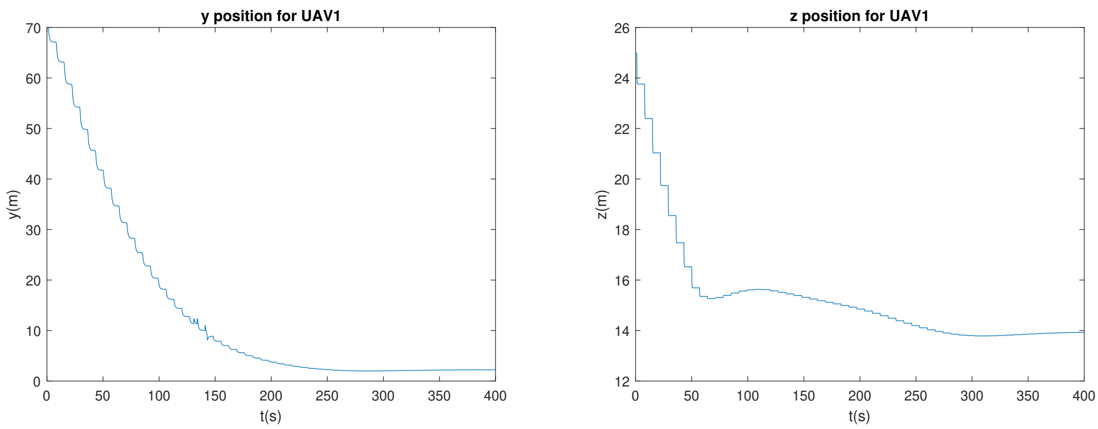


FIGURE 4.9: z and y positions for UAVs

We notice the UAVs converge to the optimal position where the industrial stack is located; the coordinates of the optimal position are (1,0.2409,20.0217) founded by the UAV4, where the concentration is 0.0018 μg/m3. The convergence time is estimated to be the 60s. The convergence time depends mainly on the initial conditions of the UAVs, which proves the performance of algorithm PSO. We also note the importance of the collision avoidance law when the UAVs become very close.

After evaluating the performance of the control laws implemented in a single UAV and ensuring that the algorithm can find the optimal position for the pollution source, we implement the control laws and UAV dynamics in the algorithm to see if they can follow the desired trajectories. We take into account the stabilization time of the UAVs in each position. The figure shows the evolution of the trajectories for UAV1.



