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**Conception et réalisation d'un robot
pendulaire inversé**

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

Dedication

Shams of Tabriz said "when you step into the zone of love, language as we know it becomes obsolete" my lovers no matter what I say I will never thank you enough. I have the honor to dedicate this work to

My parents FRIH Habib and ZIANE Fatiha, my dears, I owe you everything because you made me as I am, without you I would not have achieved anything.

My brothers Taki, Yacine and the flower of the family my sister Marwa, you have been supporting me all the time.

To my soul-mate, you always support me and motivate me, I'm great full for having you in my life.

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Abstract

This work is dedicated study, designing, and realise the inverted pendulum on a cart system is unpredictable and non-linear. In this thesis, we described the inverted pendulum generally, and talked about the types and the domains based on the inverted pendulum, after that, we explained the dynamics of the system with a partial feedback linearisation and the dynamics of the dc motor also included a study of constraints, stability and designing a controller LQR and PID. Finally, we designed the prototype of the system, we created an Arduino code including a PID controller eventually we talked about the electronics that we used so anyone can recreate our system for research. Our main goal in this thesis is to provide a system for the laboratories to test control strategies.

Keywords Inverted pendulum, partial feedback linearisation, PID control, LQR control.

ملخص

هذا العمل مخصص لدراسة وتصميم وانجاز نظام النواس المقلوب على عربة، هذا النظام لا يمكن التنبؤ به وغير خطي . في هذه الاطروحة ، وصفنا نظام النواس المقلوب بشكل عام بذكر انواعه واستخداماته في مختلف المجالات الصناعية، بعد ذلك شرحنا ديناميكيات النظام مع خطية ردود الفعل الجزئية وديناميكيات محرك التيار المستمر وشمل ايضا دراسة حول القيود التي قد نواجهها في النموذج الحقيقي ، ثم قمنا بدراسة حول استقرار النظام. في النهاية تحدقنا عن الالكترونيات التي قمنا باستخدامها حتي ستمكن لأي شخص من إعادة انشاء نظامنا للبحص . هدفنا الرئيسي في هذه الرسالة هو توفير نظام للمختبرات من اجل اختيار استراتيجيات التحكم.

كلمات مفتاحية نواس مقلوب، حطية ردود الفعل الجزئية، متحكم بي اي دي ، متحكم

تربيعي خطي

Résumé

Ce travail est consacré à l'étude, la conception et la réalisation d'un pendule inversé sur chariot, ce système est imprévisible et non linéaire. Dans ce mémoire nous avons décrit le pendule inversé en général, cité quelques types et les domaines basés sur le pendule inversé, en suite nous avons expliqué la dynamique du système avec une linéarisation partielle par bouclage et la dynamique du moteur à courant continu qui comprend également l'étude des contraintes, nous avons fait l'étude de la stabilité et la conception d'un contrôleur LQR et PID. Par la suite, nous avons conçu le prototype du système et créé un code arduino comprenant un contrôleur PID. Finalement nous avons parlé de l'électronique que nous avons utilisé afin que n'importe qui puisse recréer notre système pour la recherche. Notre objectif principal dans cette thèse est de fournir un système permettant aux laboratoires de tester des stratégies de contrôle.

Keywords Pendule inversée, linéarisation par rétroaction partielle, contrôleur PID , LQR contrôleur

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General introduction

In the seventeenth century, a group of philosophers collect to complete an investigation, Galileo the leader of this group found a way to challenge and test knowledge, this process may have started with a very simple object of the pendulum, in 1610 Galileo wrote a marvelous property of the pendulum making all its vibrations large or small in equal times. After Galileo's research, we come to the Dutch scientist Christiaan Huygens who built the first pendulum clock in 1656 [1], then the temperature compensated pendulum (the gridiron pendulum) was developed by the British clockmaker John Harrison in 1726 [2], next it was Foucault pendulum and many other pendulums were invented until the newest inverted pendulums.

There are many kinds of inverted pendulums such as simply inverted pendulums, rotated pendulums, the inverted pendulum in a cart, and many other types. The inverted pendulum does represent many real-world systems, basically, any system that requires vertical stabilization has similar dynamics to an inverted pendulum, there are many companies that use the concept and the dynamics of the inverted pendulum, like the Segway-ninebot, NASA, shadow robot company, etc. To understand the inverted pendulum principle, we have to answer some questions like, how does it work? Where do we use it? And what are the types of it?

Our idea is to study the inverted pendulum on a cart, its mechanics aspect, used cases, the different technologies used to build it and how can we program its stability using automatic

rules such as regulation, commands, etc. In order to develop our inverted pendulum which can be used as a maniple in practical work of regulation.

In this work, in the first chapter, we will define the inverted pendulum and its functioning principle to make it easy for the readers to understand it, the under-actuated mechanical systems like our pendulum system have similar problems to those found in industrial applications, such as external shocks and/or non-linear behaviour so we will talk about the systems that based or similar to the inverted pendulum on a cart. Finally, we will talk about different types of pendulums from the simplest to the weirdest.

In chapter two we will discuss and try to explain the dynamics and the mathematical modeling of the inverted pendulum from the dynamics equations that we will get we will extract the state space representation and from this last one we will extract the feedback linearisation, the modeling of the inverted pendulum it is not enough we need the modeling of the actuator (DC motor), Finally we will make a small study of constraints to know the obstacle in the real model.

In the third chapter, we will talk about the PID controller and the effect of each parameter, then we will talk about some tuning methods, Finally, we will try to use the partial feedback linearisation to design an LQR controller for the inverted pendulum system.

In the last chapter, we will talk about the process of the realisation, the electronics that we used, the designing of the prototype the simulation of the PID, finally, the main objective is the realization and the application of different methods of stabilization in an inverted pendulum

Presentation of inverted pendulum

1.1 Introduction

The inverted pendulum system is a favorite experiment in control system labs, it occupies an important place in the industry, which has been the subject of great discussion throughout the last few years, it is also a simplified representation of a lot of systems like rockets, there are many types of inverted pendulums and it is used in many domains.

1.2 Definition

The inverted pendulum is an unstable system consequently the stability and sensitivity are a strong function of flexure stiffness [3], which is affected by external factors, it has a single input and multiple outputs, and it is a non-linear system due to its dynamic structure and friction forces, and this dynamic made it a very interesting and widely studied system in the community of automation engineers, which explains the existence of several aspects and inverted pendulum shapes. For that, it's a didactic tool and it is often used to test the performance and robustness of new control laws.

1.3 Function principles

The principle of operation is very simple in theory when the pendulum is in the stable equilibrium point (simple pendulum), the cart moves to the right and left, due to these movements it will create a kinetic force that pouch potential energy to increase making the stem move (rotate) until it reaches the unstable equilibrium point or just reach stabilisation domain, it exactly when balancing begins.

When the pendulum tilts to the left, the carriage catches it by moving to the left and vice-versa until the pendulum is stabilized and we use the control commands to ensure accuracy, speed, and stabilization.

1.4 Domain based on inverted pendulum

In this section, we cite some real-world applications of this concept in various domains:

1.4.1 Segway

The Segway is a two-wheeled self-balancing personal transportation machine, invented by dean Kamen (Figure 1.1) in 2001 [4], the dynamics of the Segway are identical to the inverted pendulum, it's built to stay balanced, designed to mirror the process of human walking, there are many models of Segway, the first in Figure 1.2 (Aug 2015) was controlled by hands, unlike the second in Figure 1.3 (2021) who was controlled by bending.



Figure 1.1: dean kamen



Figure 1.2: Segway x2 SE



Figure 1.3: ninebot by segway

1.4.2 Space

Nasa and SpaceX use the design and the dynamic of the inverted pendulum in the rocket's domain, the launch [5], and the landing of a rocket are similar to the inverted pendulum concepts. There is an inverted pendulum thrust stand for high-power electric propulsion was created to make the launch of the rocket more stable.



Figure 1.4: launching a rocket

1.4.3 Construction

The new construction techniques in the field of civil engineering use the principle of the inverted pendulum for creating a certain resistance to the buildings against earthquakes by integrating a kind of resort below them in the vertical position according to the laws of the inverted pendulums [6].

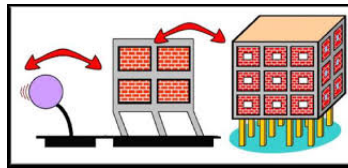


Figure 1.5: anti-seismic building

1.4.4 The humanoid robots

Humanoid robots are robots resembling the human body and they get highly efficient in maintaining standing balance under different perturbations by modeling techniques used in the linear inverted pendulum model [7].

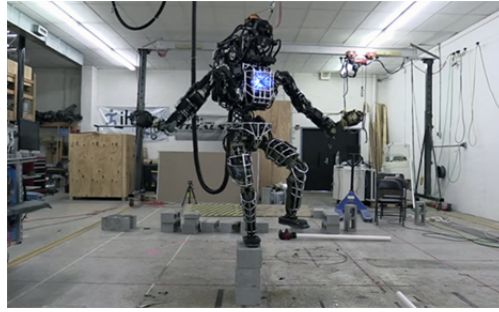


Figure 1.6: Humanoid robots

1.4.5 Self-balancing robots

The invention of self-balancing robots has been a massive milestone in the history of robotics. It works on the same phenomena as that of an inverted pendulum, it needs to maintain its upright position [8].

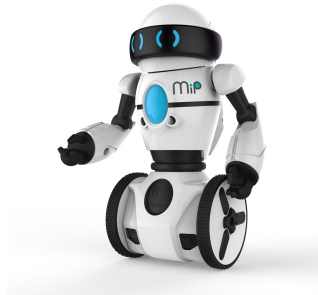


Figure 1.7: self-balancing robots

1.4.6 Human body

The man is seen as a double inverted pendulum in series, whose two main axes of rotation are the ankles and the hips [9]. When we are in a standing position, our joints work non-stop to keep us there.

Maintaining balance is a human ability. In order to maintain a vertical posture, the nervous system stimulates the musculoskeletal system to continuous motion. This movement is in a rocked slightly sagittal plane and frontal reminds their behavior in the inverted pendulum.

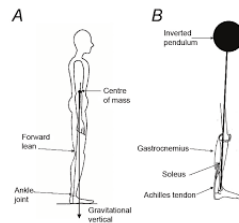


Figure 1.8: Biomechanical model for human balance and walking as an inverted pendulum.

1.4.7 Metronome

Is a device that produces an audible click or other sounds at a regular interval that can be set by the user, typically in beats per minute (BPM). Metronomes may include synchronized visual motion. Musicians use the device to practice playing with a regular pulse.

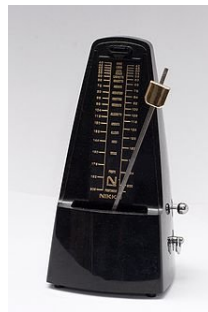


Figure 1.9: metronome

1.4.8 Medical

Robotics for the disabled embodies all the technological applications in the field of robotics that seek to improve the criterion of autonomy for each type of disability. Leg impairments arise from injuries, cancer, arthritis, and other conditions, to reduce the degree of disability the engineer [10] (like EVOM* company which is in the study phase of the possibility of creating a new automated wheelchair that can provide the disabled with better comfort in their daily lives.) created the VERTICALIZER (the stander) based on the physics and the dynamic of the inverted pendulum.

EVOM: it is a joint-stock company active for 9 years, it is specialized in the sector of engineering and technical studies. In the year 2017, it realizes a turnover of 37 800,00 €.



Figure 1.10: ibot



Figure 1.11: verticalizer

1.4.9 Games

Boxing bot

There is a game of robot boxing named boxing bot which was an application of mobile inverted pendulum systems, the boxing bot has the combined structure of a mobile robot and an inverted pendulum system.

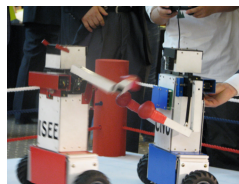


Figure 1.12: boxing bot

Space gun

It's a game in MOSTALAND, there are two parts that behave just like the simple pendulum, and in the balance point unstable they behave like the inverted pendulum.



Figure 1.13: space gun game in mostaland

1.4.10 Drones

A drone is an unmanned aircraft. A drone is a flying robot that can be remotely controlled using software-controlled flight plans in its systems, that work in conjunction with on-board sensors and a global positioning system (GPS). So to pouch the drone to stay stable and maintain its position we need to apply a control similar to the inverted pendulum.

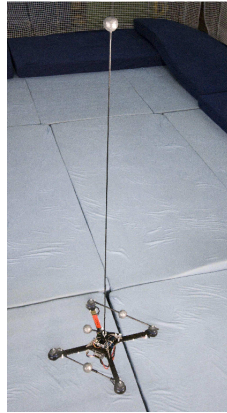


Figure 1.14: A flying inverted pendulum on a drone

1.4.11 Overhead crane

Overhead cranes are widely used in industry for their capability of moving heavy payloads despite their relative simplicity and low cost. Much research effort has been spent in the last thirty years, in devising suitable controllers for these systems, in order to increase their safety and productivity. In fact, the main control requirement, in addition to minimizing the traveling time or the energy, is to reduce the residual oscillation which is in general a difficult task for a human operator [11] . However, it has to be recognized that the simplest approach is to apply an open-loop controller, in many cases, considering the crane as a simple pendulum is not sufficient in order to obtain accurate modeling, which is particularly relevant when open-loop techniques are implemented. Indeed, a double pendulum model can capture much more precisely the dynamics of an overhead crane in those cases where the mass of the hook and the distance between the centers of mass of hook and payload are not negligible.



Figure 1.15: overhead crane

1.4.12 Self-balancing unicycle

The Self-Balancing Unicycle is a fully-powered vehicle that zips along at a speed of 12,5 miles per hour. With a rechargeable electric motor and gyro-stabilized pitch control based on inverted pendulum stabilization, riding the self-balancing unicycle is much easier than an ordinary unicycle and has tons of fun.



Figure 1.16: self-balancing unicycle

1.4.13 Stability of ships and submarines

To stabilize the ships and submarines we need to understand the mechanism of the inverted pendulum [12].

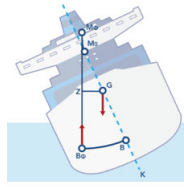


Figure 1.17: ship stability

1.4.14 Researches

Because of its complexity, the inverted pendulum is considered one of the best systems for testing modern control laws and gives the best place to see the impact of the theoretical work, and this is our main goal in this work.

1.5 Types of pendulums

1.5.1 Simple pendulum

A simple pendulum is composed of weight, or bob, hanging freely from the end of a string or bar. Gravity pulls the bob in a downward arc, causing it to swing. This type of pendulum is the most common and can be seen in clocks, metronomes, and seismometers. It was created around 1602 by Galileo Galilei.



Figure 1.18: simple pendulum

1.5.2 Newton pendulum

Newton's pendulum or Newton's cradle is an instrument that demonstrates the conservation of energy and the conservation of momentum, was created in 1687 by Newton.

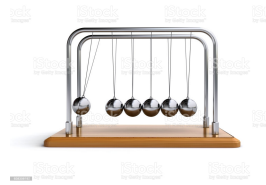


Figure 1.19: newton pendulum

1.5.3 The elastic pendulum

It is just like the simple pendulum but instead of the string there is a spring, it's used in bungee jumping.

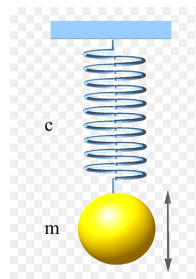


Figure 1.20: the elastic pendulum

1.5.4 The Foucault pendulum

A Foucault pendulum is a type of simple pendulum that swings in two dimensions. This pendulum was first developed by Jean-Bernard Leon Foucault in 1851. Once the Foucault pendulum is set in motion, its swing will tend to rotate clockwise in a circle over about a day and a half. Foucault's pendulum was the first demonstration of the rotation of the earth that did not require astronomical observation [13].



Figure 1.21: the Foucault pendulum

1.5.5 Double pendulum

A double pendulum consists of two simple pendulums, one suspended from the other. It is also called a chaotic pendulum since its motions become more chaotic the larger, they are. A double pendulum acts similar to a simple pendulum for small motions but becomes less predictable as the motions increase in size. The motion of the first pendulum tends to throw the second one in unexpected ways. Double pendulums are used primarily in mathematical simulations.



Figure 1.22: double pendulum

1.5.6 The torsion pendulum

A torsion pendulum is analogous to a mass-spring oscillator. Instead of a mass at the end of a helical spring, which oscillates back and forth along a straight line, however, it has a mass at the end of a torsion wire, which rotates back and forth.



Figure 1.23: torsion pendulum



Figure 1.24: torsion pendulum clock

1.5.7 Classical inverted pendulum (inverted pendulum on a cart)

It's just like the simple pendulum, but in a cart, it consists to apply a force on the cart to push the pendulum to maintain its upright position.

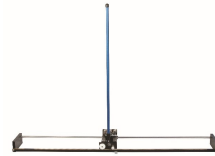


Figure 1.25: inverted pendulum on a cart

1.5.8 Cascading inverted double pendulum

It has the same principle as the simple pendulum, the only difference lies in the fact that it has two freely rotating rods.



Figure 1.26: Cascading inverted double pendulum on a cart

1.5.9 Parallel inverted double pendulum

In this case, the cart supports two independent rods.

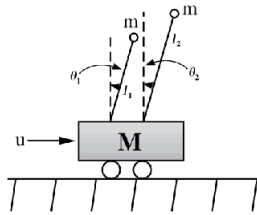


Figure 1.27: parallel inverted double pendulum

1.5.10 Furuta's inverted pendulum (rotary inverted pendulum)

It is composed of an arm operated by rotation in the horizontal plane, at its end is added an inverted pendulum mounted in an unstable balance. The infinite rotation of the arm ensures the stabilization and maintenance of the pendulum around the vertical at the point of unstable equilibrium.



Figure 1.28: rotary inverted pendulum

1.5.11 The flywheel inverted pendulum

It is a system composed of two mechanical bodies: an inverted pendulum in free rotation around a pivot linked to the frame and an actuated flywheel (inertia wheel) whose center of mass coincides with the end of the pendulum. The principle of operation of this system is based on the rotational movement of the steering wheel of inertia which causes, by the dynamic effects that it induces, the rotation of the pendulum. The purpose of the control is to stabilize the pendulum around the vertical at the point of unstable equilibrium and maintain it in this state.



Figure 1.29: the flywheel inverted pendulum

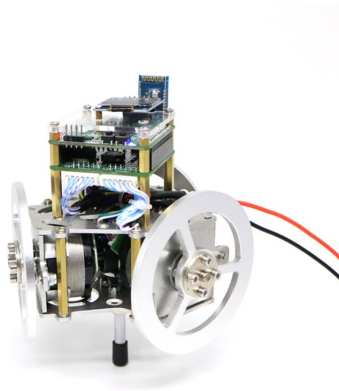


Figure 1.30: 2D flywheel inverted pendulum

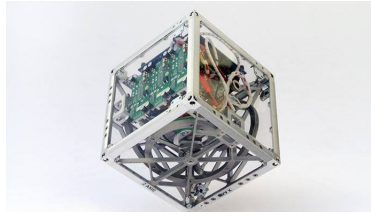


Figure 1.31: 3D flywheel inverted pendulum

1.6 Conclusion

This chapter makes us understand the different types of the inverted pendulum and it is an important system that is used in many fields so much so that we can't accomplish the best degree of stabilization without understanding better the principle of the inverted pendulum system. From the types that we had talked about we chose the inverted pendulum on a cart to study its dynamics in the next chapter.

Chapter 2

The mathematical modelling of an inverted pendulum on a cart and the DC motor

2.1 Introduction

After we present the inverted pendulum system and its applications, we need to design a controller to stabilize the pendulum in its upright position, and to do that we need to derive the mathematical model of an inverted pendulum on a cart. There are two methods to derive the mathematical model of the system, the first is by applying Newton's laws, and the second is by applying Lagrange's equations, in this work we will do it by the first method. The most common device used as an actuator in mechanical control is the DC motor, we need a DC motor to drive the cart to move the cart to stabilize the pendulum, so we need to find the model of the DC motor. In the modeling of the system, we also need to study the constraints.

2.2 Derivation of equations of motion for the inverted pendulum system

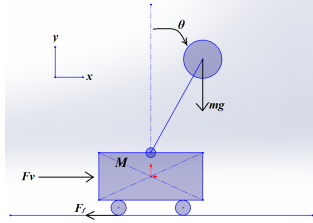


Figure 2.1: schematic of the inverted pendulum on a cart

Firstly, we need to describe the parameters that characterize the system shown in Figure 2.1 so where M is the mass of the cart in kilograms, m is the mass of a pendulum in kilograms, l is the length of a rod that doesn't have a mass, d_0 is the friction coefficient between a cart and ground, I is the inertial moment of a pendulum with respect to the center of gravity, g is the acceleration of gravity in $m.s^{-2}$, θ is the angle of a pendulum from the vertical line in radians, x is the displacement of a cart, and $F_v = u$ is the input force applying to a cart in newton. Let x_g be the horizontal component of co-ordinates of the Centre of Gravity and y_g be the vertical component of coordinates of the Center of gravity.

$$x_g = x + l \sin \theta \quad (2.1)$$

$$y_g = l \cos \theta \quad (2.2)$$

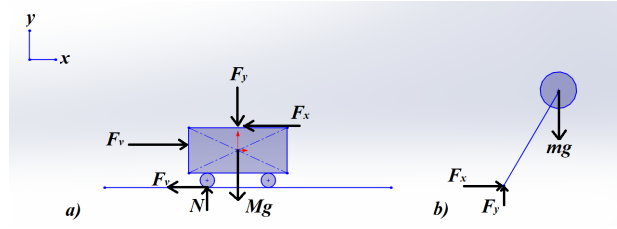


Figure 2.2: Free Body Diagrams of (a) the Cart and (b) the Pendulum

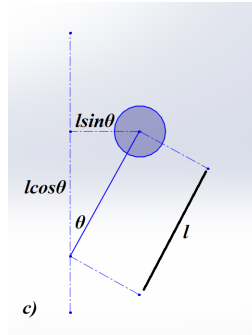


Figure 2.3: Free Body Diagrams of (c) distances

Let's analyze the translational motion first. Using Newton's First law of motion we get that the net force exerted on an object equals the product of mass and its acceleration.

$$F = m.a$$

2.2.1 Horizontal forces

So for the horizontal motion of the cart we have

$$\Sigma F = M \frac{d^2 x}{dt^2} \quad (2.3)$$

As you can see in fig 2.2 there are only 3 forces acting in the horizontal direction

$$F_v - F_x - F_f = M \frac{d^2 x}{dt^2} \quad (2.4)$$

We can write the friction force F_f as

$$F_f = d_0 \frac{dx}{dt} \quad (2.5)$$

Chapter 2. The mathematical modelling of an inverted pendulum on a cart and the DC motor

as the same, the horizontal motion of the pendulum F_x can be written as

$$F_x = m \frac{d^2 x_g}{dt^2} \quad (2.6)$$

By determining the derivative of x_g (in equation 2.1) , surely we can simplify the derivative in the equation 2.6 .

$$\begin{aligned} \frac{dx_g}{dt} &= \frac{d(x + l \sin \theta)}{dt} \\ &= \frac{dx}{dt} + l \cos \theta \frac{d\theta}{dt} \end{aligned} \quad (2.7)$$

And also by differentiating Equation 2.7, we get the second order derivative

$$\begin{aligned} \frac{d^2 x_g}{dt^2} &= \frac{d}{dt} \left(\frac{dx}{dt} + l \cos \theta \frac{d\theta}{dt} \right) \\ &= \frac{d^2 x}{dt^2} + l \frac{d}{dt} \left(\cos \theta \frac{d\theta}{dt} \right) \\ &= \frac{d^2 x}{dt^2} + l \left(\frac{d \cos \theta}{dt} \frac{d\theta}{dt} + \cos \theta \frac{d^2 \theta}{dt^2} \right) \\ &= \frac{d^2 x}{dt^2} + l \left(-\sin \theta \left(\frac{d\theta}{dt} \right)^2 + \cos \theta \frac{d^2 \theta}{dt^2} \right) \\ &= \frac{d^2 x}{dt^2} - l \sin \theta \left(\frac{d\theta}{dt} \right)^2 + l \cos \theta \frac{d^2 \theta}{dt^2} \end{aligned} \quad (2.8)$$

Substituting Equation 2.8 into 2.6 gives the horizontal motion of the pendulum

$$F_x = m \left(\frac{d^2 x}{dt^2} - l \sin \theta \left(\frac{d\theta}{dt} \right)^2 + l \cos \theta \frac{d^2 \theta}{dt^2} \right) \quad (2.9)$$

Combining Equation 2.9 and 2.5 into 2.4, give

$$\begin{aligned} M \frac{d^2 x}{dt^2} &= F_v - m \left(\frac{d^2 x}{dt^2} - l \sin \theta \left(\frac{d\theta}{dt} \right)^2 + l \cos \theta \frac{d^2 \theta}{dt^2} \right) - d_0 \frac{dx}{dt} \\ M \frac{d^2 x}{dt^2} &= F_v - m \frac{d^2 x}{dt^2} + ml \sin \theta \left(\frac{d\theta}{dt} \right)^2 - ml \cos \theta \frac{d^2 \theta}{dt^2} - d_0 \frac{dx}{dt} \end{aligned} \quad (2.10)$$

the horizontal motion of the cart can be written as

$$(M + m)\ddot{x} + d_0\dot{x} = F_v + ml \sin \theta \dot{\theta}^2 - ml \cos \theta \ddot{\theta} \quad (2.11)$$