(A) y position for UAV1

(B) z position for UAV1

FIGURE 4.10: y and z positions for UAV1

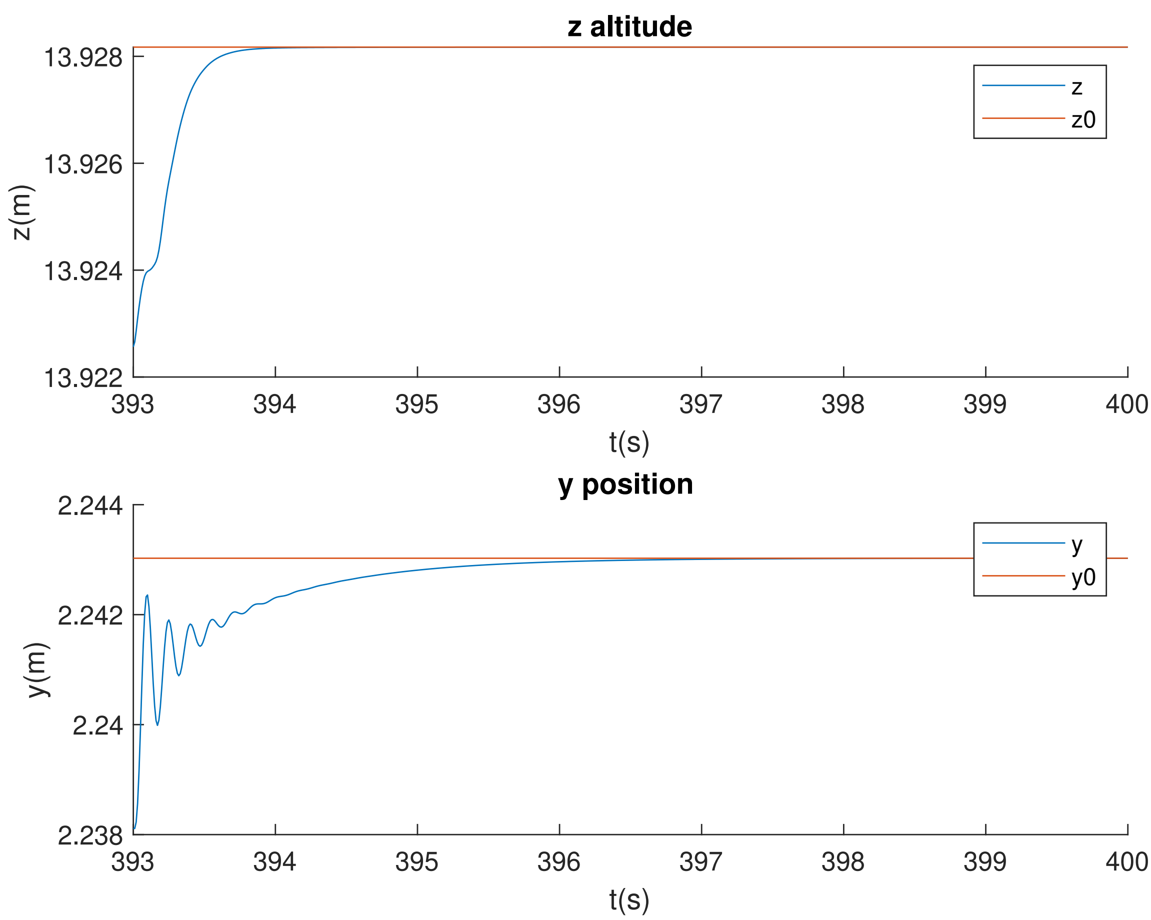


FIGURE 4.11: y and z positions for UAV1 between two time samples

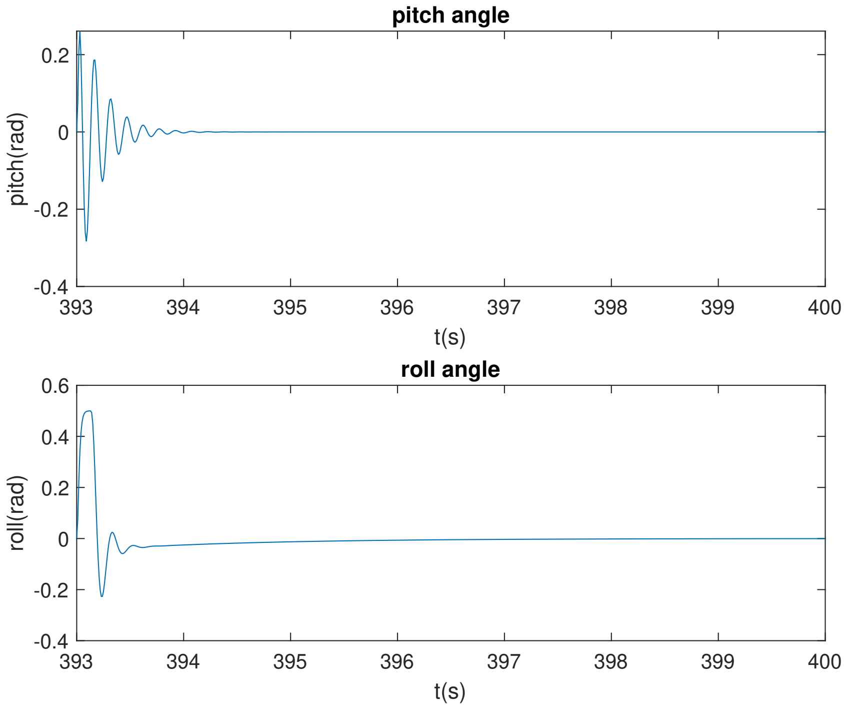


Figure 4.12: Pitch and roll angles

The system responds well, the trajectories follow the references. The convergence time is estimated at 400s to stabilize the system at each sample. As can be seen in Figure 4.11, there is some oscillation in the response of the y position before it stabilizes. A disturbance was introduced at time 150s for the variable y, the controller was able to reject the disturbance. As the drone changes position every second, there are oscillations in the pitch response before the system enters the steady state, as shown in Figure 4.12.