Vertical forces

For the vertical motion, we have

$$F_y - mg = m \frac{d^2 y_g}{dt^2} \quad (2.12)$$

As the same work that we did in the horizontal case, the derivative on the right in Equation 2.12 can be written as

$$\frac{dy_g}{dt} = \frac{d(l \cos \theta)}{dt} = -l \sin \theta \frac{d\theta}{dt} \quad (2.13)$$

$$\begin{aligned} \frac{d^2 y_g}{dt^2} &= \frac{d}{dt} \left(-l \sin \theta \frac{d\theta}{dt} \right) \\ &= -l \left(\frac{d \sin \theta}{dt} \frac{d\theta}{dt} + \sin \theta \frac{d^2 \theta}{dt^2} \right) \\ &= -l \left(\cos \theta \left(\frac{d\theta}{dt} \right)^2 + \sin \theta \frac{d^2 \theta}{dt^2} \right) \\ &= -l \cos \theta \left(\frac{d\theta}{dt} \right)^2 - l \sin \theta \frac{d^2 \theta}{dt^2} \end{aligned} \quad (2.14)$$

replacing the Equation 2.14 into the Equation 2.12 we get

$$F_y - mg = m \left[-l \cos \theta \left(\frac{d\theta}{dt} \right)^2 - l \sin \theta \frac{d^2 \theta}{dt^2} \right]$$

So the vertical reaction force F_y can be written as

$$F_y = mg + m \left[-l \cos \theta \left(\frac{d\theta}{dt} \right)^2 - l \sin \theta \frac{d^2 \theta}{dt^2} \right] \quad (2.15)$$

The relationship between the moment applied to an object and its angular acceleration

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is given by the following relationship

$$\Sigma \bar{M} = I \frac{d^2\theta}{dt^2} \quad (2.16)$$

where I is the angular momentum of the object and \bar{M} is the moment due to a given force and defined as

$$\bar{M} = \vec{F} \times \vec{r} \quad (2.17)$$

where \vec{F} is the force vector, \vec{r} is the position vector of the object concerning the point about which the moments begin summed. For the pendulum, the equation 2.16 can be written as

$$F_y l \sin \theta - F_x l \cos \theta = I \frac{d^2\theta}{dt^2} \quad (2.18)$$

By substituting Equation 2.15 for F_y and Equation 2.9 for F_x into Equation 2.18 we get

$$\begin{aligned} & \left\{ mg + m \left[-l \cos \theta \left(\frac{d\theta}{dt} \right)^2 - l \sin \theta \frac{d^2\theta}{dt^2} \right] \right\} l \sin \theta \\ & - \left\{ m \left[\frac{d^2x}{dt^2} - l \sin \theta \left(\frac{d\theta}{dt} \right)^2 + l \cos \theta \frac{d^2\theta}{dt^2} \right] \right\} l \cos \theta = I \frac{d^2\theta}{dt^2} \end{aligned} \quad (2.19)$$

with some simplifications it become

$$\begin{aligned} & mgl \sin \theta - ml^2 \sin \theta \cos \theta \left(\frac{d\theta}{dt} \right)^2 - ml^2 \sin^2 \theta \frac{d^2\theta}{dt^2} \\ & - ml \cos \theta \frac{d^2x}{dt^2} + ml^2 \cos \theta \sin \theta \left(\frac{d\theta}{dt} \right)^2 - ml^2 \cos^2 \theta \frac{d^2\theta}{dt^2} = I \frac{d^2\theta}{dt^2} \end{aligned} \quad (2.20)$$

$$mgl \sin \theta - ml^2 (\sin^2 \theta + \cos^2 \theta) \frac{d^2\theta}{dt^2} - ml \cos \theta \frac{d^2x}{dt^2} = I \frac{d^2\theta}{dt^2} \quad (2.21)$$

$$mgl \sin \theta - ml \cos \theta \frac{d^2x}{dt^2} = (I + ml^2) \frac{d^2\theta}{dt^2} \quad (2.22)$$

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The final equation for the angular position is given as

$$(I + ml^2) \frac{d^2\theta}{dt^2} = mgl \sin \theta - ml \cos \theta \frac{d^2x}{dt^2}$$

$$(I + ml^2)\ddot{\theta} = mgl \sin \theta - ml \cos \theta \ddot{x} \quad (2.23)$$

Therefore the equations of motion for the inverted pendulum on a cart is

$$\begin{cases} (M + m)\ddot{x} + d_0\dot{x} = F_v + ml \sin \theta \dot{\theta}^2 - ml \cos \theta \ddot{\theta} \\ (I + ml^2)\ddot{\theta} = mgl \sin \theta - ml \cos \theta \ddot{x} \end{cases}$$

For our system, the relationship between the force due to the voltage can be written as

$$F_v = \gamma V \quad (2.24)$$

where γ is the conversion factor and V is the applied voltage in volts

The above equations can be arranged with respect to \ddot{x} and $\ddot{\theta}$

$$\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \frac{1}{\Lambda} \begin{bmatrix} -ml \cos \theta & I + ml^2 \\ M + m & -ml \cos \theta \end{bmatrix} \begin{bmatrix} mgl \sin \theta \\ F_v + ml \dot{\theta}^2 \sin \theta - d_0\dot{x} \end{bmatrix}$$

where $\Lambda = (I + ml^2)(M + m) - (ml \cos \theta)^2$

Define the state variables to obtain the state space model

$$x_1 = x$$

$$x_2 = \dot{x}$$

$$x_3 = \theta$$

we set the inertia moment of a pendulum I as 0 then the state space equation of an inverted pendulum on a cart is as follows

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{M + m \sin^2 x_3} \{-mg \sin x_3 \cos x_3 + mlx_4^2 \sin x_3 - d_0x_2\} + \frac{1}{M + m \sin^2 x_3} F_v \\ \dot{x}_3 &= x_4 \end{aligned} \quad (2.25)$$

$$\begin{aligned} \dot{x}_4 &= \frac{1}{l(M + m \sin^2 x_3)} \{-mlx_4^2 \sin x_3 \cos x_3 + d_0x_2 \cos x_3 + (M + m)g \sin x_3\} \\ &\quad + \frac{-\cos x_3}{l(M + m \sin^2 x_3)} F_v \end{aligned}$$

Nb: We define the position of a cart (x_1) and the angle of a pendulum (x_3) as outputs of the system.

2.3 The transfer function of the inverted pendulum on a cart

a transfer function represents the relationship between a single input and a single output at a time [14]. We linearise the equations of motion that we get by small-angle approximation

$$\theta \dot{\theta} = 0; \cos \theta = -1; \sin \theta = \theta$$

we get

$$F_v - d_0\dot{x} = (M + m)\ddot{x} - ml\ddot{\theta} \quad (2.26)$$

$$(I + ml^2)\ddot{\theta} = mgl\theta + ml\ddot{x} \quad (2.27)$$

Taking Laplace Transform of the equations (2.26) and (2.27) we get

$$F_v(s) - d_0sX(s) = (M + m)s^2X(s) - mls^2\theta(s) \quad (2.28)$$

$$-m ls^2 X(s) + (I + ml^2)s^2\theta(s) = mgl\theta(s) \quad (2.29)$$

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To find our first transfer function for the output $\Theta(s)$ and an input of $F_v(s)$ we need to eliminate $X(s)$ from the equations 2.28. Solve the equation 2.29 for $X(s)$ we get.

$$X(s) = \left[\frac{I + ml^2}{ml} - \frac{g}{s^2} \right] \Theta(s) \quad (2.30)$$

Then substitute the above (equation 2.30) into the equation 2.28.

$$F_v(s) = (M + m) \left[\frac{I + ml^2}{ml} - \frac{g}{s^2} \right] \Theta(s)s^2 - ml\Theta(s)s^2 + d_0 \left[\frac{I + ml^2}{ml} - \frac{g}{s^2} \right] \Theta(s)s$$

We get the first transfer function by extracting Θ as the output so

$$P_{pend}(s) = \frac{\Theta(s)}{F_v(s)} = \frac{\frac{ml}{q}s}{s^3 + \frac{d_0(I+ml^2)}{q}s^2 - \frac{(M+m)mgl}{q}s - \frac{d_0mgl}{q}}$$

where,

$$q = [(M + m)(I + ml^2) - (ml)^2]$$

Second, the transfer function with the cart position $X(s)$ as the output.

$$P_{cart}(s) = \frac{X(s)}{F_v(s)} = \frac{\frac{(I+ml^2)s^2 - mgl}{q}}{s^4 + \frac{d_0(I+ml^2)}{q}s^3 - \frac{(M+m)mgl}{q}s^2 - \frac{d_0mgl}{q}s}$$

2.4 Partial feedback linearisation

Definition of partial feedback linearisation

[15] Consider non-linear single input system

$$\dot{x} = f(x) + g(x)u$$

where $x \in \mathfrak{R}^n, u \in \mathfrak{R}$, in a neighbourhood $U_{x^*} \subset \mathfrak{R}^n$ of an equilibrium point x^* corresponding to $u = 0$, i.e. $f(x^*) = 0$. f and g are assumed to be smooth vector fields defined on \mathfrak{R}^n with

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$g(x^*) \neq 0$. The system is said to be locally partial state feedback linearisable with index $r \leq n$ if it is locally feedback equivalent to the partially linear system

$$\dot{\xi} = \chi(\xi, z), \xi \in \mathfrak{R}^{n-r}, z \in \mathfrak{R}^r,$$

$$\dot{z} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix} z + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} v.$$

After we get the mathematical model, we simplify it by partial feedback linearisation, which divides it into a dominant subsystem and dominated subsystem.

Equation 2.25 is represented as a control affine form.

$$\begin{aligned} \dot{x}_1 &= x_2 = f_1(x) \\ \dot{x}_2 &= f_2(x) + g_2(x)u \\ \dot{x}_3 &= x_4 = f_3(x) \\ \dot{x}_4 &= f_4(x) + g_4(x)u \end{aligned} \tag{2.31}$$

Where u is F_v and

$$\begin{aligned} f_2 &= \frac{1}{M + m \sin^2 x_3} \{-mg \sin x_3 \cos x_3 + mlx_4^2 \sin x_3 - d_0x_2\} \\ g_2 &= \frac{1}{M + m \sin^2 x_3} \\ f_4 &= \frac{1}{l(M + m \sin^2 x_3)} \{-mlx_4^2 \sin x_3 \cos x_3 + d_0x_2 \cos x_3 + (M + m)g \sin x_3\} \\ g_4 &= \frac{-\cos x_3}{l(M + m \sin^2 x_3)} \end{aligned} \tag{2.32}$$

with $f_2(0) = f_4(0) = 0, g_2(0) \neq 0, g_4(0) \neq 0$.

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According to the definition, we apply partial feedback linearisation to the system 2.31

i.e. set input u as

$$u = \frac{1}{g_4(x)}[-f_4(x) + v].$$

we set u like this to eliminate $g_4(x)$ and $f_4(x)$ from the \dot{x}_4 . Then the system is transformed as follows

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_2(x) + \frac{g_2(x)}{g_4(x)}[-f_4(x) + v] \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= v \end{aligned} \tag{2.33}$$

The acceleration of the cart was described by the second equation of de 2.33. by using the equations in 2.32 we rewrite the equation 2.33 give

$$\begin{aligned} \dot{x}_2 &= \frac{1}{M + m \sin^2 x_3} \left\{ \frac{1}{\cos x_3} [(M + m)g \sin x_3 - mg \sin x_3 \cos^2 x_3] \right\} - \frac{v}{\cos x_3} \\ &= \frac{1}{M + m \sin^2 x_3} \left\{ \frac{1}{\cos x_3} [(M + m)g \sin x_3 - mg \sin x_3(1 - \sin^2 x_3)] \right\} - \frac{v}{\cos x_3} \\ &= \frac{1}{M + m \sin^2 x_3} \left[\frac{1}{\cos x_3} (Mg \sin x_3 + mg \sin^3 x_3) \right] - \frac{v}{\cos x_3} \\ &= \frac{1}{M + m \sin^2 x_3} \left[\frac{g \sin x_3}{\cos x_3} (M + mg \sin^2 x_3) \right] - \frac{v}{\cos x_3} \\ &= \frac{1}{\cos x_3} (g \sin x_3 - v). \end{aligned}$$

the system 2.33 become as follows.

$$Part1 \left\{ \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{\cos x_3} (g \sin x_3 - v) \end{aligned} \right. \tag{2.34}$$

$$Part2 \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = v \end{cases} \quad (2.35)$$

Looking into the above system, the relationship between input v and output x_3 is linear and the states of part 1 do not appear in part 2 explicitly, while part 1 depends on part 2 too much. If the state variables of part 2 approach 0, part 1 becomes a linear system,

$$\dot{x}_1 = x_2 \quad (2.36)$$

$$\dot{x}_2 = gx_3 - v$$

Therefore part 2 is the dominant subsystem that determines the aspect of the whole system, and part 1 is the dominated subsystem.

2.5 Modelling of DC motor

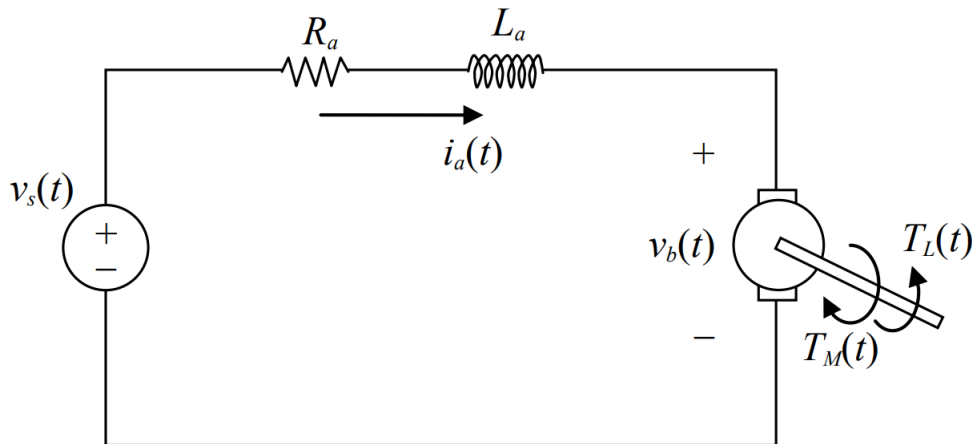


Figure 2.4: schema electric of DC motor

The electrical circuit of the DC motor is shown in figure 2.4, where R_a is the armature resistance and L_a winding leakage inductance, we define $i_a(t)$ as the armature current, $v - b(t)$ is the back emf voltage and $v_s(t)$ is the voltage source. we use Kirchoff's voltage law to describe

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the electrical equation of the DC motor [16]

$$R_a i_a(t) + L_a \frac{di_a(t)}{dt} + v_b(t) = v_s(t) \quad (2.37)$$

the back emf voltage $v_b(t)$ given as

$$v_b(t) = k_b w(t) \quad (2.38)$$

where k_b is the back emf constant and $W(t)$ is the angular velocity $\omega(t) = \dot{\theta}_m$ and θ_m is the angular position. In addition, the torque T_M generated by a DC motor is proportional to the armature current and the strength of the magnetic field. In our work, we will assume that the magnetic field is constant and, therefore, that the motor torque is proportional to only the armature current i_a by a constant factor K_T as shown in equation 2.39 where

$$T_M(t) = k_T i_a(t) \quad (2.39)$$

where K_T is the torque constant.

in our system we suppose the input voltage $v_s(t) = V_s$ is a constant so $i_a(t)$, $\omega(t)$ and $T_M(t)$ become respectively I_a , Ω and T are constant. From 2.37 to 2.39 we get

$$R_a I_a + k_b \Omega = V_s \quad (2.40)$$

$$T = k_T I_a \quad (2.41)$$

after the law the conservation of power

$$V_s I_a = T \Omega + R_a I_a^2 \quad (2.42)$$

Substituting v_s in (2.40) into (2.42) yields

$$T = k_b I_a \quad (2.43)$$

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From (2.41) and (2.43), we know that $k_T = k_b$. From (2.38), we can rewrite (2.37) and (2.39) as

$$R_a i_a(t) + L_a \frac{di_a(t)}{dt} + k\omega(t) = v_s(t) \quad (2.44)$$

$$T_M(t) = k i_a(t) \quad (2.45)$$

where $k = k_b = k_T$. Besides, if the DC motor is used to drive an external torque $T_L(t)$ of payload then its mechanical behavior is described as

$$J_M \frac{d\omega(t)}{dt} + B_M \omega(t) = T_M(t) - T_L(t) \quad (2.46)$$

where J_M is the rotor moment of inertia and B_m is the frictional coefficient. the dynamic equations of the dc motor are equation 2.46 and 2.44

2.6 DC motor transfer function

We apply the Laplace transform to the modeling equations 2.46 and 2.44 ,they become

$$s(J_M s + B_M)\Theta_m(s) = K_i$$

$$(L_a s + R_a)I_a(s) = V_s(s) - K s\Theta_m(s)$$

we consider the armature voltage is the input and the rotational speed is the output, we eliminate $I(s)$ between the two equations we get

$$P(s) = \frac{\dot{\Theta}_m}{V_s(s)} = \frac{K}{(J_M s + B_M)(L_a s + R_a) + K^2}$$

2.7 The study of constraints

In our work we encounter obstacles that hinder results, we consider them as constraints, the schematic of the inverted pendulum in the previous section is the perfect model, but in the

Chapter 2. The mathematical modelling of an inverted pendulum on a cart and the DC motor

reality, we have another model (see Fig 2.5)

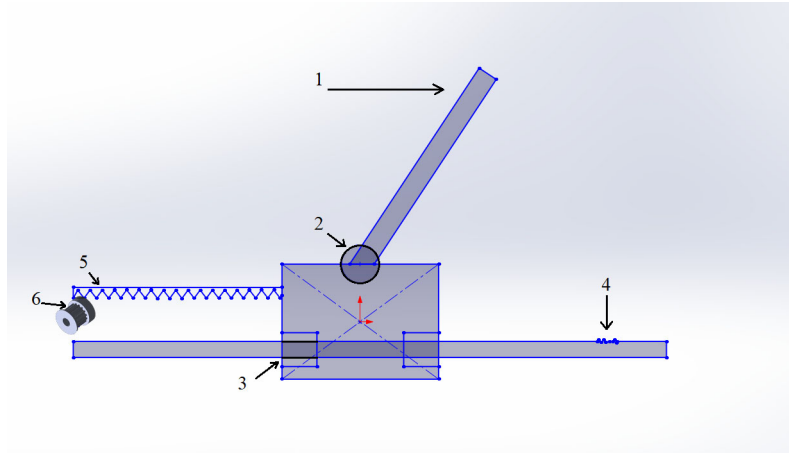


Figure 2.5: schematic of the real model with constraints

where

1 is the environmental constraint represented in air disturbance.

2, 3, 4, 5, and 6 are all physical constraints.

2 and 3 are linear friction between the linear bearing and the rails (3), and the friction between the pendulum and the joint (2).

4 it is the surface of the rail (see Fig 2.7), it affects the movement of the cart and makes some undesirable vibrations but we can overcome it by using oil.

5 represents the belt that is moved by a DC motor to move the cart, it is the link that helps us to apply a force so we can control the system, we consider it a constraint because its teeth can be damaged (see Fig 2.6) so it can make us lose the acceleration, so it can affect the controller.

6 is a teeth pulley, it also affects the belt. also if it does not stick well to the motor, the motor may turn on its self



Figure 2.6: The surface of the rails

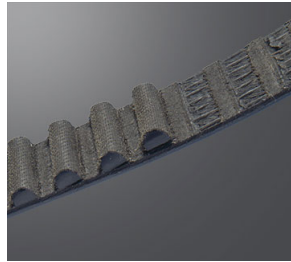


Figure 2.7: damaged belt teeth



Figure 2.8: Printing failure

There are some physical constraints like the axes of the rail not being perfectly parallel, also the pieces that are constructed by the 3D printer may have errors and problems in the construction and the dimensions (see fig 2.8), and the length of the rail is limited in 50cm so the cart can bump into the limits.

Far from physical constraints we have electronic constraints motion them as the error of the sensors, and the limits of the controller that we use (for example Arduino).

In the study of the constraint maybe you confuse between the types of the motor, why we used the DC motor and not the stepper motor, the answer is the DC motor has a very short response

time and quick acceleration, unlike the stepper motor will stop if pouch him to perform high rotation per minute.

2.8 Conclusion

We got our mathematical model by using Newton's laws, we even made a model for the DC motor and these models are going to help us to understand the system and its movement however they will help us to design a controller.

In this modeling, we use partial feedback linearisation because the feedback linearisation has the effect of separating the system into two parts: one linear and controllable and the other nonlinear and uncontrollable. We extracted the transfer function because it will help us to design a PID controller. The study of the constraints helped us to understand the real model of the system that we will need in the realization chapter

Chapter 3

Regulation and control

3.1 Introduction

To maintain stabilization, control is necessary, there are many types of control, in this work, we will discuss and test some of the themes. In our system we must use non-linear control because non-linear control is a current topic, with a variety of powerful methods, we can't use Linear control methods because they are valid only in the vicinity of the operating point, also a linear controller is likely to perform very badly or to be unstable because the non-linearities of the system cannot be properly compensated. The objective of the control in this chapter is to drive the pendulum from the pendent position to the upright position and to keep the pendulum in that position. We use linear control in the stabilization process because we have an approximation and linear model around the point of equilibrium.

3.2 PID controller

Proportional-Integral-Derivative (PID) control method. This type of controller is widely used in industry and does not require an accurate model of the plant or process being controlled. it is combined from three controllers in such a way that it produces a control signal.

As a feedback controller, it delivers the control output at desired levels, and it's traditionally used in process control applications, due to its flexibility and reliability. This type of

control is used to drive a system in the direction of an objective location like in our system.

3.2.1 P-controller

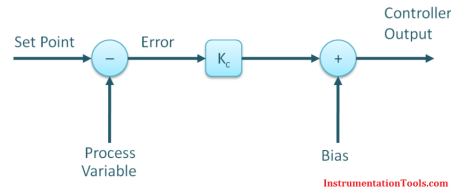


Figure 3.1: P-controller

The proportional or P- controller uses the present state of the system, giving an output that is proportional to the current error $e(t)$. It compares the desired or set point with the actual value or feedback process value. The resulting error is multiplied by a proportional constant to get the output. If the error value is zero, then this controller output is zero. This controller requires biasing or manual reset when used alone. This is because it never reaches the steady-state condition. It provides a stable operation but always maintains the steady-state error. The speed of the response is increased when the proportional constant K_p increases. The control signal $u(t)$ is

$$u(t) = K_p e(t)$$

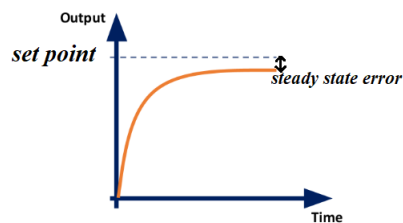


Figure 3.2: (a) P-controller response with small k_p

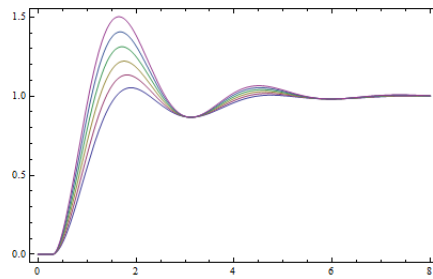


Figure 3.3: (a) P-controller response with different values of big k_p

In these pictures, we see the p-controller response, in the first (a) we have a response with a small K_p , when the p-controller is small it will not get us to the set point. And in the second (b) we use big k_p which will make an overshoot, every time we increase the k_p the overshoot increases, even though the steady-state error stays.

Applications of p-controller

The proportional controller is often used in cases when the system dynamics are unknown, the system is unstable and the required control performance in steady-state operation is not demanding. This controller is common in the design of cascaded control systems, and it is needed for integrating processes. It is used also in operational amplifiers.

The limitation of P-controller

Using P-controller is simple, but often insufficient, it has several limitations, because if k_p is small, error $e(t)$ can be large, if it's large, the system may oscillate i.e. the system becomes unstable especially if the process is *2nd* order or higher, even if the system is stable, and eventually settle down to the steady-state value, the system output may exhibit large overshoot transient behavior and take a long time to settle to its final output value, exhibiting large overshoots. Finally, using P-only control may not give us the tolerance to perturbation required.

3.2.2 Proportional-Integral (PI) control

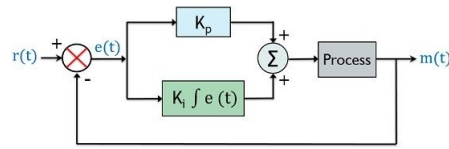


Figure 3.4: Block diagram for PI control

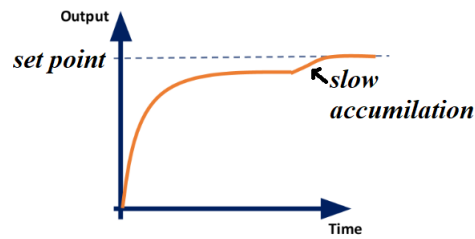
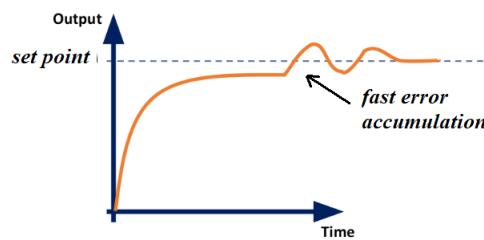
We can add an integral term that is dependent on the integral of the error over time to the proportional controller, and become a new controller known as a PI controller, it is an improvement to the p-controller, its control signal is :

$$u(t) = k_p e(t) + k_i \int e(\tau) d\tau$$

PI controller accumulates error to produce a constant output $u(t)$ that is sufficient to drive the plant to give the desired output value $y(t)$. When that happens, $e(t) = 0$, and yet the output $y(t) = r(t)$ is maintained. It can be seen that this cannot be achieved with P-only controller. It has many advantages but the main advantages are: the accuracy and pi-control eliminate the steady-state error, and can help with the stability of the system, especially if we increase k_p . NB: the integral term in practical electronics and programming we implement is as a summation. The s-domain representation of PI controller can be given as:

$$G(s) = k_p + k_i \frac{1}{s}$$

A PI controller will work with the P term gain $K_p = 0$ (i.e. I-only control), but its response to a step, input would be slower. The integral action will take time to reach the desired value.