4.4 Conclusion

The simulation of synergetic control and implementation of the PSO algorithm for UAV fleet trajectory planning are presented in this chapter. The procedure of synthesis of synergetic control for the quadrotor UAV in an environment with external disturbances with the use of complete nonlinear models developed in the second chapter has ensured the asymptotic stability of the system in closed loop, and the rejection of disturbances by using the law of synthesis of adaptive control systems "integral adaptation" in particular. The asymptotically stable equilibrium can be easily proved by applying Lyapunov methods to the resulting closed-loop system of the quadcopter. For the purpose of planning the drone fleet's trajectories. The PSO algorithm's result is satisfactory; the drones have converged on the optimal position for detecting the polluting source. The convergence time is determined by both the initial conditions. The control laws were successfully implemented, with positive results for each agent.

GENERAL CONCLUSION

Drones in training are becoming the focus of interest for researchers around the world. The use of multiple unmanned aerial vehicles (UAVs) is highly recommended for military, government, and commercial tasks such as intelligence, surveillance and reconnaissance, border patrol, and environmental emergency detection. The goal of our thesis is to control and plan the trajectories of the UAV fleet to converge on the most polluting source in a region. We started our thesis with a general overview of UAV fleets with their applications and cited the different approaches and strategies in the literature review to control UAVs in formation. Next, we employed a decentralized structure to control each drone using PD and synergetic controllers to granulate the stability of each agent in the fleet. Then, we modeled the dispersion of the air pollution generated by the factories by the the Gaussian plume model or function. For the planning of the trajectories of the fleet of drones, we used the PSO algorithm to maximize the dispersion function while avoiding collisions between agents. The results got by the synthesis of the control laws and implementation of the PSO algorithm are satisfactory. The drones converged towards the desired positions while ensuring their stability and the algorithm found the optimal position with good accuracy.

Our study can be developed by the design and implementation of distributed path planning algorithm for a fleet Of UAVs by implementing command laws in the flight controllers.

APPENDIX

A

LYAPUNOV STABILITY

Consider the autonomous system:

$$\dot{x} = f(x) \tag{A.1}$$

where $f : D \to R^n$ is a locally Lipschitz map from a domain $D \subset R^n$ into R^n Suppose $\overline{x} \in D$ is an equilibrium point of A.1, that is, $f(\overline{x}) = 0$. Our goal is to characterize and study stability of $\overline{x}$. For convenience, we state all definitions and theorems for the case when the equilibrium point is at the origin x = 0.

Definition 1

The equilibrium point x=0 of A.1 is stable if, for each $\epsilon > 0$,there is $\delta = \delta(\epsilon) > 0$ such that $|x(0)| < \delta \Rightarrow ||x(t)|| < \epsilon$ for all $t \geq 0$ unstable if not stable: asymptotically stable if it is stable and can be chosen such that $||x(0)|| < \delta \Rightarrow \lim_{x \to +\infty} x(t) = 0$.

Let $V : D \to R$ be a continuously differentiable function defined in a domain $D \subset R^n$ that contains the origin The derivative of V along the trajectories of A.1 denoted $\dot{V}$ is given by

$$\dot{v}(x) = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} x_i = [\frac{\partial V}{\partial x_1}, \frac{\partial V}{\partial x_2}, ..., \frac{\partial V}{\partial x_n}] \begin{bmatrix} f_1(x) \\ f_2(x) \\ . \\ . \\ f_n(x) \end{bmatrix} = \frac{\partial V}{\partial x} f(x) \tag{A.2}$$

If $\dot{V}(x)$ is negative, V will decrease along the trajectory of A.1 passing through x. A function V(x) is positive definite if V(0) = 0 and $V(x) > 0$ for $x \neq 0$. It is positive semi-definite if it satisfies the weaker condition $V(x) \geq 0$ for $x \neq 0$. A function V(x) is negative definite or negative semi-definite if -V(x) is positive definite or positive semi-definite, respectively.

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