Figure 3.5: PI control response with small k_i Figure 3.6: PI control response with big k_i

In the first, we enter a small k_i so the errors accumulate slowly until some of the errors reach a value the system can notice and correct, and in the second with a big value for k_i so it accumulates the errors and multiple them with high value so the system notices it faster so reacts faster and may make overshoots as we see in figure 1.6.

The limitation of PI-control

The PI control is capable of correcting various uncertainties to maintain the reference current until its output is saturated. After saturation, the PI is not able to maintain current and the effect of this on the torque production needs to be clearly understood for the optimal control of the drive system. The PI controller may increase the overshoot because sometimes when you increase the k_p the controller accumulates the errors more so he overshoots the set point, Decreases settling time

3.2.3 Proportional-Derivative control

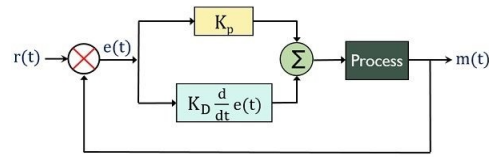


Figure 3.7: Block diagram for PD control

We can add another term for the p-control to include the rate of change of the error , the output of the controller is now proportional to the error $e(t)$ (with a gain of k_p) and to the derivative of the error $\dot{e}(t)$ (with gain of k_d), this known as a proportional-derivative (PD) controller:

$$w(t) = k_p e(t) + k_d \dot{e}(t)$$

The derivative term $\dot{e}(t)$ in computers such as Raspberry Pi is usually calculated by taking the difference between current error value $e[n]$ and the previous error value $e[n - 1]$

$$n = k_d(e[n] - e[n - 1])/\Delta t$$

The main advantages of the PD controllers are : it can reduce the overshoot of a proportional controller because the PD controller takes into account the rate of change in error. it can also improve the system's tolerance to external disturbances. Even it reduces the time of the stability. The limitation of the PD control is it doesn't affect on the steady-state error.

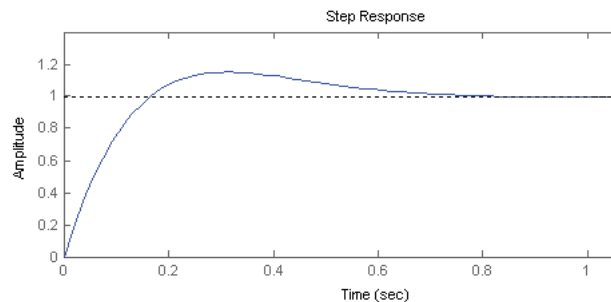


Figure 3.8: Step response with PD controller

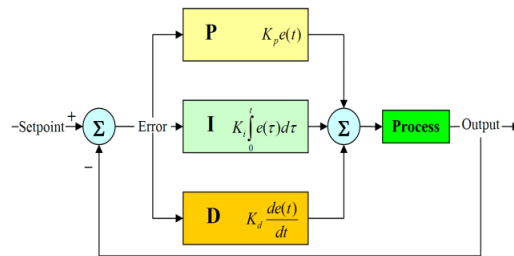


Figure 3.9: block diagram of PID control

3.2.4 PID control

We can combine all three terms to form a PID controller with

$$u(t) = k_p e(t) + k_i \int e(\tau) d\tau + k_d \dot{e}(t)$$

the PID controller has the advantages of all three types of feedback control (P, I, and D) it removes the steady-state error due to me. Reduce the number of overshoots due to I and D, improve the transient response to make it faster, improve stability of the system, and better perturbation tolerance.

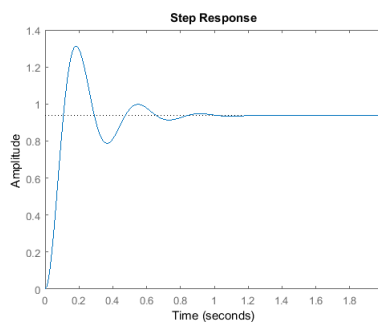


Figure 3.10: step response for PID controller

	Overshoot	Rise/settling time	Steady-state error
$\uparrow k_p$	\uparrow overshoot	\uparrow the response	\downarrow the steady-state error
$\uparrow k_d$	\downarrow overshoot	\uparrow the response	doesn't effect
$\uparrow k_i$	\uparrow overshoot	\downarrow settling time	make the error zero

Table 3.1: The effect of P, I, and D on the overshoot, time of response, and the steady-state error

The effect of P, I, and D Before we consider how to tune a PID controller, you should know the effect of each of increasing K_p , K_i , and K_d on the feedback system behavior in terms of overshoot, settling time and steady-state error (i.e. $e(t)$ value after the system settles to final condition).

3.2.5 Tuning methods

PID control structure is therefore very general. Designing a controller for a given plant or process requires us to determine the correct value for K_p , K_i , and K_d . This process is known as “tuning”. Different types of tuning methods are developed to tune the PID controllers and require much attention from the operator to select the best values of proportional, integral, and derivative gains. Some of these are [17]

Trial and Error Method In this method, we can tune the controller while he is working, we have to set K_i and K_d values to zero and increase the proportional term (K_p) until the system reaches oscillating behavior. Once it is oscillating, adjust K_i (Integral term) so that oscillations stop, and finally adjust D to get a fast response.

Process Reaction Curve Technique It is an open-loop tuning technique. It produces a response when a step input is applied to the system. Initially, we have to apply some control output to the system manually and have to record the response curve. After that, we need to calculate the slope, dead time, and the rise time of the curve, and finally substitute these values in P, I, and D equations to get the gain values of PID terms.

NB: inflection point is the point where the curve stops increasing and starts decreasing.

T_u is lag time. T_a is processed reaction time. Δc_p is variable change.

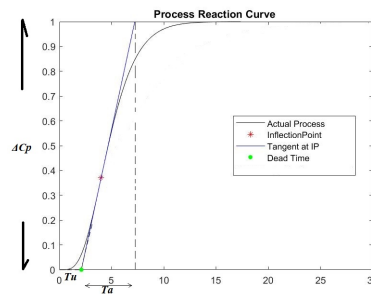


Figure 3.11: process reaction curve

the reaction rate

$$N = \frac{\Delta c_p}{T_a}$$

the lag reaction

$$R = \frac{NL}{\Delta c_p}$$

Zeigler-Nichols method A popular method for tuning P, PI, and PID controllers is the Ziegler–Nichols method. This method starts by zeroing the integral and differential gains and then raising the proportional gain until the system is unstable. The value of K_P at the point of instability is called K_{MAX} ; the frequency of oscillation is f_0 . The method then backs off the proportional gain a predetermined amount and sets the integral and differential gains as a function of f_0 . The P, I, and D gains are set according to Table

	k_p	k_i	k_D
P controller	$0.05K_{max}$	0	0
PI controller	$0.45K_{max}$	$1.2f_0$	0
PID controller	$0.6K_{max}$	$2f_0$	$0.125/f_0$

Table 3.2: Settings for P, I, and D Gains According to the Ziegler–Nichols Method

3.2.6 PID controller for inverted pendulum on a cart

The control of CIP uses two separate controllers, one control for the Pendulum angle and the other control for the Cart position but only one control action is allowed for the system. Hence the control action for the two subsystems are combined with a single control action, F . When a force is given to push the Cart to the left side will cause the Pendulum to move to the right side. Hence the force provided to move the Cart and Pendulum in the same direction should be with opposite sign. The Pendulum angle is controlled by a PID controller and the Cart position is controlled by a PD controller. The Pendulum angle control is F_p the Cart position control is F_c . The total control is represented as F .

3.3 Designing a controller

In this section, we will design a controller which stabilizes the system asymptotically, as we see in chapter 2 the dynamics of the inverted pendulum on a cart are locally equivalent to equation 2.34 and 2.35. Hence we concentrate on the additional control input v and assume that the angle of the pendulum x_3 is restricted within $(-\pi/2, \pi/2)$ [15] If we set $v = -\alpha x_3, -\beta x_4$, where $\alpha > 0$ and $\beta > 0$ are constants, part 2 in 2.34 and 2.35 is exponentially stable, but part 1 may diverge. If we set $v = -\gamma x_1 - \delta x_2 - \alpha x_3 - \beta x_4$ with appropriate gain set $\{\gamma, \delta, \alpha, \beta\}$, the linearised system consisting of part 2 in 2.34 and 2.35 is exponentially stable also, but that is nothing but a linear controller. We want to design a controller which stabilizes the dominant subsystem, prevents the divergence of the dominated subsystem, and finally stabilizes the whole system. Set the additional control input as follows

$$v = -\alpha x_3 - \beta x_4 + \rho \mu,$$

where $\mu = \gamma x_1 + \delta x_2$ with constants $\gamma > 0, \delta > 0$. Here ρ needs to have such properties that it prevents the divergence of the dominated subsystem as long as the dominant subsystem is stabilized, and that it converges to an appropriate constant when the system becomes almost linear.

Theorem 1 For given α, β if there exist constant a, b, c such that

$$\begin{bmatrix} a/2 & c/2 \\ c/2 & b/2 \end{bmatrix} > 0,$$

$$\begin{bmatrix} ac & (-a + \alpha b + \beta c)/2 \\ (-a + \alpha b + \beta c)/2 & \beta b - c \end{bmatrix} > 0,$$

then using

$$\rho = \frac{x_{34}^T N x_{34}}{(\gamma|x_1| + \delta|x_2| + \varepsilon_1)(c|x_3| + b|x_4| + \varepsilon_2)} - \omega,$$

$\|x_{34}\|$ can be ultimately bounded sufficiently small, where

$$\omega = \exp\left[\frac{-(\gamma x_1 + \delta x_2)^2}{\Delta_1^2}\right] \exp\left[\frac{-(cx_3 + bx_4)^2}{\Delta_2^2}\right]$$

, $\varepsilon_1, \varepsilon_2 \geq 1$ and $\Delta_1 \gg 1, 0 < \Delta_2 < 1$.

Proof. We put a Lyapunov function candidate for x_3 and x_4 as follows

$$V_{34} = x_{34}^T M x_{34},$$

$x_{34} = [x_3 x_4]^T$. Derivative is the function with respect to time along the trajectory

$$\begin{aligned} \dot{V}_{34} &= ax_3x_4 + bx_4(-\alpha x_3 - \beta x_4 + \rho\mu) + cx_4^2 + cx_3(-\alpha x_3 - \beta x_4 + \rho\mu) \\ &= -\alpha cx_3^2 - (\beta b - c)x_4^2 - (-a + \alpha b + \beta c)x_3x_4 + \rho\mu(cx_3 + bx_4) \\ &= -x_{34}^T N x_{34} + \rho(\gamma x_1 + \delta x_2)(cx_3 + bx_4) \end{aligned}$$

$$= x_{34}^T N x_{34} \left[\frac{(\gamma x_1 + \delta x_2)(c x_3 + b x_4)}{(\gamma |x_1| + \delta |x_2| + \varepsilon_1)(c |x_3| + b |x_4| + \varepsilon_2)} - 1 \right] - \omega(\gamma x_1 + \delta x_2)(c x_3 + b x_4).$$

In case $(\gamma x_1 + \delta x_2)(c x_3 + b x_4) \geq 0$, $\dot{V}_{34} < 0$. In case $(\gamma x_1 + \delta x_2)(c x_3 + b x_4) < 0$, because $y \times \exp[-\frac{y^2}{\Delta}]$ has maximum value $\pm \sqrt{\frac{\Delta^2}{2e}}$ at $y = \pm \sqrt{\frac{\Delta^2}{2}}$, $|\omega(\gamma x_1 + \delta x_2)(c x_3 + b x_4)| \leq \frac{\Delta_1 \Delta_2}{2e}$
 $|(\gamma |x_1| + \delta |x_2| + \varepsilon_1)(c |x_3| + b |x_4| + \varepsilon_2)|$ Hence \dot{V}_{34} is bounded as follows

$$\dot{V}_{34} \leq x_{34}^T N x_{34} \left[\frac{-|(\gamma x_1 + \delta x_2)(c x_3 + b x_4)|}{(\gamma |x_1| + \delta |x_2| + \varepsilon_1)(c |x_3| + b |x_4| + \varepsilon_2)} - 1 \right] + \frac{\Delta_1 \Delta_2}{2e}.$$

In order to $\dot{V}_{34} \leq 0$ $x_{34}^T N x_{34} > \frac{\Delta_1 \Delta_2}{2e} > \frac{\Delta_1 \Delta_2}{2e[1+\phi(x)]}$,

where

$$\phi(x) = \frac{|(\gamma x_1 + \delta x_2)(c x_3 + b x_4)|}{(\gamma |x_1| + \delta |x_2| + \varepsilon_1)(c |x_3| + b |x_4| + \varepsilon_2)}$$

is restricted within $[0,1]$. That means

$$\dot{V}_{34} < \left(\frac{\Delta_1 \Delta_2}{2e} + \varepsilon \right) [-1 - \phi(x)] + \frac{\Delta_1 \Delta_2}{2e} = -\varepsilon - \phi(x) \left(\frac{\Delta_1 \Delta_2}{2e} + \varepsilon \right) < 0,$$

whenever $x_{34}^T N x_{34} > \frac{\Delta_1 \Delta_2}{2e}$, where $\varepsilon > 0$. We can assign $\Delta_1 \Delta_2$ small arbitrarily, so $\|x_{34}\|$ can be bounded small sufficiently. Moreover there exist class K function, α_1 and α_2 and continuous positive definite function W such that $\alpha_1(\|x_{34}\|) \leq V_{34} \leq \alpha_2(\|x_{34}\|)$ and $\dot{V}_{34} \leq -W(x_{34})$, whenever $x_{34}^T N x_{34} > \frac{\Delta_1 \Delta_2}{2e}$. Hence x_{34} is ultimately bounded [4]. Meanwhile the existence of positive definite matrices M, N can be shown by checking the leading principle minors of the matrices.

$$a > 0, ab - c^2 > 0, ac > 0, ac(\beta b - c) - (-a + ab + \beta c)^2/4 > 0.$$

The set α, β, a, b, c which satisfies the above inequalities exist as follows

$$\alpha > 0,$$

$$\beta > 0,$$

$$a > 0,$$

$$\frac{2\alpha a + a\beta^2}{2a^2} - \frac{1}{2}\sqrt{\frac{4\alpha\beta^2 a^2 + a^2\beta^4}{a^4}} < b < \frac{2\alpha a + a\beta^4}{2a^2} + \frac{1}{2}\sqrt{\frac{4\alpha\beta^2 a^2 + a^2\beta^4}{a^4}}$$

and

$$\frac{a\beta + \alpha\beta b}{2a^2} - 2\sqrt{-\frac{\alpha a^2 - 2\alpha^2 ab + \alpha^3 b^2 - \alpha a\beta^2 b}{(4a + \beta^2)^2}} < c$$

$$c < \frac{a\beta + \alpha\beta b}{4a + \beta^2} + 2\sqrt{-\frac{\alpha a^2 - 2\alpha^2 ab + \alpha^3 b^2 - \alpha a\beta^2 b}{(4a + \beta^2)^2}}$$

for example $\alpha = 14, \beta = 5, a = 9, b = 2, c = 2$. * Theorem 1 implies that the states of the dominant subsystem can be bounded small arbitrarily and that they never escape once the states come into the boundary. Then the system ?? becomes almost linear, or precisely speaking the system becomes a linear system with uncertainty. We can divide the system ?? into the linear part and the The uncertainty part is as follows

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = gx_3 - \tilde{u} + \left(-gx_3 + \tilde{u} + g \tan x_3 - \frac{v}{\cos x_3}\right)$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \tilde{u} + (-\tilde{u} + v)$$

in state space form

$$\dot{x} = Ax + B\tilde{u} + h(x), \quad (3.1)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & g & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}, h(x) = \begin{bmatrix} 0 \\ -gx_3 + \tilde{u} + g \tan x_3 - \frac{v}{\cos x_3} \\ 0 \\ -\tilde{u} + v \end{bmatrix},$$

$$\tilde{u} = -k^T x, k^T = [\gamma \delta \alpha \beta].$$

We investigate the robustness of the system 3.1 in the presence of some non-linear perturbation $h(x)$. Here the pair (A, B) is controllable. The performance index to be minimized is

$$J = \int_0^{\infty} e^{2\eta t} [x^T Q x + u^T R \tilde{u}] dt,$$

where the weighting matrices Q and R are positive definite and $\eta > 0$ is a constant. The optimization of the performance index with the system $\dot{x} = Ax + B\tilde{u}$ yields the state feedback control

$$\tilde{u} = -k^T x, k^T = R^{-1} B^T P,$$

Where $P > 0$ is the solution of the following algebraic Riccati equation

$$(A + \eta I)^T p + p(A + \eta I) - P B R^{-1} B^T P + Q = 0$$

The resulting closed loop system is given by

$$\dot{x} = (A - B R^{-1} B^T P)x + h(x)$$

if $h(x) = 0$ the system is stable, moreover some bounds on $h(x)$ preserves the stability of the system.

Theorem 2 Let

$$D = 2Q + (A + \eta I)^T P + P(A + \eta I)$$

and let $\|\cdot\|_E, \lambda_{max}(\cdot), \lambda_{min}(\cdot)$ denote the euclidean norm, maximum eigenvalue, and minimum eigenvalue respectively. If the uncertainty term $h(x)$ satisfies the condition

$$\frac{\|h(x)\|_E}{\|x\|_E} < \frac{\lambda_{min}(D)}{2\lambda_{max}(P)} + \frac{\eta\lambda_{min}(P)}{\lambda_{max}(P)}. \quad (3.2)$$

Then the system $\dot{x} = Ax + B\tilde{u} + h(x)$ is asymptotically stable.

Proof Choose the Lyapunov function as $V = x^T P x$. Derivative the function with respect to time along the trajectory,

$$\begin{aligned}\dot{V} &= \dot{x}^T P x + x^T P \dot{x} \\ &= x^T [(A - BR^{-1}B^T P)^T P + P(A - BR^{-1}B^T P)] x + 2h^T P x \\ &= x^T [A^T P + PA - 2PBR^{-1}B^T P] x + 2h^T P x\end{aligned}$$

Using algebraic Riccati equation and

$$D = PBR^{-1}B^T P + Q$$

yields

$$\begin{aligned}\dot{V} &= x^T (-2\eta P - D)x + 2h^T P x \\ &= -x^T D x - 2\eta x^T P x + 2h^T P x\end{aligned}\tag{3.3}$$

Meanwhile, the uncertainty term satisfies the following inequalities.

$$h^T P x \leq \|h\|_E \|P x\|_E \leq \lambda_{\max}(P) \|h\|_E \|x\|_E.$$

By the condition 3.2

$$\begin{aligned}h^T P x &\leq \left[\frac{\lambda_{\min}(D)}{2\lambda_{\max}(P)} + \frac{\eta\lambda_{\min}(P)}{\lambda_{\min}(P)} \right] \lambda_{\max}(P) \|x\|_E^2 \\ &= \left[\frac{1}{2}\lambda_{\min}(D) + \eta\lambda_{\min}(P) \right] \|x\|_E^2.\end{aligned}\tag{3.4}$$

Applying the above relation 3.4 to 3.3 yields

$$\dot{V} \leq -x^T D x - 2\eta x^T P x + 2 \left[\frac{1}{2}\lambda_{\min}(D) + \eta\lambda_{\min}(P) \right] \|x\|_E^2$$

$$= -x^T[(D - \lambda_{\min}(D)I) + 2\eta(P - \lambda_{\min}(P)I)]x. \quad (3.5)$$

It is easy to see that, because $D - \lambda_{\min}(D)I$ and $P - \lambda_{\min}(P)I$ are positive semi-definite matrices, $\dot{V} \leq 0$ which proves that x is bounded, ρ and v are bounded as well, and finally \dot{x} is bounded. We can conclude that $\dot{x}^T \tilde{D}x + x^T \tilde{D}\dot{x}$ is bounded also, where $\tilde{D} = (D - \lambda_{\min}(D)I) + 2\eta(P - \lambda_{\min}(P)I)$. On the other hand, 3.5 implies that

$$\lim_{x \rightarrow \infty} \int_0^t x^T \tilde{D}x d\tau \leq - \lim_{x \rightarrow \infty} \int_0^t \dot{V}(\tau) d\tau \leq V(0) - \lim_{x \rightarrow \infty} V(t) < \infty.$$

That means $x^T \tilde{D}x$ is integrable. According to Brablat's lemma, because $\dot{x}^T \tilde{D}x + x^T \tilde{D}\dot{x}$ is bounded, $\lim_{x \rightarrow \infty} x^T \tilde{D}x = 0$ which implies x converges to zero as $t \rightarrow \infty$.

3.4 Conclusion

Designing a controller is an important thing to reach our goals to control the angle of the pendulum and the position of the cart, we used the partial feedback linearisation in chapter 2 to help us to analyze the stability, and we proposed a controller composed of the nonlinear controller for the dominant subsystem and the linear quadratic controller. We discussed the PID control, its effect on the system, and how we can tune it.

Chapter 4

Experimentation

4.1 Introduction

The inverted pendulum on a cart is a clear example of a mechanical under-actuated system that has had an interest among researchers of control theories. Although it is a purely academic system, the system facilitates the study, application, and analysis of different control strategies. Our main objective of this thesis is two create a system that can be used in the laboratory to test the control method. By using the mathematical model that we get of the system in chapter 2 (DC motor model and the cart-pendulum) taking into consideration the study of the constraints, and using the PID controller that we discussed in chapter 3 to control the DC motor and the system, we will get the realisation

4.2 Designing the system

There are many designing tools and programs like AutoCAD, sketch, and TinkerCAD, in our work we will use SolidWorks. It is a solid modeling computer-aided design (CAD) and computer-aided engineering (CAE) application published by Dassault Systèmes. according to In 2017, there were over 2.3 million active users at over 234,800 companies in 80 countries that use Solidworks [18]

4.2.1 Designing the supports

first, we designed the supports for the DC motor shown in fig 4.1, the encoder, and the switch limits. see Fig 4.2

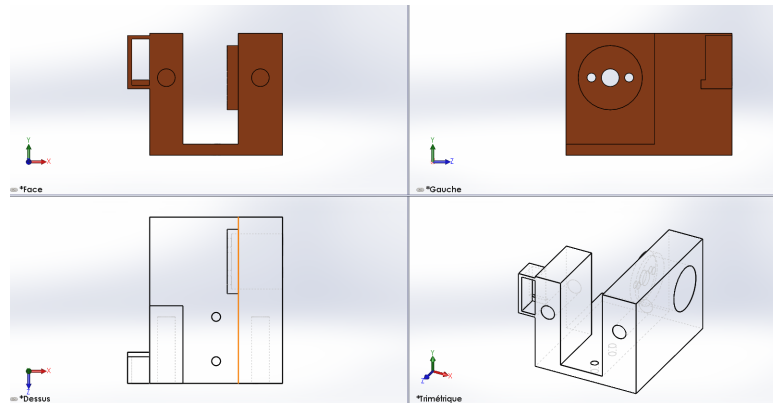


Figure 4.1: Design of the motor support

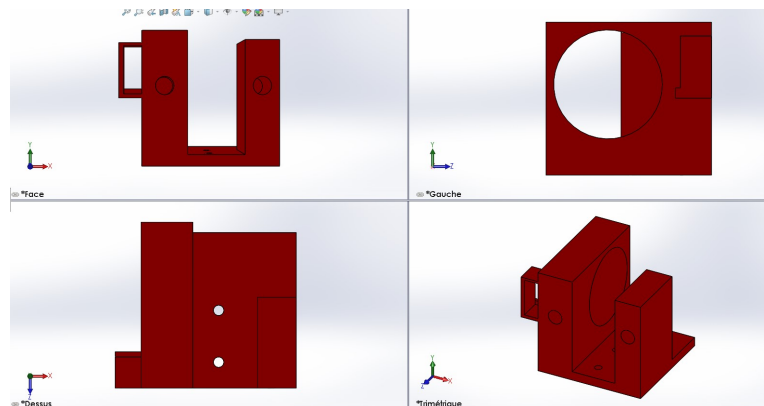


Figure 4.2: design of the support for the encoder

4.2.2 Designing the pendulum

In order to print the pendulum by 3D printer we had to design it see fig 4.3, we designed it as simple as possible taking into consideration the screw hole to fix the encoder

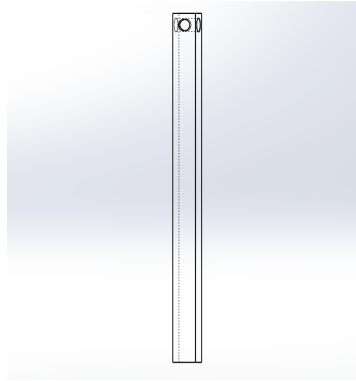


Figure 4.3: design of the pendulum

4.2.3 Designing the cart

The first prototype of the cart shown in the fig 4.4 was created at the beginning of the study, it is a cart that has 4 linear bearings and at the top has an extension so we can fix in it the encoder.

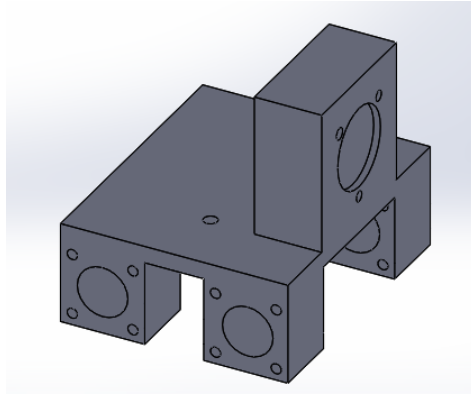


Figure 4.4: First prototype of the cart

After we removed two linear bearings to decrease the friction and overcome the axes constraint. we did some improvements in the design see fig 4.5 In the old prototype, we had 4

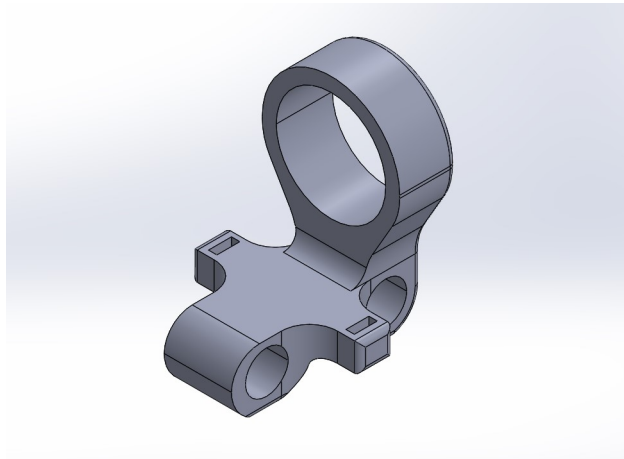


Figure 4.5: The second prototype of the cart

linear bearings, and we were fixing the belt with screws, but in the new one, we made extensions to fix the belt

4.3 Electronics

4.3.1 Arduino mega 2560

In our work, we used the open source electronic platform Arduino shown in the fig 4.6

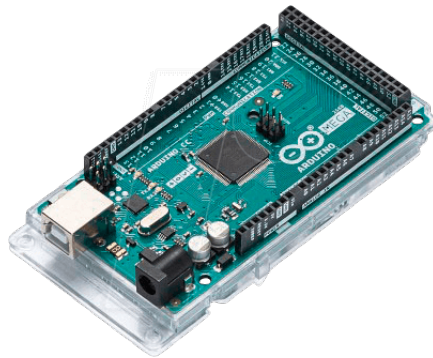


Figure 4.6: Arduino mega 2560

4.3.2 DC motor

The Direct Current motor is an electrical device designed to convert electrical energy into mechanical energy [19], it is able to start, stop and reverse immediately because it is better and necessary in industrial machinery than other types of motors.

The DC motor components

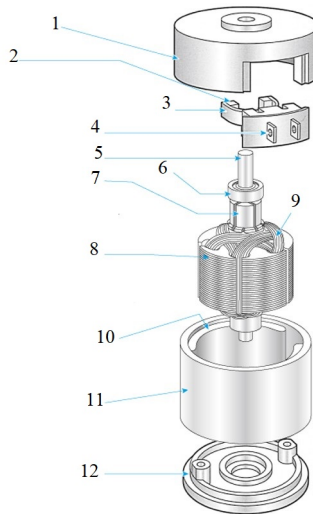


Figure 4.7: component of DC motor

The DC motor contains different parts that cooperate with each other to make the motor work, the components shown in Fig 4.7 are

1 is the bracket that covers the motor from the back.

2 is the carbon brush it plays a critical role, it makes a sliding contact to transmit the electric current to the commutator.

3 is a spring its role is to keep the brush in contact.

4 is terminal, it is the positive and negative input.

5 is the shaft, it is a cylindrical component that extrudes out from the motor, rotates and allows as to connect the motor with a pulley or another component.

6 is the bearing, it is made to support and locate the rotor, one of its roles is to transfer the loads from the shaft to the motor and minimize frictional losses.

7 is the commutator, it makes contact with the brush and applies electric current to the windings.

8 and 9 together are the armature winding, it is the main current-carrying in which the electromotive force is induced.

10 is the field magnet (like in our work) and in other motors is the field winding, it is the one that produces a magnetic field in the machine.

11 is the yoke, it is the external structure that covers the dc motor.

12 is the bracket in the front.

The DC motor working principles

The electrical current gets to the motor from its terminal going to the brushes which pass the current into the commutator and from the commutator it gets into the armature field so this last one generates an electromagnetic field, also we have the magnetic field from the field magnet, finally, the interact of this two magnetic field produces a torque on the armature, which causes it to revolve.

Advantages and Disadvantages

Advantages Firstly the advantages of the brushed DC motor

easy to control its speed, and it can be controlled on a wide range.

when the motor is running at a constant speed, it doesn't require a drive circuit.

start and stop almost immediately.

high speed and acceleration compared to the stepper motor.

a high starting torque.

Disadvantages there are some disadvantages of the DC motor like

existence of electrical and acoustic noise

it has a short life.

Types of motors

There are many types of motors that are used in the industry among them

Brushed DC motor

Brushless DC motor Brushless DC motors have a permanent magnet rotor and a coil wound stator. The benefit of a brushless DC motor is the elimination of wear of brushes because very little heat is produced by the rotating magnet. It produces a higher torque per watt but they are expensive.

AC motors I motion of them two

Induction motors

Synchronous motors

Stepper motor

Hysteresis motor

Driver of the DC motor

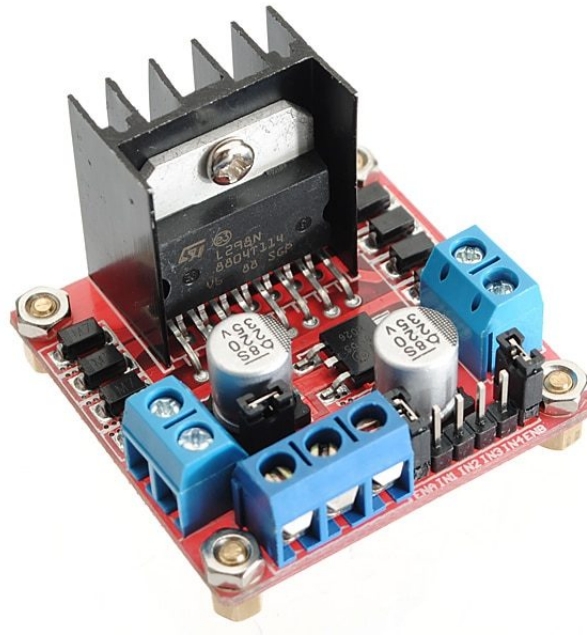


Figure 4.8: motor driver L298N

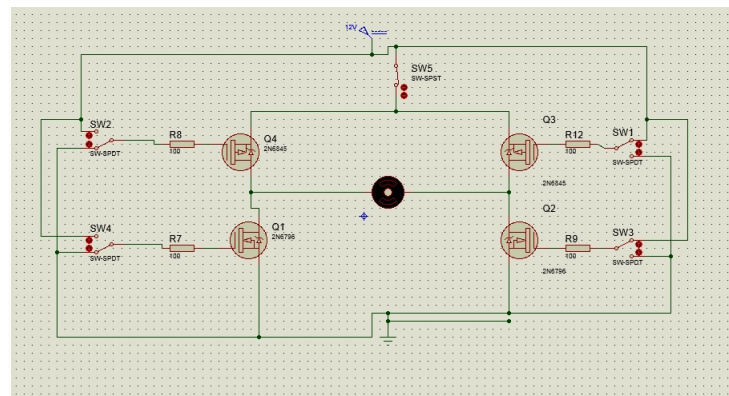


Figure 4.9: H bridge on proteus

The dc motor driver is a type of amplifier or power modulator that integrate between the controller and a DC motor. In our work we simulated the H-bridge (is an electronic circuit that switches the polarity of a voltage applied to a load) using proteus see fig 4.9, so we can drive the DC motor by H-bridge but in the realization, we used the driver l298n see fig 4.8 it composed by two H-bridge. When the input is positive-negative the current will pass through Q_4 and Q_2 , which makes the motor turn right. and if the input is negative-positive the current

will pass through Q_3 and Q_1 and that makes the motor turn left.

4.3.3 Rotary encoder



Figure 4.10: Rotary encoder

The rotary encoder shown in fig 4.10 is an electro-mechanical device that converts the angular position into digital output signals, it can measure distance, speed, and position. We use it to sense for us the pendulum angle, we can use it also to give us the linear position of the cart

The working principle of the rotary encoder

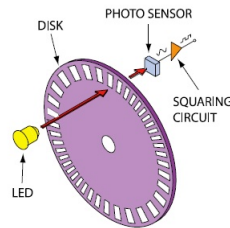


Figure 4.11: Rotary encoder structure

As we see in the fig 4.11 when the shaft rotates the disk the light from the LED can pass through the shutters, so the shutters in the rotation movement block and unblock the light, and thank this the photo-sensor can generate a square signal and then interpreted in a number of signals per revolution, the most of encoders like the one that we used to use two output channels A and B to sense the position and the direction of rotation

4.3.4 Limit switch

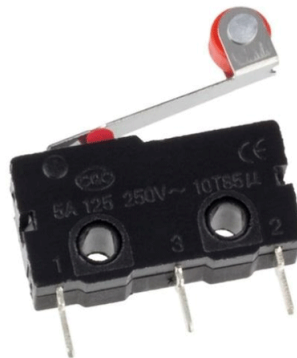


Figure 4.12: Limit switch

we can use the limit switch shown in the fig 4.12 in case of the cart bump into the limit of the rail so it closes the limit switch and that involve a code to work which forces the cart to go back to the center of the rail.

4.3.5 Ultrasonic sensor

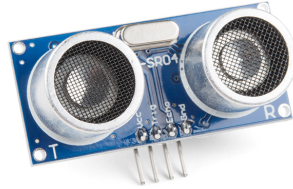


Figure 4.13: Ultrasonic sensor

The ultrasonic sensor is shown in fig 4.13 it is an instrument that measures the distance to an object using ultrasonic sound waves. We can use it to measure the cart position instead of the encoder

4.3.6 Connections

The connections of the electronic parts are explained in fig 4.14, the encoder has 4 connections pin: the red should be connected to the 5v source, and the black should be connected to the ground and the green and the white should be connected to pins 1-2. The L298N driver has : 3 pin and 3 energy sources, one in the 5v and the other in 12 v and the last in the ground, and has to output should be connected to the dc motor. The limit switch has two connections: the first should be connected to the Arduino pin and the other should be connected to a resistance (for protection) then to the ground.

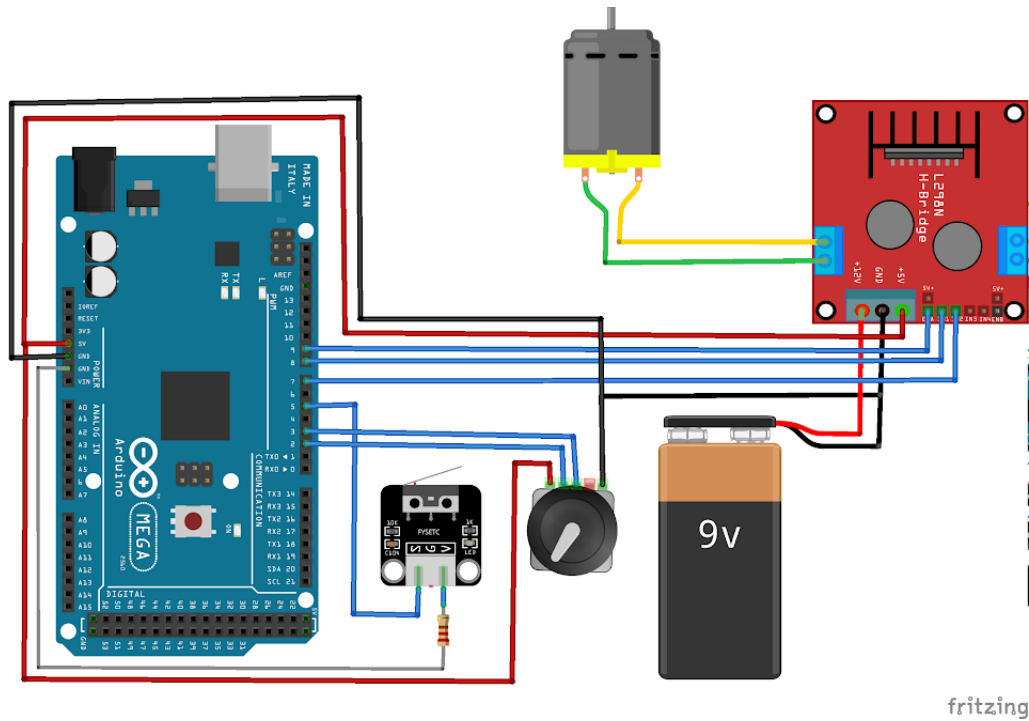


Figure 4.14: Connections of the electronic parts

4.4 simulation of DC motor speed control using PID

we need to use the PID regulator that we talked about in chapter 3, to control the DC motor and the pendulum. For deep knowledge of the PID parameters (K_p , K_i and K_d) we need to simulate the system's transfer function with PID and tune the PID using Simulink see fig 4.15

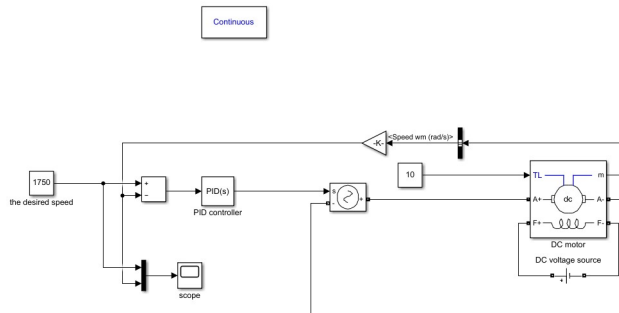


Figure 4.15: The Simulink model

with random PID parameters, the response is (see fig 4.16)

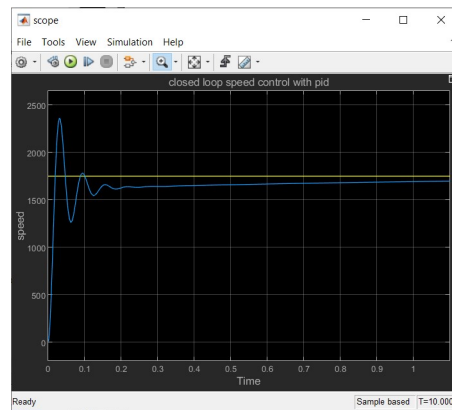


Figure 4.16: Response with random PID parameters

As you see we observe that the response of the DC motor oscillates with a big overshoot reach the 2400 turns per minute and that may disturb the stability of the system and make us lose the energy.

After using the tuning in the Simulink and getting the right parameters the response becomes (fig 4.17)

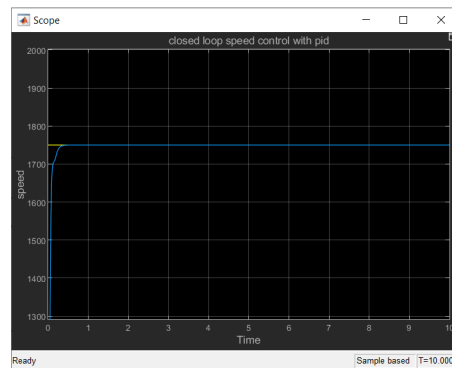


Figure 4.17: Response with tuned PID parameters

4.5 Assemblage and Final prototype

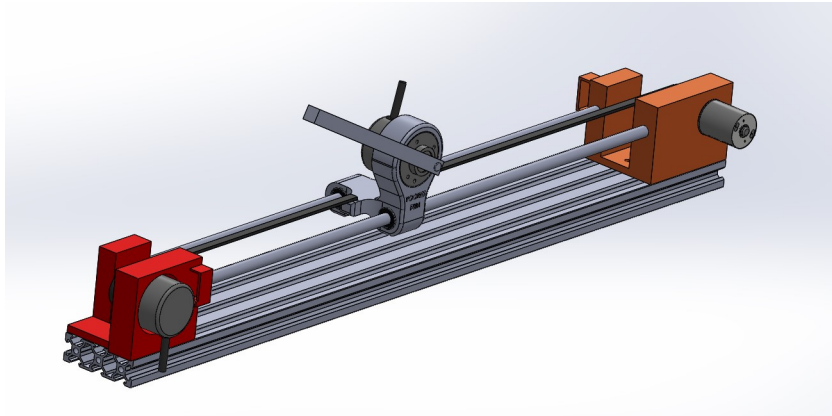


Figure 4.18: Assemblage of all pieces

after the realization and the amelioration of all the pieces, we assemble them as you see in the fig 4.18 This is the final realization of our system (see fig 4.19)

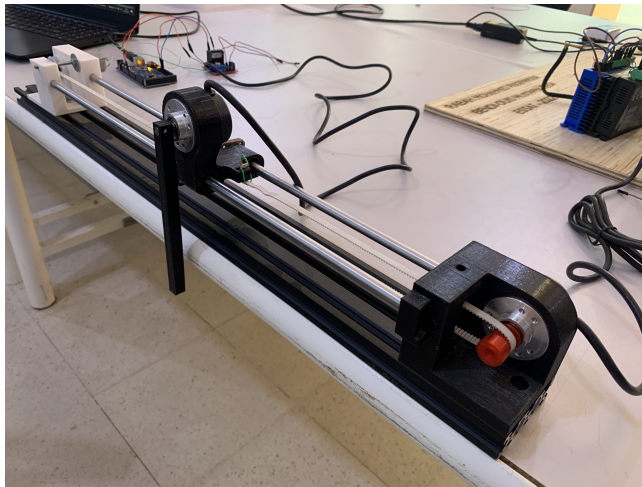


Figure 4.19: The real photo of the realization

4.6 Arduino algorithm

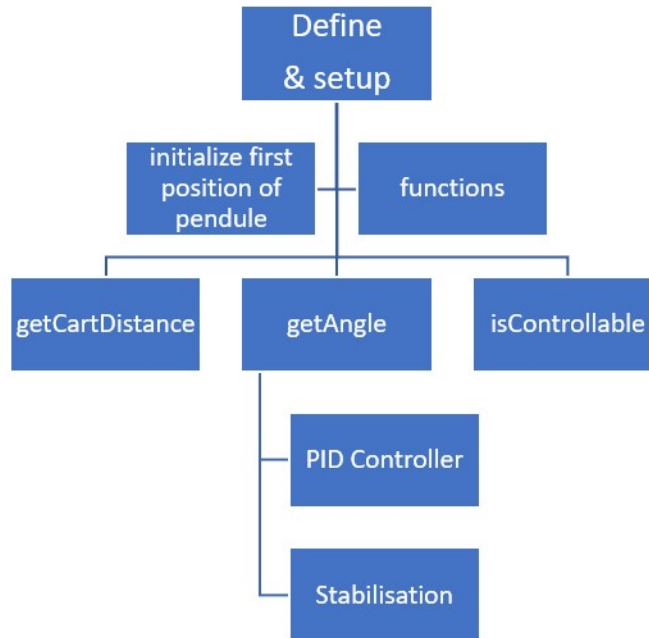


Figure 4.20: The organizational chart of the Arduino code

The code of the arduino can be explained as shown in the organizational chart (see fig 4.20), First you have to define your variables and their pins. Secondly, you have to create and call functions such as move motor function, and call the PID and encoder functions from the library. Third we create `getCartDistance` function to know the place of the cart, `getAngle` function to know the angle of the pendulum, `isControllable` function to know if the pendulum is in the stabilization range $[5\pi/6; 7\pi/6]$, we create the PID function for the angle of the pendulum because our main code is to stabilise the pendulum in the up right position. Finally we create the stabilisation code by calling all the previous functions and if the angle is in the range $[5\pi/6; 0]$ the cart will move to the right, and if the angle is in the range $[0/6; 7\pi/6]$ the cart will move to the right.

4.7 Conclusion

We try our best to explain the process of the realization in this chapter we talked about the electronics that we used, we talked about the prototype design in Solidworks, and the improvement in the design, and we simulate the PID for tuning its parameters.

General conclusion

Firstly in this work we present and define the inverted pendulum on a cart system, we talked superficially about how it works, then to give the reader how important the inverted pendulum system we motioned some industrial system that is similar to the inverted pendulum or dependent on its principle. We went through the types of pendulums, later we explained the dynamics of the cart, pendulum, and the DC motor, then we mentioned the obstacle that probably we bump into them in the real model. By and by the modeling that we got we managed to linearise it by partial feedback and that gave us the capability to design a controller, we analysed the stability of the system through the Lyapunov function, we can't talk about the controllers without mention the PID controller, so we tried to explain the effect of its parameters, Eventually, we used the PID controller to control the speed of the DC motor and then we realized the system that we have been talking about.

We accomplished our main goal in this work by the realisation, this thesis will help researchers so they can reprocess it, and may develop it into many inverted pendulums on a cart by providing them with the designs and the electronics that are needed. And also it will help the student and the next generations to see in front of them the effect of different control laws so they can test any control method and compare it, this realisation will help the university and research labs to provide all the under-actuated system in just one small system.

I realised an inverted pendulum on a cart in this final project. in the future, I will try to stabilise it better than now, and I will put it in the school laboratory to provide a better system for

testing control laws. I will compare it to the old rotary encoder that had been realised by a previous student at my school. Eventually, I will realize a double inverted pendulum on a cart.

Perspective In this work, we created one inverted pendulum on a cart, in future work we will try to apply many control laws, and we will control it using AI. then we will try to develop it into two inverted pendulums on a cart.

Appendix A

Arduino program

```

#include <PID_v2.h>
#include <Encoder.h>
#include <L298N.h>

// Libraries
// #include <PIDController.h>
// Objects
//PIDController pid;

#define MOTOR_ENCODER_PPR 1600 // 400 pulse per revolution
#define SHAFT_R 0.00573 // get cart distance
#define PENDULUM_ENCODER_PPR 1600
#define THETA_THRESHOLD PI/10
#define POSITION_LIMIT 8500

bool initial = true; // Bool Variable pour mise en position du pendule
volatile long encoderValue = 0; // Initial value for motor encoder
volatile long lastEncoded = 0; // previous value for motor encoder

```

```
volatile long refEncoderValue = 0;           // Initila value for pendul encoder
volatile long lastRefEncoded = 0;           // previous value for pendul encoder

float theta = 0.;                           // angle du pendule
float last_theta = 0.;                       // previous angle

const unsigned int Rot_ENCoder1 = 3;        // White pin for rotatif encoder
const unsigned int Rot_ENCoder2 = 2;        // green pin for rotatif encoder

const unsigned int Lin_ENCoder1 = 18;
const unsigned int Lin_ENCoder2 = 19;

// Create Encoder instance
Encoder myEnc(Rot_ENCoder1, Rot_ENCoder2);   // MyEnc : encoder rotatif object
Encoder CartmyEnc(Lin_ENCoder1, Lin_ENCoder2); // cartmyEnc : encoder for cart object

// Motor Pin definition
const unsigned int IN1 = 7;
const unsigned int IN2 = 8;
const unsigned int EN = 9;

//PID Declaration
double Setpoint,Input,Output;
//double Kp = 2, Ki = 5, Kd = 1; // P,I,D Values
double Kp = 1500, Ki = 200, Kd = 100;       // PID for cart
```

```
double thKp = 2, thKi = 5, thKd = 1;          // PID FOR Theta

// Create PID instance
//PID_v2 cartPID(Kp, Ki, Kd, PID::Direct);
PID_v2 pendulumPID(thKp, thKi, thKd, PID::Direct);

// Create one motor instance
L298N motor(EN, IN1, IN2);

// Function to move motor
void move_motor(int value){
    motor.setSpeed(fabs(value));

    if(value<0){
        motor.forward();
    }
    else{
        motor.backward();
    }

// if(value == 0){
//     // Stop
//     motor.stop();
// }else {
//     if(d){
```



```

//          Serial.println("Forward");
//          // motor.forward();
//      }
//      else {
//          Serial.println("BackWard");
//          //motor.backward();
//      }
//  }

}

float getCartDistance(long pulses, long ppr) {
    return 2.0 * PI * pulses / ppr * SHAFT_R;
}

float getAngle(long pulses, long ppr) {
    float teta = 2.0 * PI * pulses / ppr;
    if(teta > PI) teta = teta-(2*PI);
    else if(teta<-PI) teta = teta+(2*PI);
    return teta;
}

boolean isControllable(float theta) {
    return !(fabs(theta) < (PI-THETA_THRESHOLD))&& ;
}

```

```

void setup() {

    pendulumPID.Start(Input, // input
                      0,      // current output
                      PI);    // setpoint

    pendulumPID.SetOutputLimits(0, 255);
    pendulumPID.SetMode(AUTOMATIC);

    // pid.begin();           // initialize the PID instance
    // pid.setpoint(PI);     // The "goal" the PID controller tries to "reach"
    // pid.tune(2, 5, 1);    // Tune the PID, arguments: kP, kI, kD
    // pid.limit(0, 255);    // Limit the PID output between 0 and 255, this is important t

    Serial.begin(9600);
    Serial.println("Basic Encoder Test:");

}

void loop() {

    // initialize first position of pendule
    encoderValue = CartmyEnc.read();
    if (initial){
        while(encoderValue<POSITION_LIMIT){
            move_motor(-100);
            encoderValue = CartmyEnc.read();
            Serial.println(encoderValue);
        }
    }
}

```

```

    motor.stop();
    initial = false;
}

refEncoderValue=myEnc.read();

Serial.println(encoderValue);
    if (refEncoderValue != lastRefEncoded) { // program for stabilization

        lastRefEncoded = refEncoderValue;
        theta = getAngle(refEncoderValue,PENDULUM_ENCODER_PPR);
        // Serial.println(theta);
        if(isControllable(theta)){
            Input = fabs(theta);
            Output = pendulumPID.Run(theta);
            if(theta<0)Output = 0 - Output;
            Serial.print("output ");
            Serial.println(Output);
            move_motor(Output);
        }
        else { /// program for swing up

    }
}

delay(5);
}

```

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